



## Extended Reactivity Trace Curves for Nuclear Power Control with No Power Shooting

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### ملصقة

في هذه الورقة، نقدم مفهوم جديد يسمى بمنحنيات التتبع للتفاعلية (RTC) لأجل التحكم في قدرة المفاعلات بدون شطط في القدرة. المفهوم يستند على دراسات حديثة لبرنارد وزملائه حول الفترة الديناميكية للمفاعلات النووية. لقد تم دراسة منحنيات التتبع من خلال محاكاة مفاعل نووي ذي مجموعة واحدة من النيوترونات البطيئة باستخدام ثابت الانحلال الاستاتيكي، كما تم أيضا محاكاة مفاعل نووي ذي ست مجموعات من النيوترونات البطيئة باستخدام ثابت الانحلال الديناميكي. لقد تم مناسبة (fitting) منحني التتبع للتفاعلية الخاص بنموذج الست مجموعات من النيوترونات البطيئة الى معادلة بسيطة والتي تشمل تأثير كل من ثابت الانحلال الاستاتيكي وثابت الانحلال الديناميكي. وبالتالي، تم اقتراح بصميتين للمفاعل لتحديد شكل منحني التتبع للتفاعلية (RTC) والقادر على التحكم في قدرة المفاعل. إن التكامل بين منحنيات التتبع للمفاعلية ومنحنيات كفاءة أعمدة التحكم، أنتجت ما يسمى بمخطط  $p-z-t$ . هذا المخطط يربط بين قيمة التفاعلية الكفيلة بتثبيت القدرة عند حد معين (RTC)، وموقع أعمدة التحكم في المفاعل، والزمن اللازم للتحكم.

### Abstract

This Paper introduces a new concept of Reactivity Trace Curve (RTC) for nuclear power control with no power shooting. The concept is based on recent work of Bernard et al. on the dynamic period of nuclear reactors. RTC-method is simulated for both a static effective decay constant corresponding to a one-group delayed neutrons model, and a dynamic effective decay constant corresponding to a six-group delayed neutrons model. A fitting to the RTC of a six- group reactor model resulted in a

closed form formula for the RTC that couples the effect of both static and dynamic decay constants. Hence, introducing two "fingerprints" for the reactor in concern to identify a closed form RTC formula capable of controlling the reactor power. Integration of the RTC with control rod integral curves results in the  $\rho$ -z-t diagram. This diagram relates the amount of recommended reactivity (RTC), the position of control rod, and the time required for power control.

## 1. Introduction :

In nuclear power plants, the power shooting is reduced by achieving certain percent of required power first, then the excess reactivity is gradually changed until the power levels off without any shooting. The over shooting phenomena (in power increase) indicates that the rate of power change is a function of both the existing reactivity and its rate of change. This is concluded because at the peak of overshooting, the rate of change of power achieves zero value corresponding to an infinite period, meanwhile a positive reactivity with a decreasing rate is existing. This phenomena is due to the effect of delayed neutrons on the reactor dynamics. It occurs because, the production of the delayed neutron precursors is in equilibrium with the transient reactor power, while for given finite precursors life time, they are not in equilibrium with their decay. Therefore, for power increase, the precursor concentration will always be less than what its equilibrium value would be at the transient power. Accordingly, the delayed neutrons will always be less than what it would be if the precursors were at equilibrium and the prompt neutron contribution will therefore be larger. Hence, whenever the reactor is not at steady state, the non equilibrium condition of delayed neutrons will cause a delayed time response that must be recognized and balanced Bernard et al.[1] have shown that as a result of discrepancy between the equilibrium and the actual precursors concentrations that exists during power change, the reactor power can only be kept constant by a time dependent adjustment of reactivity. This is presented mathematically by equation (1):

$$0 = \rho(t)n(t) - \Gamma \sum_{i=1}^I [\lambda_i C_i^o(t) - \lambda_i C_i(t)] \quad (1)$$

where,  $\rho(t)$  is the reactivity,  $n(t)$  is reactor power,  $\Gamma$  is the neutron generation time,  $\lambda$  is the decay constant of the  $i$ -th group of delayed

neutron precursors,  $C_i''(t)$ , and  $C_i(t)$  are the equilibrium and instant precursors concentrations of type  $i$ , respectively. Moreover, Bernard et al.[2] introduced the dynamic period concept and derived a mathematical expression for such period which is given by:

$$\tau(t) = \frac{\beta - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + (\beta - \rho(t))\dot{\lambda}_e / \lambda_e} \quad (2)$$

where,  $\tau(t)$  is the dynamic period,  $\rho$  is time rate of change of reactivity,  $\lambda_2(t)$  and  $\dot{\lambda}_2$  are the effective time dependent (dynamic) decay constant and its derivative respectively. From such expression Bernard et al. [3] proposed the reactivity constraint technique (bounded reactivity control) for digital control of power with reduction and possible elimination of power shooting. one of the constraints is named absolute reactivity constraint and is given by:

$$-|\dot{\rho}| \leq \lambda_e(t)\rho(t) \leq |\dot{\rho}| \quad (3)$$

and, the other is the sufficient reactivity constraint which is given by:

$$-\left[ \frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}| \tau \ln(P_f / P) \right] \leq \rho \leq \left[ \frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}| \tau \ln(P_f / P) \right] \quad (4)$$

where,  $|\dot{\rho}|$  is the maximum available rate of change of reactivity that could be obtained at given rod height where the selected control rod to be moved,  $P_f$  is the desired power,  $P$  is the present power, and  $\tau$  is either the observed reactor period or the asymptotic period that corresponds to the net reactivity.

In our approach, we propose achieving, exactly, the required power and then keeping it constant thereafter with absolute elimination of power shooting. This is done by trying to identify a Reactivity Trace Curve (RTC) that makes the reactor period infinite and hence sustaining the reactor power steady at the desired power level. Such RTC approach represents a priori knowledge which should be important to the operators of nuclear reactors, as well as its high potential for automatic power control application.

## 2. Reactivity Trace Curve Concept

A sufficient condition for terminating the power transient and keeping the power constant is by setting the sum of all the terms involved in the denominator of the reactor period equation, Eq.(2),

equals to zero which is equivalent to the solution of the following differential equation:

$$\dot{\rho}(t) + \lambda_e(t)\rho(t) + (\beta - \rho(t))\dot{\lambda}_e / \lambda_e = 0 \quad (5)$$

subject to the safety constraint  $\rho < \beta$ . The solution of Eq.(5) identifies a Reactivity Trace Curve which, when it is followed, makes the reactor period infinite and hence sustaining the power at a constant level.

### 3. RTC For Static and Dynamic Effective Decay constants

Bernard et al.[4] defined the dynamic effective decay constant corresponding to multi group delayed neutrons as:

$$\lambda_e(t) = \frac{\sum \lambda_i C_i(t)}{\sum C_i(t)} \quad (6)$$

meanwhile, the static effective decay constant corresponding to one group of delayed neutrons valid for  $\rho < \beta$  is given by [5]:

$$\lambda_e = \beta \left[ \sum \frac{\beta_i}{\lambda_i} \right]^{-1} \quad (7)$$

We, next, demonstrate the concept of RTC technique based on the dynamic period for both the static and dynamic effective decay constant.

#### 3.1 Simulation of One Group Model With RTC Application

Delayed neutrons are commonly grouped into six groups, but one can effectively group them into one group provided that proper weighing is done for the calculation of the effective static decay constant  $\lambda_e$  given by Eq.(7). The one group point kinetic model, representing a nuclear reactor with ramp insertion of reactivity is given by:

$$dn / dt = n(\rho - \beta)\Gamma + \lambda_e C \quad (8)$$

$$dC / dt = \beta n / \Gamma - \lambda_e C \quad (9)$$

and

$$d\rho/dt = a \quad (10)$$

where,  $a$  is in units of reactivity per second. For such reactor model, with static decay constant, the condition for infinite dynamic period can be deduced from Eq.(5) to be:

$$d\rho / dt = -\lambda\rho(t) \quad (11)$$

Equations (8-10) were simulated for positive ramp reactivity insertion ( $a=5\text{m}\beta/\text{sec}$ ) for 20 sec, Then we replaced Eq.(10) by Eq.(11) to keep the power constant thereafter with absolute elimination of power shooting. Fig. 1 demonstrates our results. Similar results were obtained for a negative ramp insertion of reactivity and this is depicted in Fig.2 . The RTC can be identified by a closed form formula which is the solution of Eq.(11) and is given by [6]:

$$\rho = \rho_o e^{-\lambda_c t} \quad (12)$$

where,  $\rho_o$  is the end point of ramp reactivity and  $t$  is the time just after initiating the power control.

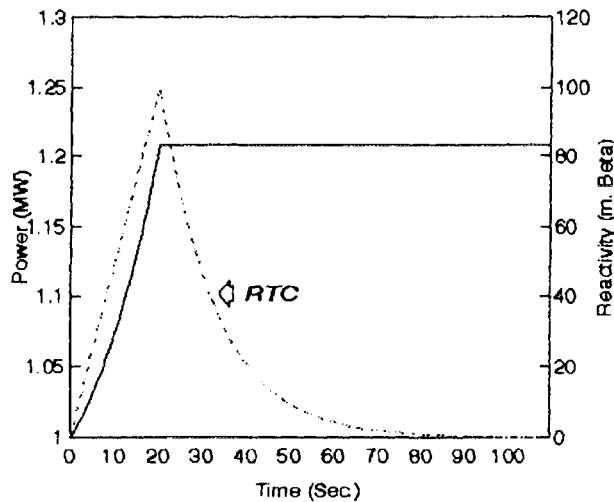


Fig. 1 One Group-RTC for Power Increase Case

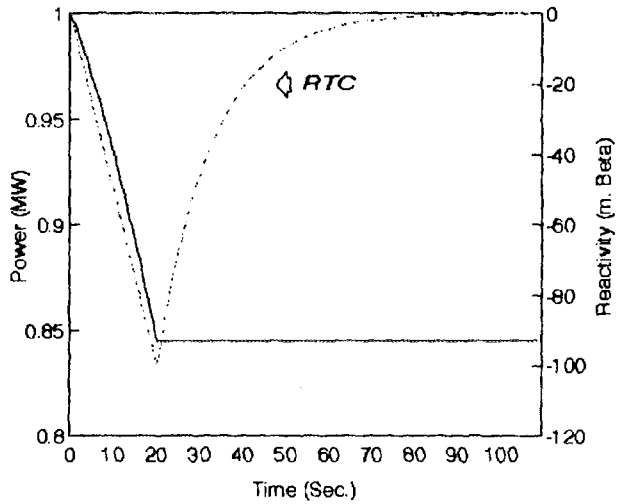


Fig.2 One Group-RTC for Power Decrease Case

### 3.2 $\rho$ -z-t Diagrams

The recommended  $\rho(t)$  to keep the power steady at desired power which is suggested by the newly developed RTC can be achieved through the movement of control rods according to their reactivity worth integral curves. Fig.(3) and Fig.(4) represent the  $\rho$ -z-t diagrams for power increase and power decrease, respectively.

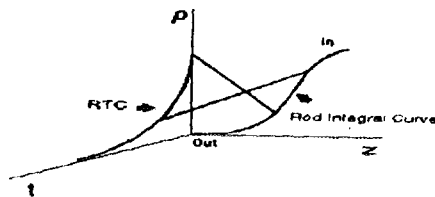


Fig.3  $\rho$ -z-t Diagram for Power Increase Case

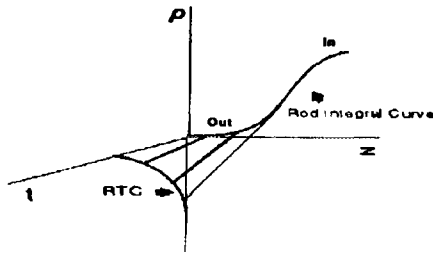


Fig.4  $\rho$ -z-t Diagram for Power Decrease Case

Such diagrams relate the amount of recommended system reactivity (RTC) responsible for keeping constant power, the corresponding position of control rods, and the time required for control. The crossing lines between the reactivity trace curves and the rod integral worth curves make correspondence between the absolute value of recommended reactivity for control, and the reactivity worth of control rod. The sequence of intersections of the crossing lines with the rod integral curve, starting from RTC initiating point, defines the with draws or insertions of the control rod accordingly. For example, in the power decrease case, Fig.(4), the sequence of intersections implies that the control rod is being withdrawn from the reactor in order to keep the power steady at a lower value. When a single control rod is not enough for the control of the power, another control rod is used and a new  $\rho$ -z-t diagram is applied, provided that one continues on the next point on the RTC and make correspondence to the rod worth of the new control rod which is put into operation.

### 3.3 Simulation of Six Group Model With RTC Application

The six group point kinetic model, representing a nuclear reactor with ramp insertion of reactivity is given by:

$$dn / dt = n(\rho - \beta) / \Gamma + \sum_{i=1}^6 \lambda_i C_i \quad (13)$$

$$dC_i / dt = \beta_i n / \Gamma - \lambda_i C_i \quad i = 1, 2, \dots, 6 \quad (14)$$

and

$$d\rho/dt = a \quad (15)$$

where,  $\mathbf{a}$  is in units of reactivity per second. For such reactor model the condition for infinite dynamic period is represented by the following RTC-differential equation:

$$d\rho / dt + \lambda_e(t)\rho(t) + (d\lambda_e / dt)(\beta - \rho(t)) / \lambda_e = 0 \quad (16)$$

with the dynamic effective decay constant  $\lambda_e(t)$  is given by Eq.(6) and its rate of change is given by:

$$\dot{\lambda}_e = \frac{(\sum C_i)(\sum \lambda_i \dot{C}_i) - (\sum \lambda_i C_i)(\sum \dot{C}_i)}{(\sum C_i)^2} \quad (17)$$

Equations (13-15) were simulated for positive ram reactivity insertion ( $\mathbf{a} = 5\text{m}\beta/\text{sec}$ ) for 20 sec, Then we replaced Eq.(15) by Eq.(16),with its supporting equations, to keep the power constant thereafter with absolute elimination of power shooting. Fig.5 demonstrates our result. A negative ramp insertion for power decrease case is presented in Fig. 6 . one, clearly, sees the absolute elimination of powershooting by following the RTC.

### 3.4 Reactor Fingerprints For Closed Form RTC-Formula

The logarithm of reactivity of RTC for six group reactor model and that for one group reactor model are plotted on Fig.7. The close investigation of the figure shows that the six group RTC can be represented by two slopes (fingerprints), one slope represents the one-group RTC static decay constant  $\lambda_e$  and the other represents another constant  $\gamma_e$ .The resultant RTC can then be represented by the following equation:

$$\rho_{\text{RTC}} = \begin{cases} \rho_{01}e^{-\lambda_e t}, & 20 < t \leq 30 \\ \rho_{06}e^{-\gamma_e t}, & t > 30 \end{cases} \quad (18)$$



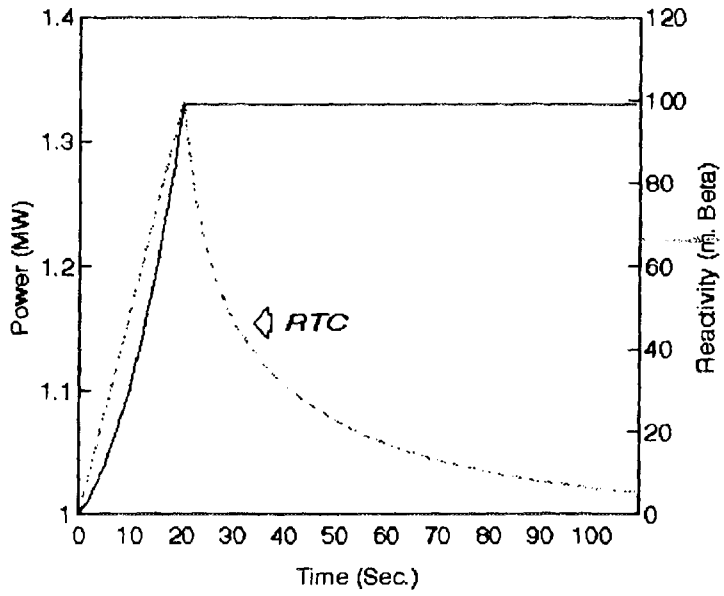


Fig.5 Six Group RTC for Power Increase Case

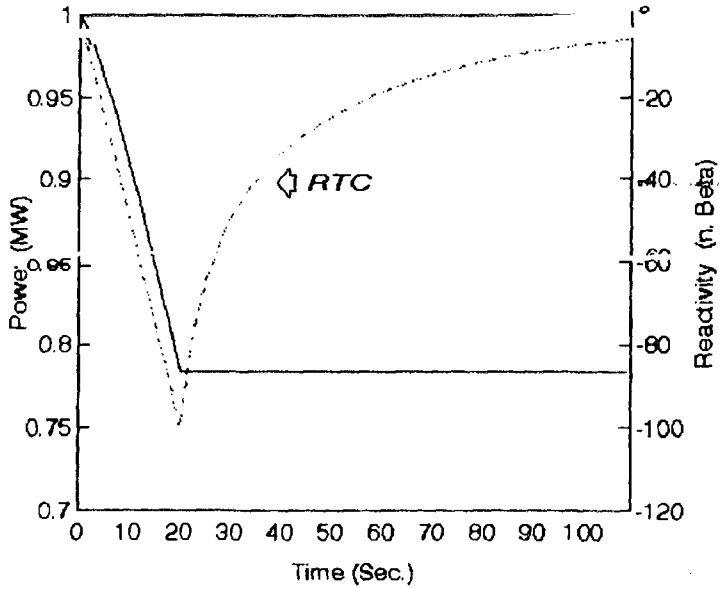


Fig.6 Six Group RTC for Power Decrease Case

Equations (13-15) were simulated for positive ram reactivity insertion ( $a = 5m\beta/\text{sec}$ ) for 20 sec. To control the power after that time, Eq.(15) is replaced with the following equation:

$$d\rho / dt = \begin{cases} -\lambda_e \rho(t), & 20 < t \leq 30 \\ -\gamma_e \rho(t), & t > 30 \end{cases} \quad (19)$$

Fig.8 demonstrates our results. It can be seen that there is a slight over shoot in the power, this is because a better fitting should be found to replace  $\lambda_e$  at early stage of RTC application. Nevertheless, one concludes that at the initiation of RTC control, only static decay constant  $\lambda_e$  fingerprint plays its major role, whereas, after some time the dynamic decay constant  $\lambda_e(t)$  comes in to effect throught the  $\gamma_e$  fingerprint. Further studies to select appropriate fingerprints for minimum overshoot is under investigation.

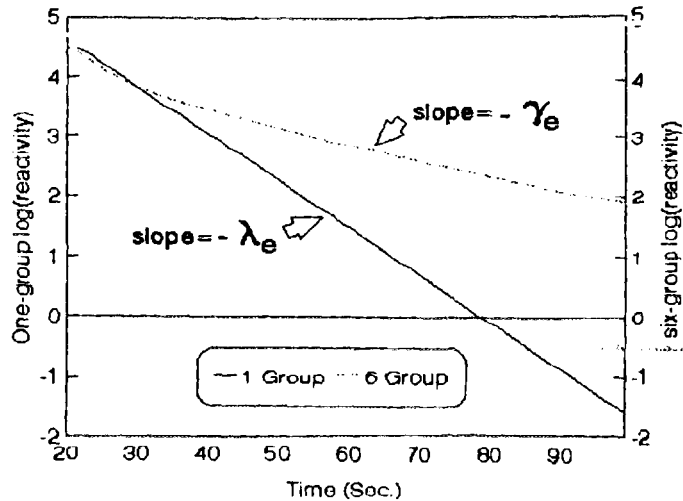


Fig. 7 Logarithmic RTC's For Fingerprints

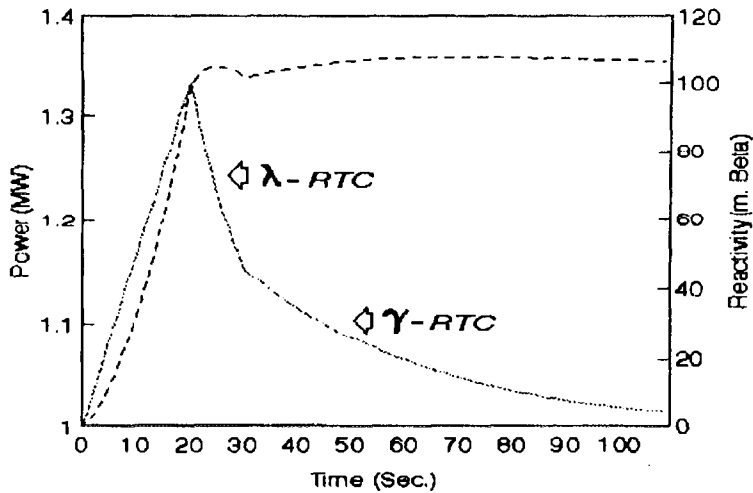


Fig.8 Six Group Fitted RTC with fingerprints for Power Increase Case

#### 4. Conclusion

In this paper a new concept has been introduced for reactor power control with absolute elimination of power shooting. The concept is based on defining a Reactivity Trace Curve (RTC) which when followed during either power increase or power decrease assures infinite period and hence keeping power at a constant level. For low power reactors (research reactors) where no presence of temperature feedback our simulation studies encourages us to apply the technique experimentally. For high power reactors, one has to justify the approach with the inclusion of feedback effects. This later study is being under investigation by the authors.

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