Application of Finite Element Numerical Technique to Nuclear Reactor Geometries

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خلاصة

معرفة توزيع درجات الحرارة في عناصر وقود المفاعلات النووية من الأمور ذات الأهمية البالغة لضمان عدم تجاوز درجة الحرارة حدود الأمان المسموح بها أثناء التشغيل. هذا البحث يناقش استخدام طريقة العناصر المتناهية العددية (FE) لحل معادلة انتقال الحرارة في بعدين في أشكال هندسية ذات علاقة بالمفاعلات النووية. الحل يبدأ بتحويل معادلة انتقال الحرارة الى معادلة تكاملية I ويبحث عن دالة تجعل التكامل I نهاية صغرى ومن ثم نحصل على حل معادلة انتقال الحرارة.

في هذا البحث نعطي نبذة عن تطبيق نظرية العناصر المتناهية في انتقال الحرارة ونقدم برنامج حاسوب استخدم لتطبيق النظرية على أشكال هندسية بسيطة وعلى عناصر الوقود لمفاعلين مبردين بالغاز. تم الحصول على نتائج جيدة في الحالتين باستخدام عدد معقول من العناصر المتناهية.

Abstract

Determination of the temperature distribution in nuclear reactor elements is of utmost importance to ensure that the temperature stays within safe limits during reactor operation. This paper discusses the use of Finite Element numerical technique (FE) for the solution of the two dimensional heat conduction equation in geometries related to nuclear reactor cores. The FE solution starts with variational calculus which considers transforming the heat conduction equation into an integral equation $I(\theta)$ and seeks a function that minimizes this integral and hence gives the solution to the heat conduction equation.

In this paper FE theory as applied to heat conduction is briefly outlined and a 2-D program is used to apply the theory to simple shapes and to two gas cooled reactor fuel elements. Good results are obtained for both cases with reasonable number of elements.

1. Introduction:

Numerical techniques are used frequently for the solution of the general heat conduction equation especially in cases where the equation is non-linear or involves more than one space variable. Finite difference techniques are straight forward in simple geometries where coarse nodal spacing gives reasonable results. However, when the geometry is not simple and has irregular shape finite difference will prove combersome and requires special treatment. In such cases Finite Element (FE) numerical techniques are superior and can handle any geometrical shape provided that the boundary conditions are known.

This paper discusses the use of Finite Element numerical technique (FE) for the solution of the two-dimensional heat conduction equation in geometries related to nuclear reactor cores.

The transient heat conduction equation in 2-D with internal heat generation is given by [1]

$$\frac{\partial}{\partial x}\left[k(x,y)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[k(x,y)\frac{\partial T}{\partial y}\right]+G'''(x,y)=\rho C_P\frac{\partial T}{\partial \theta} \qquad (1)$$

symbols are defined in the nomenclature

The solution of this equation subject to the boundary conditions prevailing at the boundaries determines the temperature distribution in the region of interest.

2. Theory of Finite Elements :

Due to space limitations only a brief outline of FE theory will be given here. More details can be found in the cited references [2,3,4].

2.1 Finite Element Formulation

The application of the Finite Element (FE) method to Eq. 1, considers the variational formulation of the heat-conduction problem. The analysis begins by considering the function $I(\theta)$ given by

$$I(\theta)=\frac{1}{2}\int\int_A\left[k(\frac{\partial T}{\partial x})^2+k(\frac{\partial T}{\partial y})^2-2G'''T+\rho C_p\frac{\partial T^2}{\partial\theta}\right]dxdy$$

$$-\frac{1}{2}\int_{Bh}h(2T\infty T-T^2)ds-\int_{Bq}q''Tds \qquad (2)$$

Where the integrals are carried out over the region of interest, Fig. 1. The double integral over A is over the entire region. The single integral over Bh is carried out only along that portion of the boundary where there is a convective boundary. The integral over Bq is along that portion of the boundary where a heat flux input has been specified. The values of k, q''', ρ Cp,

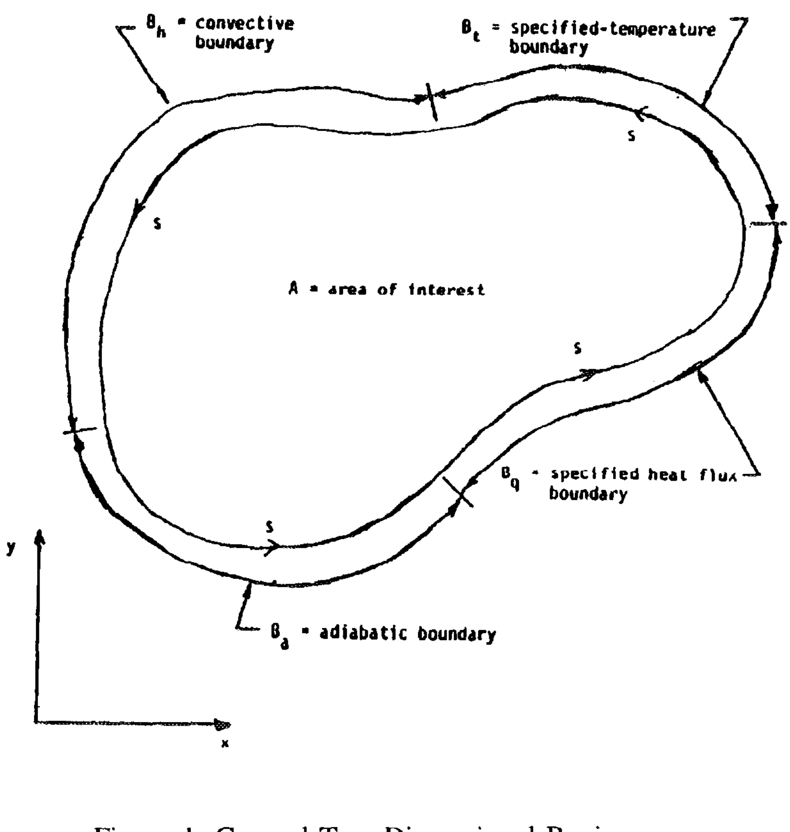

Figure 1: General Two-Dimensional Region

h, $T\infty$ and qn" in Eq. 1 may be functions of position but not of temperature or time. The value of the integral $I(\theta)$ depends upon the temperature distribution within the region, $T(x,y,\theta)$. In general, $I(\theta)$ will have a different value for each temperature distribution that is substituted into Eq. 1. We are interested in a particular temperature distribution that minimizes $I(\theta)$ at every time θ. It is proven [1] that this particular temperature distribution is also the solution to the two dimensional conduction problem (Eq. 1).

The analysis considers a **trial function** $T'(x,y,\theta)$ that satisfies all specified temperature boundary conditions but does not necessarily minimize $I(\theta)$. This trial function can be written as

$$T'(x,y,\theta) = T(x,y,\theta) + \in \cap(x,y) \qquad (3)$$

where $T(x,y,\theta)$ is the desired (but yet unknown) solution that minimizes $I(\theta)$. The function $\cap(x,y)$ is an arbitrary function except that $\cap=0$ along all specified temperature boundaries. The product $\in \cap(x,y)$ is called a **variation** of $T(x,y,\theta)$ and the search for the solution is part of the subject of calculus of variations hence the name variational formulation.

Since $\cap(x,y)$ is an arbitrary function, the trial function expressed in Eq.(3) includes all possible temperature distributions at time θ which satisfy all specified temperature boundary conditions. The trial function $T'(x,y,\theta)$ will equal the desired function $T(x,y,\theta)$ only when the parameter $\in$ is equal to 0.

substituting the trial function Eq. 3 into Eq. 1 gives

$$I(\theta,\in) = \frac{1}{2}\iint_A \left[k\left(\frac{\partial T'}{\partial x}\right)^2 + k\left(\frac{\partial T'}{\partial y}\right)^2 - 2G'''T' + \rho C_P \frac{\partial T'^2}{\partial \theta} \right] dxdy$$

$$-\frac{1}{2}\int_{BH}^{h} (2T\infty T' - T'^2)ds - \int_{Bq} q''nT'\, ds \qquad (4)$$

where I is also a function of the parameter $\in$. In order to have I be a minimum value for every time θ the derivative of I with respect to $\in$ must be zero. This must occur when $\in=0$ since this is when $T'=T$.

Actually this relation will only ensure that I(θ)will be an extreme value (maximum or minimum). For I(θ) to be a minimum the second derivative must be positive. Manipulations and rearrangements [1] and the application of Greens theorem lead to

$$\left.\frac{\partial I}{\partial \in}\right]_{\in=0} = -\int\int_A \cap \left[\frac{\partial}{\partial x}(kT_x) + \frac{\partial}{\partial y}(kT_y) + G'' - \rho C_P \frac{dT}{d\theta}\right] dxdy$$

$$+\int_B \cap \left[kT_y dx + kT_x dy\right] - \int_{Bh} \cap h(T\infty - T)ds - \int_{Bq} \cap q''_n \, ds = 0 \quad (5)$$

Further manipulations and rearrangements [1,2,3] we give

$$\left.\frac{\partial I}{\partial \in}\right]_{\in=0} = -\int\int_A \cap \left[\frac{\partial}{\partial y}(kT_y) + \frac{\partial}{\partial x}(kT_y) + G'' - \rho C_P \frac{\partial T}{d\theta}\right] dxdy \quad (6)$$

Since $\cap = \cap(x, y)$ is completely arbitrary (except that $\cap = \theta$ along Bt), the only way we can guarantee that the double integral will be θ is if the term in brackets is identically zero for all x and y. Thus T(x,y,θ) must satisfy

$$\frac{\partial}{\partial x}(kT_x) + \frac{\partial}{\partial y}(kT_y) + G'' = \rho C_P \frac{\partial T}{\partial \theta} \quad (7)$$

observe that this is the heat conduction equation in two dimensions (Eq. 1). Therefore, we see that the function T(x,y,θ) that extremizes I also satisfies the heat conduction equation. This extreme value of I is indeed a minimum which can be verified by considering the second derivative [1]. It can now be seen that our task of obtaining an approximate solution to the two dimensional heat conduction equation will require the approximation of integrals.

2.2 Application of Finite Element Theory:

A brief outline of the application of the finite element theory to a sample problem will be illustrated next.

2.2.1 Discretizing the Spatial Problem:

The finite element method for solving the heat conduction equation approximates and minimizes the integral I(θ) given by Eq. (2). I(θ) contains an integral over the area of interest and two integrals along the boundary. We will approximate these integrals by choosing a set of nodal points that may be placed at any convenient locations on the 2-D region and its boundaries. Each node is given a number and the nodal points are connected to form a network of triangular finite elements as shown in Fig. 2. Each element is also assigned a number (1,2,3, ... 70 in this example).

The function $I(\theta)$ may now be approximated as a sum of integrals over each triangular element and sums along each boundary segment. That is we may approximate Eq. (2) as

$$I(\theta)=\sum_{e=1}^{E}\frac{1}{2}\iint_{A^{(e)}}\left[kT^{X2}+kT^{y2}-2G'''T+\rho C_p\frac{\partial T^2}{\partial\theta}\right]dxdy$$
$$-\sum_{b=1}^{Bh}\frac{1}{2}\int_{Bh_{(b)}}h_{(b)}(2T\infty T-T^2)ds-\sum_{b=1}^{Bq}\int_{Bq_{(b)}}q''_{(b)}Tds \qquad (8)$$

Where E is the total number of elements , Bh is the number of convective boundary segments, and Bq is the number of specified non-zero heat-flux boundary segments. The only approximation in this expression is due to the fact that the boundary line segments are only an approximation to the true boundary.

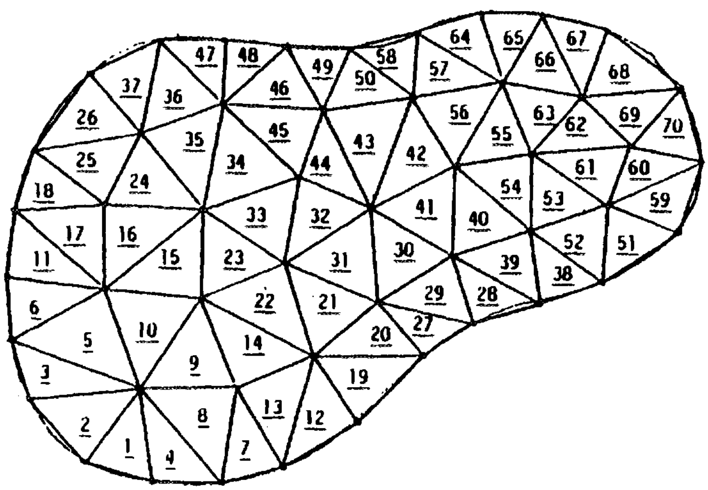

Figure 2 : Fe Discretization of 2-D Region

A 2-D finite element computer program is used to perform the necessary approximation and minimization. This FE program is implemented on a personal computer for this study and it handles steady state or transient problems in any 2-D geometry. The program was tested for simple rectangular geometries where the exact solutions are available for comparisons and good agreement is obtained with reasonable number of nodal points and elements [4].

Fig. 3 shows a comparison between the exact analytical solution and the solution obtained by present program for a simple 2-D square problem with internal heat generation. It can be seen that the FE solution approaches the exact solution as the number of nodes is increased and that a reasonable number of 15 nodes gives acceptable results.

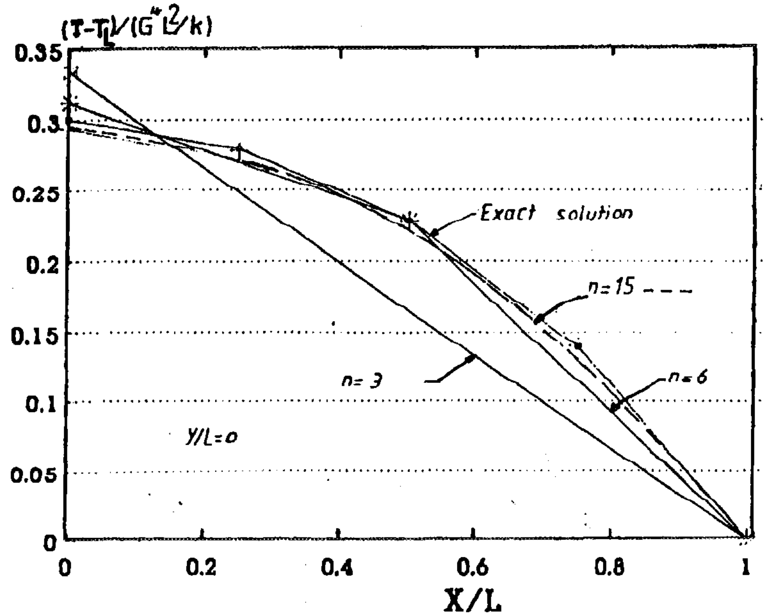

Figure 3 : Comparison of FE Solution with Analytical Solution for a Simple Square 2 - D Region

3. Application of FE to REA Reactor Fuel Elements :

The FE program is applied to determine the temperature distribution in several nuclear reactor fuel elements [4]. Due to space limitations only two cases will be presented in this paper. The first is the fuel element of an early MAGNOX type gas cooled research reactor, EDF-1, and the second is the fuel element of a High Temperature Gas Cooled Reactor (HTGR). In both cases the input data was taken from the available literature [5,6].

3.1 EDF-1 Fuel Element.

The core of the French research reactor EDF-1 is cylindrical with 8.3 m diameter, 9.0 m height and is divided into two zones. Zone A is square with 1.8 m side and Zone B is peripheral with 4.15 m radius. The core lattice is square with pitchs of 19.6 cm in zone A and 22.4 cm in zone B. The reactor uses natural uranium as the fuel, MAGNOX as the cladding, graphite as the moderator and CO_2 as the coolant [5]. The fuel element is a hollow cylinder with 14 mm inside diameter and 35 mm outside diameter and the cladding thickness is 1.8 mm. Fig. 4 shows the finite element grid used for the thermal analysis of the unit cell chosen from geometrical symmetry. The cell is discretized into 34 finite triangular elements using 27 Nodes. The convective heat transfer coefficient between the cladding surface and CO_2 coolant is calculated from the data available [5]. Fig. 5 gives the temperature distribution in the fuel and cladding as obtained from the present FE program. No data is available for the exact temperature distribution and comparison is made for the maximum fuel temperature. The value from the present analysis is 940°F compared with the quoted value of 1020°F [5] with a difference of about 8%.

3.2 (HTGR) Fuel Element:

Next we apply FE to the determination of the temperature distribution in the fuel element of a 3000 MWt High Temperature Gas cooled nuclear Reactor (HTGR) [6]. The fuel is a mixture of enriched Uranuim and Thorium in the form of carbide or oxide individually clad with multilayer ceramic coatings. Helium is the coolant and Graphite doubles as the moderator and core structural material. The fuel and coolant hole configuration is shown in Fig. 6 which also shows the triangular unit cell used for the present analysis. The heat transfer coefficient is calculated from the operating conditions of the fuel element. The temperature distribution in the fuel and moderator is compared in Fig. 7 with the 1-D solution given in reference [6]. It can be seen that the present 2-D temperature distribution lies lower than the

1-D solution with similar shape. This is expected since the 1-D solution assumes that heat is conducted in one direction only. The Value of the maximum fuel temperature from the two solutions is within about 7%.

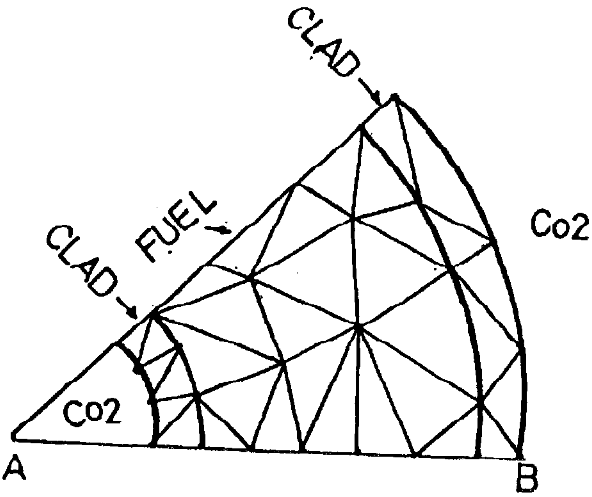

FIGURE 4: FE CELL EDF-1

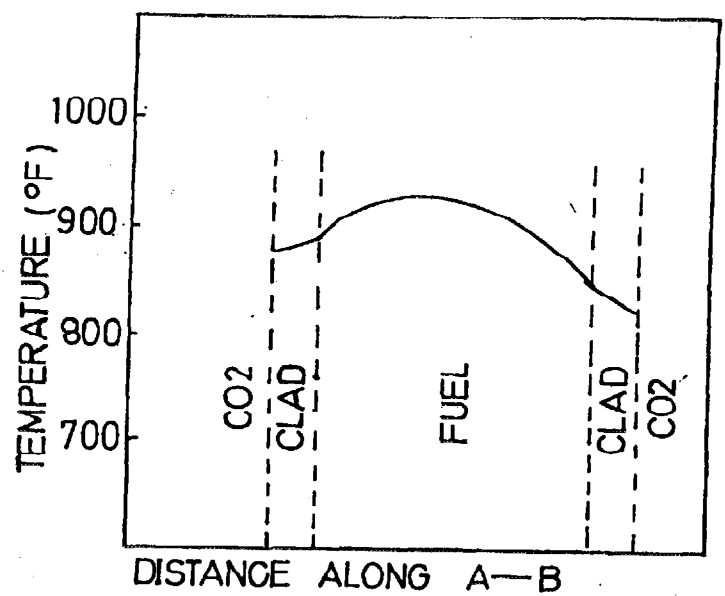

Figure 5 : Temperature Distribution in EDF-1 Fuel element

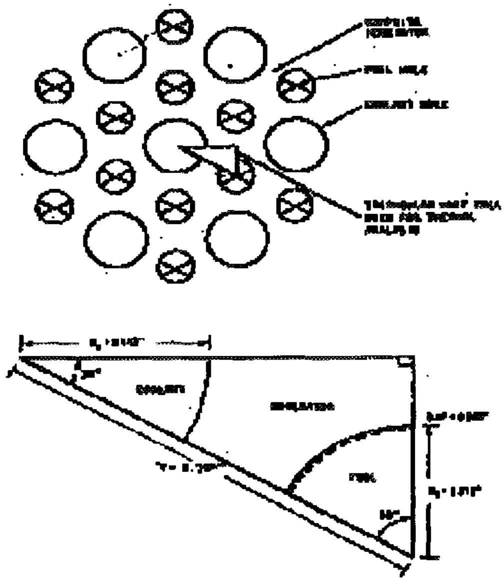

Figure 6 : HTGR Fuel and Coolant Hole Configuration

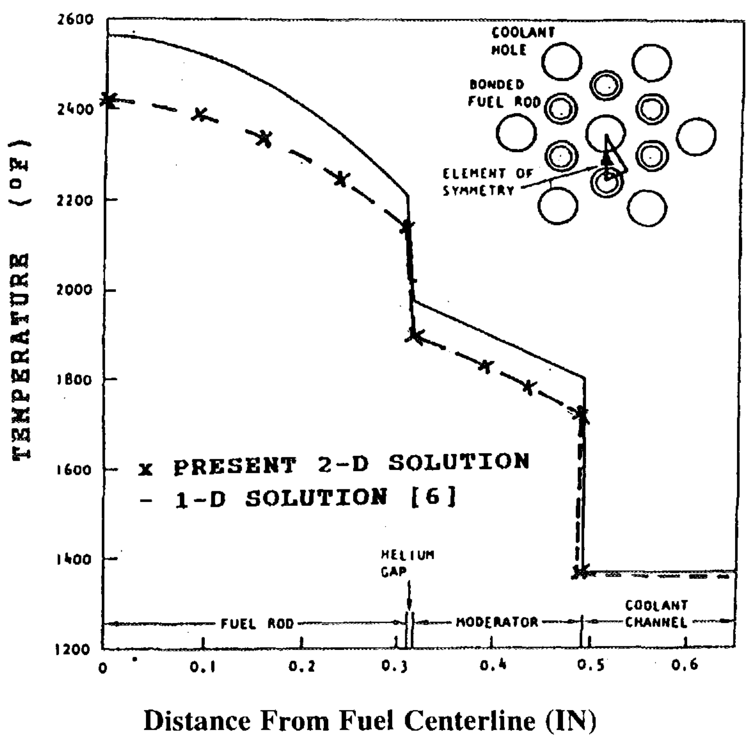

Figure 7 : Temperature Distribution in HTGR Fuel Element

Conclusion

From the results of this study we can conclude that using a 2-D finite element technique gives good accuracy for the temperature distribution in reactor fuel elements with reasonable number of nodal points and elements.

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Nomenclature

A. Area

B. Boundary

Bh. convective boundary

Bq. heat flux boundary

Bt. constant temperature boundary

C_p. specific heat

G'''. Volumetric heat generation

h. heat transfer coefficient

k. thermal conductivity

q''. heat flux

T. Temperature

T_x. derivatives with respect to x

T_y. derivative with respect to y

T_∞ fluid temperature

Greek:

θ. time

$\in$. Eq. 3

$\cap$. Eq. 3

f.· Density

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