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MEASUREMENT IN QUANTUM PHYSICS

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Abstract

The conceptual problems in quantum mechanics –including the collapse of the wave functions, the particle-wave duality, the meaning of measurement– arise from the need to ascribe particle character to the wave function, which describes only the wave aspects. All these problems dissolve when working instead with quantum fields, which have both wave and particle character. The predictions of quantum physics, including Bell's inequalities, remain unchanged from the standard treatments of quantum mechanics.

More than half a century after the inception of quantum mechanics one basic conundrum remains unresolved [1]. Quantum mechanics is supposed to provide the complete knowable description of Nature. However, it is precisely in the connection between theory and Nature, in the theory of measurement relating the system and aparatus, that a consistency gap seems to exist. This problem concerns the so-called "collapse of the wave function" which presumably takes place upon performing a measurement: the wave function of the system, which before the measurement may have consisted of a linear superposition of states, after the measurement collapses into one particular state, corresponding to a particular eigenstate of the measuring apparatus. This collapse, together with the "propagation of a signal faster than light" in the Einstein-Podolski-Rosen setup [2], is needed to account for the experimental facts. However, this collapse is not

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compatible with the mathematics of quantum mechanics. We shall demonstrate that this problem can be resolved when the framework of quantum field theory is employed, even though it can not be resolved within quantum mechanics.

Debates and discussions on this and related problems still take place today; from small, semi-popular papers [3] to highly technical large-scale programmatic treatments [4, 5], to extensive discussions in recent books [6, 7]. Bell, in the Introduction of Ref. [1], remarks that "... despite numerous solutions of the [measurement] problem..., a problem of principle remains". (See also Feynman, Ref. [8].) Bell's "problem of principle" contains in essence two components; one is the above mentioned "collapse of the wave function", the other is known as "the Schrödinger cat". The former is associated with the initial interaction between the system and the apparatus, the latter with the chain of interactions transmitting the information about the outcome of the measurement from the initial interaction to "the pointer", which in Schrödinger's example is taken to be a cat. In the present paper we shall treat the first of these problems as the more fundamental of the two. We will employ the concept of density matrix to address the second in a paper elsewhere.

All the extant proposals to overcome Bell's "problem of principle" always necessarily break the framework of quantum theory, see [1, 3, 4, 5, 6, 7, 8] . Among the attempts, the deBroglie-Bohm pilot wave hypothesis [9] supplements the wave function by the "particle function" $X(t)$, called by the authors a "hidden variable". This work in fact provided the stimulus for Bell to derive the famous Bell inequalities [10] which allow for the distinction by experiment between at least a large class of hidden-variable theories and quantum physics. The experiments by now have come out in favor of the quantum physics predictions [11].

For completeness we begin by collecting the few concepts of quantum field theory needed in the present context. Here one need not go beyond the lowest order, i.e., no Feynman graphs containing loops need be considered. This limited theory -which we would like to call "quantum physics"- is known to pose no mathematical difficulties [12]. We also demonstrate in which way quantum mechanics is a sub-field of quantum physics.

Essentially *all of the concepts* needed in the description of the measurement are present already in the case of a two-slit experiment. The interference pattern arises in response to the wave aspect, as a probability distribution, while the particle aspect manifests itself in forbidding coincidences: in a weak beam situation only one counter at a time can register an event. No "collapse of the wave function" [13] is needed or takes place. The sensitivity of the interference between different reaction channels to an intervening measurement also emerges in the quantum physics description.

The Einstein-Podolski-Rosen (EPR) experiment [2], discussed next, is the simplest setup allowing for two-particle coincidences. Again no conceptual difficulties arise in its description in quantum physics.

The present paper does not address the question of the logical superstructure, denoted "the interpretation of quantum mechanics" in Refs. [4, 5]; we refer the reader to these papers. The point that measurement inescapably is an irreversible process already on the quantum level [14] is not important for our discussion here and we shall ignore it. It is, however, central in a complete description, including the measuring apparatus, in the resolution of the above second component of the measurement problem, which we will discuss in another paper.

Quantum Physics. In quantum physics the state under consideration is described by its state vector, $|S\rangle$. For example, $|S(x_1, x_2, t)\rangle$ represents a state such that at time t the system had two particles, one located at x_1, the other at x_2. The state vector for the system which has no particles, "the vacuum" is denoted $|V\rangle$. The field operator, denoted by $\Psi(x,t)$, interrogates the state vector for the presence of a particle at the point (x,t) in the form (we shall use units such that $\hbar$ and $c = 1$)

$$\Psi(x,t)\;|S(y,t)\rangle \;=\; \delta(x\;-\;y)\;|V\rangle \tag{1}$$

with

$$\Psi(x,t)\;|\;V\rangle \;=\; 0 \quad . \tag{2}$$

Hence one calls Ψ a "particle annihilation operator". $\Psi(x,t)$ and $\bar{\Psi}(x,t)$ are defined to obey the anti-commutation relations (commutation relations for Bosons)

$$\left[\Psi(y,t),\;\bar{\Psi}(x,t)\right]_+ \;=\; \delta(x\;-\;y) \quad , \tag{3}$$

where the 3-space δ-function implies the structure of a point particle. Comparing (1) and (3) one sees that

$$|S(x,t)\rangle \;=\; \bar{\Psi}(x,t)\;|V\rangle \quad ; \tag{4}$$

accordingly $\bar{\Psi}(x,t)$ is the "creation operator". The field operator $\Psi(x,t)$ is defined to obey the appropriate equations of motion, e.g., the Schrödinger wave equation in the nonrelativistic case

$$(i\,\partial_t - H)\,\Psi(x,t) \;=\; 0 \quad . \tag{5}$$

Equations (3) and (5) mean that $\Psi(x,t)$ has particle character which nevertheless propagates as a wave; it has simultaneously both particle and wave characteristics.

One may separate Ψ into the particle and the wave aspects in terms of an expansion in a complete set of c-number functions, say $\psi_n(x,t)$ which obey the wave equation (5) *together with the boundary conditions appropriate to the system*:

$$\Psi(x,t) \;=\; \sum_n b_n\;\psi_n(x,t) \tag{6}$$

for the field and

$$\bar{\Psi}(x,t) \;=\; \sum_n b_n^\dagger\;\bar{\psi}_n(x,t) \tag{7}$$

for the hermitian conjugate field. Following from Eq. (3), the quantities $b_n,\ b_{n'}^\dagger$ must obey the anti-commutation relations

$$\left[b_n,\ b_{n'}^\dagger\right]_+ \;=\; \delta_{n,n'} \quad , \tag{8}$$

$$\left[b_n^\dagger,\ b_{n'}^\dagger\right]_+ \;=\; 0 \;=\; [b_n,\ b_{n'}\ \ ,]_+ \tag{9}$$

which then leads to the completeness relation

$$\sum_n \bar{\psi}_n(x,t)\;\psi_n(y,t) \;=\; \delta(x\;-\;y) \quad . \tag{10}$$

Furthermore, from Eqs. (2) and (6) it follows that

$$b_n\;|V\rangle \;=\; 0 \quad . \tag{11}$$

The basic concept of quantum mechanics is the wave function, denoted as $w_n(x,t)$. To compute the wave function one must solve the Schrödinger equation (5) with appropriate boundary conditions imposed. How does this wave function emerge from quantum physics? We make the ansatz

$$w_n(x,t) \;=\; \langle V \mid \Psi(x,t) \mid S_n\rangle \quad , \tag{12}$$

where

$$|S_n\rangle \;=\; b_n^\dagger \;|V\rangle \tag{13}$$

is the state vector for the system labeled by n. Equation (12) together with (6) yield

$$w_n(x,t) \;=\; \psi_n(x,t) \tag{14}$$

which is consistent, since both $w_n(x,t)$ and $\psi_n(x,t)$ fulfill the same equation and the same boundary conditions.

This equation (14) demonstrates why the quantum mechanics predictions which are based only on the quantum mechanics function $w_n(x,t)$ are correct even though this function describes only "the wave aspects" of quantum physics. But *it lacks the particle aspects* which have been lost in the interrogation (12). These would be needed to describe particle emission or absorption processes. The quantum physics function $\Psi_n(x,t)$ describes a particle propagating through space and time. This propagation is that of a wave and thus precludes the possibility of assigning a trajectory to that motion. In contrast, quantum mechanics inherently lacks the particle concept: the wave function $w_n(x,t)$ describes nothing propagating.

Measurement in Quantum Physics: The Two-Slit Experiment. Any measurement requires an interaction between "the system" and the "measuring device", and in quantum physics every interaction involves the emission or absorption of a particle (recall the interaction term $\bar{\Psi}\gamma^\mu A_\mu \Psi$ of quantum electrodynamics). *Thus the measurement process lies outside of the framework of quantum mechanics*, which does not encompass the particle aspects which have been lost in the interrogation Eq. (12).

Consider the arrangement for the two-slit experiment. It consists of a photon source, an intervening screen with slits, and an array of detectors behind the slit-screen. The photon field is given by the solution of Maxwells equations together with the boundary conditions required to account for the source, slits, screen, etc., which in the decomposed form of Eqs. (6), (7) is (we change notation from above and suppress the vector character of the photon):

$$\varphi^{(k)}(x) \;=\; \sum_n a_n^{(k)}\; f_n^{(k)}(x) \quad , \tag{15}$$

with $k = 2$ if both slits are open, and $k = 1$ if only one slit is open. The two sets of solutions, $f_n^{(2)}(x)$, $f_n^{(1)}(x)$, are different since the boundary conditions for the two cases are different.

Subsequently, the interference patterns described by these two solutions are different. In these fields, $f_n^{(k)}(x)$ concerns the wave aspects, while $a_n^{(k)}$ concerns the particle aspects: $a_n^{(k)\dagger}$ creates, while $a_n^{(k)}$ annihilates, a particle in the state described by the wave function $f_n^{(k)}(x)$.

There are two parts in the action of the detector: (i) the interaction with the photons, and (ii) the registration of a "count" and the transmission of the data to the user, and so on. The action (i) of the detector m tests for the presence of a particle by interrogating the state vector at the space-time point x_m (within the resolution of the detector), For instance, the probability amplitude for detecting a particle in state n with $|S_n^{(k)}\rangle = a_n^{(k)\dagger}|V\rangle$, is

$$\langle V|\varphi_n^{(k)}(x_m)|S_n^{(k)}\rangle = f_n^{(k)}(x_m) \quad . \tag{16}$$

We collect the description of the action (ii) of the detector in an appropriate quantum operator η, which contains the reaction of the measuring apparatus and also includes the detector efficiency. Then the detector operator can be modeled as

$$D_m = \sum_n \eta_m^n \varphi_n^{(k)}(x_m) \quad . \tag{17}$$

The extent of the sum over n depends on the selectivity of the detector. The actual construction of the detector is of no importance; as an example, the absorption of the photon may result in the ionization of an atom, and the emitted electron may initiate a discharge as in a proportional counter. Thus the experimental apparatus, and by extension the experimenter, can be considered to be "part of the system", and represented in the detector operator η. A detail account of the action (ii) is the subject of a forthcoming paper.

The probability amplitude for the response of the detector m thus is [15]

$$A_m^{(k)} = \langle d_f| \otimes \langle V|D_m|S_n^{(k)}(x_m)\rangle \otimes |d_i\rangle \sim \langle d_f|\eta_m^n|d_i\rangle \; f_n^{(k)}(x_m) \quad . \tag{18}$$

As expected, the detector responds to the interference pattern of the photon field, described by $f_n^{(k)}(x_m)$. Here $\langle d_f|\eta|d_i\rangle$ denotes the expectation value describing the detector response, from state $|d_i\rangle$ before the interaction to $|d_f\rangle$ the state after detection. As always, the probability for counter m to respond is $W_m \sim \left|A_m^{(k)}\right|^2 \sim \left|f_n^{(k)}(x_m)\right|^2$, which would be the same in a quantum mechanics treatment.

Both the particle and the wave aspect contributed in reaching this result. The solution to Maxwells equation, the factor $f_n^{(k)}(x_m)$, provides the wave aspect. The particle aspect, the factor

$a_n^{(k)}$ contained in D_m, absorbs or annihilates the photon, and this takes place in a local manner, precisely at the (four-)point x_m in the detector m.

The need for the "wave function collapse" in quantum mechanics arises as follows. As long as the wave function does not vanish at the position of counter m', i.e., if $f_n^{(k)}(x_{m'}) \neq 0$, the probability for this counter to respond, $W_{m'} \sim \left|A_{m'}^{(k)}\right|^2 \sim \left|f_n^{(k)}(x_{m'})\right|^2$, does not vanish, which is independent of whether or not counter m has responded. However, once one detector has registered a photon, then no other detector can respond since the particle already has been absorbed. To prevent counter m' to respond if counter m has registered the particle, the only way to achieve this in quantum mechanics is to put $f_n^k(x_{m'})$ to zero. This is called "the collapse of the wave function", as evidently this process is not described by quantum mechanics.

In contrast, in quantum physics the proper result arises fully naturaly. Thus, the expression for the probability amplitude, say A_c, of a coincidence in detectors m and m', is

$$A_c \sim \left\langle V \left| D_{m'} \, D_m \right| S_n^{(k)}(x_{m''}) \right\rangle \sim \left\langle V \left| D_{m'} \right| V \right\rangle \eta_m \quad . \tag{19}$$

The probability for a coincidence thus vanishes because $D_{m'} \mid V\rangle = 0$, cf., Eq. (11). No collapse of a wave function needs be invoked. Also, since this result arises from the action of the particle operators, the distance between conters m and m' can be arbitrarily large.

In order to try to check "which slit the photon passed through" one places a detector in the slit, say at x_s; it would have to record a Compton scattering event. In this detection process the original photon is absorbed, and a new photon is emitted, having a new energy and a new radiation pattern appropriate to the new geometry, i.e., radiation from within the slit. The Compton detector function then would have the form

$$D_s \sim \eta_s \, \varphi^{(-)(s)}(x_s) \, \varphi^{(k)}(x_s) \quad , \tag{20}$$

where $\varphi^{(-)(s)}(x_s)$ is the emision part of $\varphi(x_s)$ for a photon with the new radiation pattern, constructed in analogy to Eq. (15). η_s is as previously the operator describing the reaction of the detector, here the recoiling atom. Of course, the new radiation pattern is different from the old radiation pattern; the two-slit interference pattern has been replaced by the single-slit interference pattern [16].

It is worth emphasizing that the two-slit interference pattern will disappear upon the Compton scattering of the photon even if nobody actually observes the counter, or even if the counter is broken. Such processes, of course, take place all the time; they are called "collimator scattering" and contribute to the experimental background. That means that the photon needs not "know that it has been observed" to lose coherence. Nature in quantum physics "has an objective existence; it exists by itself" independent of measurement.

Measurement in Quantum Physics: The E-P-R Experiment. In the EPR experiment at time $t = t_0$ two particles of spin $s = 1/2$, coupled to total spin $S = 0$, are emitted in opposite directions, go through a series of polarizers and analyzers to be finally absorbed in two widely separated detectors. The setup can be changed at random after the particles have been emitted and have become separated by such a distance that they cannot communicate without violating relativistic causality.

Denoting spin "up" and "down" by the indices $+$ and $-$ respectively, where we take the quantization to be along the z-direction, we have for the two-particle field operator

$$\Psi(x,y) \;=\; \Psi_1(x)\;\Psi_2(y) \tag{21}$$

with

$$\Psi_1(x) \;=\; \sum_n \;(a_{n,+}\; w_{n,+}(x) \;+\; a_{n,-}\; w_{n,-}(x)\;) \quad ; \tag{22}$$

and

$$\Psi_2(y) \;=\; \sum_n \;(b_{n,+}\; v_{n,+}(y) \;+\; b_{n,-}\; v_{n,-}(y)) \quad ; \tag{23}$$

here w denotes a particle emitted "to the right," and v "to the left". The state vector then is

$$|S\rangle \;=\; (a^\dagger_{i,+}\; b^\dagger_{i,-} \;-\; a^\dagger_{i,-}\; b^\dagger_{i,+}\;)|V\rangle \quad . \tag{24}$$

The detectors are endowed with polarization analyzers; denoting the polarization by the index p the detector response is given as

$$D_1(x) \;=\; \sum_{n,p} \; a_{n,p}\; w_{n,p}(x)\; \eta_{1;n,p} \quad ; \tag{25}$$

and

$$D_2(y) \;=\; \sum_{n,p} \; b_{n,p}\; v_{n,p}(x)\; \eta_{2;n,p} \quad . \tag{26}$$

The probability for obtaining a coincidence for polarization-insensitive detectors then is given by (detector 1 is at $x = X$, detector 2 at $y = -X$)

$$A \sim \langle V|\, D_1\, D_2\, |S\rangle \quad , \tag{27}$$

which contains only the interference of terms having the spin at X opposite to that at $-X$, as can be readily deduced from (24).

Now insert a polarization-sensitive filter in arm 1, such that only the "up" state is transmitted. This filter is represented by the projection operator

$$F = a_+^\dagger\, a_+ \quad . \tag{28}$$

Thus Eq. (27) is replaced by

$$A \sim \langle V|\, D_1\, D_2\, F\, |S\rangle \quad . \tag{29}$$

Now only the term with spin "up" at X and "down" at $-X$ survives. This implies that in a coincidence the detector in arm 1 "determines" the polarization of the particle in arm 2. And it does not matter at what time the filter was inserted in the beam path, as long as the filter was in place before the arrival of the particle wave packet.

A similar analysis can be carried out for the case of a spin-flip filter,

$$T = a_+^\dagger\, a_- + a_-^\dagger\, a_+ \quad . \tag{30}$$

Replacing in (29) F by T of (30), one finds that coincidence is achieved with both detected spins are of the same "orientation". Again, as previously, the counter in one of the arms "determines" the polarization of the particle in the other arm. And, as in the two-slit experiment, the distance between the counters can be arbitrarily large since the "determination" arises from the coordinate-independent particle creation/annihilation operators.

An interesting case arises when the analyzer of the detector in arm 1 is positioned along the z-axis and that of arm 2 along the y-axis, say. The state then will appear to the detector in arm 2 as having terms not only of the form $a_+^{\prime\dagger}\, b_-^{\prime\dagger}$ but also of the form $a_+^{\prime\dagger}\, b_+^{\prime\dagger}$ (the prime refers to the y-axis orientation). Therefore no strict yes-no coincidence rules exist and only probability predictions are possible. It is precisely these probabilities which are different in quantum and in classical probability. The analysis of this situation forms the basis for the Bell inequalities. All of the Bell predictions, Ref. [10], made in the framework of quantum mechanics are in full agreement with those derivable from the above quantum physics description –except that quantum mechanics requires an acausal propagation of the signal inducing the "collapse of the wave function", even though it cannot provide for the existence of such a signal.

Summary. In a measurement both aspects of quantum physics, viz. the particle and the wave aspect, are involved: the first to provide the "yes-no" decision, the second to provide the "how much" of the measurement. If the particle aspect gives the "yes" decision, the wave aspect provides the probability of the particular outcome; hence the numerical correctness of quantum mechanics. In case of the "no" decision, in quantum mechanics a mechanism for simulating this decision must be supplied "by hand". This artifact, extraneous to quantum mechanics, and unneeded in quantum physics, is called "the collapse of the wave function". This way, in agreement with Einstein's observation, quantum mechanics itself is incomplete. Of course, the 19-th century dream of a fully deterministic description remains unfulfilled.

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[15] The use of probability amplitude here is only appropriate for the initial quantum interaction at the detector. We will address elsewhere the second aspect of measurement involving macroscopic aparatus -as in the Shrödinger cat- with the use of density matrix which gives the probability directly, not the amplitude.

[16] Whether the two-slit pattern is replaced or still persists fully or partially depends on the relative length of scattered wavelengths with respect to the slit separation.

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