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## AGEING BEHAVIOUR OF [(n-1)/n] ACTIVE REDUNDANCE SYSTEMS

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Ageing of systems becomes a real concern if intelligent maintenance is required. Determining the ageing behaviour of a system necessitates having a powerful calculating tool and knowing the ageing behavior of the basic components in the system. Consequently, time dependent failure rates are required for basic components and need to be determined for systems. As, this is a general problem in reliability analysis, only (n-1)/n active redundancy systems will be examined in the paper. Systems with (n-1)/n active redundancy are commonly used in a wide range of engineering fields. This should permit a priori improving the system reliability and availability. Still, a deeper analysis of the ageing behaviour of such systems may reveal some particular aspects.

### INTRODUCTION

System ageing becomes a real concern if intelligent maintenance is required. Both preventive and operational maintenance need a good knowledge of the ageing behavior of both systems and their components.

Failure rate time behavior is a direct indicator of the ageing of systems and of components. So, time-dependent failure rates are required. Producing time dependent failure rates by observation or by calculations is not an easy task in reliability analysis.

Only the systems with [(n-1)/n] active redundancy are considered in the paper. Systems with [(n-1)/n] active redundancy are commonly met in many engineering fields. Applications varies between fields like detection, monitoring, signal transmission, power transmission and cooling circuits.

Very often (n-1) identical units are necessary such that a given system could accomplish a well defined function (mission). Adding one more unit, actively redundant, improves generally the overall system reliability and availability. Especially, if reparations or replacements of faulty components does not necessitate the system shutdown and if the repair rate is largely higher than the failure rate.

On the contrary the situation might be quite different if components are not repairable. It will even be worst if components have high failure rates because of environment aggression (neutron induced damage, high thermal or mechanical stress, ...). In some of these cases, redundancy will be useless, specially for long mission.

Moreover, in some special fields of detection and monitoring, it is necessary to design systems with many identical units such that the system overall detection rates should be time independent, given that the components themselves suffer from ageing (showing time-increasing failure rates). This is to guarantee the detection of the real time behavior of the phenomenon under observations.

Such types of problems and many others necessitate analysing the time-dependence of the system global failure rate as a function of the components' ones.

### [n-1/n] Active Redundance

In many systems which are composed of (n-1) identical components necessary to the success of the mission (function), one more component may be added to improve the overall system availability/reliability. This is generally the case if components are highly repairable and if component reparation does not necessitate the system shutdown. It could also be the case of some systems whose components are not repairable but whose mission time is far smaller than the mean time between component failures.

By active redundancy is meant that all components available to the system are active (in operation). So, losing a component will not necessitate a start up action of another during operation. This eliminates the possibility of a failure to start-up event. Although, this will imply an increase of the operating loads per component (for still operating ones), it will be supposed that change of the operating load will certainly succeed (failure-to-load-increase probability will be equal to zero).

So, it is necessary to determine both system overall availability and system overall failure rate as functions of the component availability (and/or unavailability) and of the component failure rate. In order to do this, the Differential Model for Equivalent Parameters (DMEP) will be used<sup>1</sup>. The main interest of the DMEP is its capability to calculate an overall system failure rate which is dependent on the time.

Let S be the success of a given system that contains n identical active components. If the success of at least (n-1) out of n units is sufficient to assure the success of the system then the system availability ( $A_e(t)$ ) may be determined by :

$$A_e(t) = [1+(n-1) u(t)] \cdot a^{n-1}(t) \quad (1)$$

where, a(t) and u(t) are the component availability and unavailability at the instant 't', respectively.

In the same time, it can also be shown<sup>1</sup> that the system overall failure rate ( $\lambda_e(t)$ ) may be determined by :

$$\lambda_e(t) = n(n-1) \lambda(t) \{u(t) / [1+(n-1) u(t)]\} \quad (2)$$

where  $\lambda(t)$  is the component failure rate at the instant 't'.

Thanks to equation (2), the determination of a time-dependent overall system failure rate is possible.

### Active Redundance Benefits

In order to evaluate the real interest of such a type of redundance both the gain in the system availability and the reduction in the system overall failure rate will be examined. Comparison will be done with the system configuration where no-redundance was considered (a system with only (n-1) necessary units).

If no redundance was considered, the system would have contained only (n-1) components (necessary for its success to accomplish a given mission). In this case, the system availability [ $A_0(t)$ ] would have been equal to :

$$A_0(t) = a^{n-1}(t) \quad (3)$$

and its overall failure rate [ $\lambda_0(t)$ ] would have been equal to :

$$\lambda_0(t) = (n-1) \lambda(t) \quad (4)$$

Equations (3) and (4) provide us with reference values for the availability and for the system overall failure rate. They will thus be used to quantify the benefits achieved by the redundance. In the next section, the gain in the system availability (G) and the Reduction Factor in Failure Rate (RFFR) will be calculated.

These indicators (G and RFFR) will be thus used to assess the pertinence of a given redundance option taking into account the system ageing.

### Availability Gain

One way to evaluate the benefits of using such an (n-1)/n redundance may be to calculate the gain in the availability (G) of the system such as:

$$\begin{aligned} G &= A_e(t) - A_0(t) \\ &= (n-1) \cdot u(t) \cdot a^{n-1}(t) \end{aligned} \quad (5)$$

The gain factor G measures the difference between the system availability with and without the (n-1)/n redundance, where  $A_e(t)$  and  $A_0(t)$  are given by equations (1) and (3), respectively.

The gain in system availability due the redundance is shown in figure 1 for different redundance order n (n=3, 7 and 11) and at different values of component unavailability  $u(t) \in [0,1]$ .

First of all, it is worth observing that the gain factor is always positive ( $> 0$ ), equation (5). Thus, It is not possible to lose in terms of availability in making use of this type of redundance.

The second remark is that the gain varies with component unavailability and attains some maximum values. These maximum values are higher for redundances with high values of n

(total number of systems' components). In the case of 2/3 redundance, figure (1), gains in availability can go up to attain its maximum value of 0.30 for component unavailability around 0.30.

The 3rd remark is that at the extreme values of the component unavailability ( $\approx 0$  or  $\approx 1$ ) such redundance is not really interesting in terms of global system availability. This would be the case for systems whose components have a very small failure rate and a very short repair time (too good to justify a redundance). Or on the contrary, the system components are not repairable and have a very high failure rate (too bad to justify a redundance). In both cases the redundance may not be justified.

In all cases, one should proceed to exact calculations of availability gain taking into account the total number of components and the time profile of their unavailability to be able to judge the real interest of making use of such redundance.

### Reliability Gain

The same could also be done to determine the reduction in terms of the system unreliability. The comparison will be made on the basis of the ratio of the system global failure rate with and without redundance. This will be called the Reduction Factor in Failure Rate (RFFR). A reduction factor (H) will be determined as following:

$$\begin{aligned}
 H &= \lambda_e(t)/\lambda_o(t) \\
 &= n \cdot u(t) / [1 + (n-1)u(t)] \quad (6)
 \end{aligned}$$

As it could be expected from equation (6), for values of component unavailability less than 0.1, the reduction in failure rate due to redundance is almost linearly proportional to the component unavailability, figure (2).

It can also be observed that reduction in failure rate is higher for systems with smaller number of components.

### Aging & Failure Rate

Failure rate time behaviour is a direct indicator of systems and components ageing. A system, a sub-system or a component is said to suffer from ageing if;

$$\frac{d}{dt} \lambda > 0.$$

where,  $\lambda$  is its failure rate.

Moreover, a system, a sub-system or a component is said to be regenerated if;

$$\frac{d}{dt} \lambda < 0.$$

On the other hand if the failure rate ( $\lambda$ ) is time independent (constant);

$$\frac{d}{dt} \lambda = 0.$$

The system, the sub-system or the component may be called simple.

If the failure rate is defined as 'the conditional probability per unit time to have the first failure at  $t$ ' and if failure is defined as the non-capability to fulfill the mission that has been assigned to the system, to the sub-system or to the component. Then, ageing is a measure of the time-degradation in

the capacity of accomplishing a predefined mission (function). The failure rate of a system could then be defined as the conditional probability per unit time not to fulfill an assigned mission (function) at instant ' $t$ '. Besides, if many missions are assigned to the same system, the system may have many partial failure rates, one per mission (function).

This emphasis on the functional nature of the failure rate and consequently on the functional nature of the system ageing is necessary to understand some of the results that will be presented in the paper. This would give a comprehensible meaning to terms like success and failure.

Failure data used in reliability and risk analysis are generally the result of observations concerning the failure to accomplish some assigned functions and missions. Treating these data to extract time dependent failure rates, even for basic components, is not an easy task. This necessitates having statistically-representative samples of observed individuals and well-described observations.

Very often, adequate time-dependent failure data are not available. In this paper, only basic components with time constant failure rate have been considered. The examples presented here are issued from real applications.

Besides, the term *basic component* will be used only if failure and repair data are issued from extensive observations (experience feed-back). The component will be called *simple* if its characteristic data as failure ( $\lambda$ ) and repair rates ( $\mu$ ) are constant (time independent).

### System Failure Rate

The term *system* will be used if failure and repair data can be evaluated starting from basic components.

Reliable data issued from practical experience are generally unavailable for new technology complex systems. This is mainly because of the absence of statistically-representative observations. So, evaluating such data starting from the basic components of systems is inevitable. Consequently, adequate means of evaluations are necessary.

One of the most required pieces of data to be evaluated is the time-dependent equivalent failure rate of systems. This is mostly to analyse ageing, to optimize maintenance, to manage components replacement, to help in design options selection,... etc.

The main object of the paper is to assert the fact that system ageing is generally due to not only physical, chemical or mechanical degradation factors but it would also be due to the organization (internal structure) of the system.

A system is then characterized by both its physical and its organizational aspects.

In order to demonstrate this fact, the system equivalent failure rate needs to be evaluated as a time function that takes into account both the physical aspect (components) and the organizational aspect (internal structure ) of the system.

Many techniques and methods could be used such as Markov approach and Monte-Carlo simulation to generate system required time-dependent data. Markov approach is specially useful for systems with few possible states and with time constant transition rates between states. Monte-Carlo techniques may be more general. Although it is more difficult to apply, Monte Carlo simulation may be revealed as a powerful tool to evaluate required system time dependent data starting from its basic components.

The Differential Model of Equivalent Parameters<sup>1</sup> (DMEP) is one of the formal tools that is used to evaluate system kinetics. It has been developed at the CEA-DMT/SERMA, France. It permits calculation of the characteristic parameters of the system as time-functions. Characteristic parameters of a system are the failure rate, the repair rate and the failure-to-start probability. Besides it evaluates systems reliability and availability.

#### Aging of 2/3 Active Redundance Systems

One of the widely used (n-1)/n active redundance systems is the 2/3 one, specially in monitoring & detection, signal transmission or power transmission systems. In such a case, the system overall availability (A(t)) is given by:

$$A(t) = [1+2u(t)] \cdot a^2(t) \quad (7)$$

where u(t) and a(t) are the component unavailability and availability, respectively.

The system overall failure rate is determined by equation (8) as follows:

$$\lambda_e(t) = 6 \lambda(t) \{ u(t) / [1+2u(t)] \} \quad (8)$$

where  $\lambda(t)$  is the component failure rate.

It will be interesting to examine three different practical cases with 2/3 active redundance:

- 1° the basic component has a constant failure rate and non-reparable ( $\mu=0$ ),
- 2° the basic component has a constant failure rate and reparable ( $\mu=10^{-2}$ ), and
- 3° the basic component has a constant failure rate, is reparable ( $\mu=10^{-2}$ ) and has a non zero failure-to-start probability ( $\gamma=10^{-1}$ ).

In all three cases the basic component is supposed to be simple (constant failure and repair rates). This implies that the component unavailability will be determined by the following equation:



$$u(t) = (\lambda/\sigma)[1-e^{-\sigma t}] + \gamma e^{-\sigma t} \quad (9)$$

where  $\sigma = \lambda + \mu$  and  $\gamma$  is the failure to start probability.

### Case No1

In this case, the basic component is non-reparable ( $\mu=0$ ) and with zero failure to start probability ( $\gamma=0$ ). Substituting these values in equation (9), the basic component unavailability is then:

$$u(t) = [1-e^{-\lambda t}] \quad (10)$$

This gives a basic component unavailability varying from 0 (at  $t=0$ ) to 1 ( $t \gg 1/\lambda$ ).

Consequently, the system over all failure rate, equation (8), will vary from 0 (at  $t=0$ ) to  $2\lambda$  (at  $t \gg 1/\lambda$ ), figure (3). Although, the basic component itself does not show any ageing the system shows an ageing effect. This is what may be called a fatigue ageing. It reflects the effect of the non-reparability of the basic component that leads consequently to a time increase in its unavailability.

It may also be interesting to notice that the overall system failure rate increases steadily with the time and that it becomes even higher than the failure rate of one basic component ( $\lambda=10^{-4}/h$ ) around  $2 \cdot 10^3$  hours, figure (3).

After a long enough time ( $\gg 1/2\lambda$ ) the system global failure rate tends to its asymptotic value (twice the component failure rate)  $2 \cdot 10^{-4} /h$ .

An immediate conclusion is that systems, with non-reparable components, show an ageing effect although basic components do not.

Besides, non-reparable 2/3 active redundancy seems to be useful in short mission ( $T_{mission} \ll 1/\lambda$ ).

### Case No2

In the 2nd case, the basic component is supposed to be reparable ( $\mu=10^{-2}$ ) and to have a constant failure rate. So, equation (9) describing the component unavailability becomes as following:

$$u(t) = (\lambda/\sigma)[1-e^{-\sigma t}] \quad (11)$$

where  $\sigma = \lambda + \mu$ . This gives a basic component unavailability that varies from 0 (at  $t=0$ ) to  $(\lambda/\sigma)$  (for  $t \gg 1/\sigma$ ).

The fact that the component is reparable has led to a situation where component unavailability attains its asymptotic value in a shorter time (than in case 1, as  $1/\sigma \ll 1/\lambda$ ), figure (4).

Consequently, the system global failure rate will attain its asymptotic value in a shorter time ( $\approx 200$  hours), figure (4), besides it will not exceed the failure rate of the basic component.

Thanks to the reparability of the basic components the overall system unreliability has been improved with no risk of becoming worse than that of one component, even for long time mission.

Although the system keeps on showing an ageing effect in spite of that the component itself does not. This again may be classified as fatigue ageing.

In both cases, this system fatigue ageing was not accompanied by a component wear ( $\lambda=0$ ) but rather with a component functional degradation (time-increasing unavailability).

It may seem reasonable to deduce that general ageing is the combination of two fundamentally different effects, a physical one (degradation due to wear) and a functional one (fatigue). The latter reflects a degradation in the functional performances that may be due to the non-reparability (among other possible reasons).

In this case, redundancy seems to be useful both in short and in long missions, as far as RFFR is concerned.

### Case No3

The 3rd case is almost identical to the 2nd one with the difference that basic components had a non-zero failure to start probability ( $\gamma \neq 0$ ). Thus, the component unavailability will be determined by equation (9).

It can be shown that the component unavailability will have a time-decreasing profile. It is to say that the component availability improves with the time. This reflects the fact that this specific basic component has some evident problems to start up (e.g. diesel alternators after relatively long stand-by).

No-surprise then that the system global failure rate will have a time-decreasing profile (regeneration effect). This is exactly what can be concluded from figure 5, as the failure rate decreases with time before coming to its asymptotic value after a long enough interval ( $\gg 1/\sigma$ ).

Although there was no ageing to determine in this example it demonstrates clearly that system ageing does not reflect exclusively component physical deterioration but it could reflect rather the performances deterioration (or amelioration) of the components.

In this case, redundancy seems more useful in long missions. This becomes more and more true if failure-to-start probability becomes more and more higher.

## Fatigue Aging & FRRF

It has been demonstrated that fatigue ageing translates in some way the time-increasing behavior of basic components' unavailability.

It has been also mentioned, that using the 2/3 redundancy permits achievement of some reduction in the global system failure rate with respect to the case of 1/2 (at least two out of two components are needed for the mission). This reduction factor is given in figure (2).

If the system contains non-reparable components, case n°1, the FRRF will then decrease with time. Thus, for long missions with non-reparable system, redundancy loses its interest with time, as far as system reliability is concerned.

On the contrary, for systems with difficult-to-start components, case n°3, redundancy becomes more and more interesting with time, in terms of reliability.

The interest of FRRF is that it provides a direct measure of relative ageing of the system with respect to its basic components. This can easily be estimated during observations by calculating the ratio between the observed system number of failures and the observed components' number of failures within the same time intervals.

In the case of time independent basic components, the system is considered showing an ageing effect if the FRRF estimator increases with time. In fact this easily calculated estimator may be very useful in helping to orientate decision making in preventive maintenance.

## Maintenance & Aging

It can be stated that maintenance improves the ageing behavior of the systems in two different ways.

If the system has no particular problem in starting after a relatively long standby (almost zero failure to start probability), maintenance forces the system to attain its asymptotic failure rate values rapidly and consequently stop ageing. The time interval, needed before the system can come to its failure rate asymptotic value, is proportional to  $1/\sigma$  ( $\sigma = \lambda + \mu$ ). The faster the maintenance ( $\mu \ll \lambda$ ), the shorter will be this interval.

If the system is somehow difficult to start up ( $\gamma \neq 0$ ), an adequate maintenance will, to some extent, improve the ageing of the system. Two situations can then be distinguished. The first situation, if  $\gamma < \lambda/\sigma$ , an ageing effect will be observed at an early time in the mission and attains asymptotic values within an interval proportional to  $1/\sigma$ . The second situation is when  $\gamma > \lambda/\sigma$ , no-ageing will be observed.

A possible indicator of good maintenance may then be the absence of ageing. If not it may also be the observed transition time before attaining the system overall failure rate asymptotic value.

## Conclusion

Failure rate time behaviour is a direct indicator of ageing for systems and for components as well. If failure rate increases with time, the system will suffer from ageing. If failure rate decreases with time the system will be regenerated.

In order to be able to determine system ageing taking into account its components' ageing, time-dependent failure data are required for basic components. Working out time-dependent failure data in observing (statistically) basic components is not an easy task.

Although there is no general scheme of analysis to be proposed for the moment, the examination of some specific cases may help in deriving some pertinent conclusions of general interest. One of the specific cases is the  $(n-1)/n$  active redundancy.

Although the detailed analysis has been performed for the 2/3 systems, conclusions are yet valid for the class of systems with  $(n-1)/n$  active redundancy. In the paper, only simple basic components were considered. Simple components are those with constant failure and repair rates.

System ageing reflects components' ageing and performance degradation. In the paper, basic components are supposed simple. Consequently, observed system ageing was only due to the performance degradation of the basic components. This may be called fatigue ageing.

As component ageing itself has been intentionally suppressed in the paper and as system ageing was exclusively due to components performance, so there was no surprise to discover cases with system regeneration when basic components ameliorate their performances, case n°3.

Consequently, it seems evident to come to the conclusion that generally, system ageing and components' ageing may be of different tendencies.

As system ageing influences the gain in availability or the FRRF of systems with active redundancy, so it may happen that some redundancies become less interesting either for long time missions or for short time missions. In order to assess the pertinence of a given redundancy, system ageing analysis should then be performed.

Ageing analysis is thus a critical task in systems reliability analysis. It seems necessary to optimize maintenance, to decide replacement policy and to judge the pertinence of some design options.

## SYMBOLS USED

$u(t)$	: component unavailability	$a(t)$	: component availability
$\lambda(t)$	: component failure rate	$\mu(t)$	: component repair rate
$\sigma$	: $\lambda + \mu$	$A_e(t)$	: system availability
$G$	: availability Gai	$H$	: reduction factor in failure rate

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