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Neutrino Clouds

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Abstract

We consider the possibility that neutrinos are coupled very weakly to an extremely light scalar boson. We first analyze the simple problem of one generation of neutrino and show that, for ranges of parameters that are allowed by existing data, such a system can have serious consequences for the evolution of stars and could impact precision laboratory measurements. We discuss the extension to more generations and show that the general conclusion remains viable. Finally, we note that, should such a scalar field be present, experiments give information about effective masses, not the masses that arise in unified field theories.

1 Introduction

In the last few years, it has been suggested that neutrinos might interact weakly among themselves through the exchange of a very light scalar particle [1, 2], with possible consequences for the evolution of the Universe and for the propagation of neutrinos from distant events. In many of these discussions, one assumes that neutrinos are distributed according to the usual Big Bang scenario and Standard Model physics, the effects of scalar exchange being treated as a perturbation. In this paper we examine that assumption.

This problem is a special case of the general problem of relativistic fermions interacting through the exchange of scalar and vector bosons, and the general formalism has been worked out under the name Quantum Hadrodynamics (QHD) and applied to the study of nuclear physics [3]. We shall show that, for a wide range of parameters, neutrinos will tend to cluster, that these clusters could be of a size to affect stellar formation and dynamics and that there may be observable consequences for physics within the solar system. In fact, if the clustering is strong enough so that the density of neutrinos within a cloud is sufficiently large, terrestrial laboratory experiments can be affected.

Over the last decade, many groups studying the endpoint of the Tritium beta ray spectrum for signs of neutrino mass have reported a best fit value for the square of the anti-neutrino mass less than zero [4, 5, 6, 7, 8, 9]. Robertson et.al [4] point out that this result could be obtained by assuming a very small branch due to the absorption of relic neutrinos, provided the density of such neutrinos were some 10^{13} higher than usual cosmological values. In choosing parameter ranges for the examples in this paper, we have kept this idea in mind. We must emphasize, however, that the general feature of cloud formation and its consequence for the evolution of structure in the history of the Universe is quite robust, and must be considered whatever the eventual resolution of the anomaly in Tritium beta decay.

In this work, we only consider the effects of light scalar exchange. The effects of the known vector exchange (Z_0) are far too small (really, too short ranged) to affect these results. The exchange of a light vector particle is severely constrained by data. To avoid problems with the axial anomaly (the neutrinos do couple to the Z_0) one must either invent many new fermions or demand that the light boson couple to known leptons or quarks, which quickly leads to conflict with experiments designed to test for a fifth force [10]. Furthermore, the self shielding of a vector exchange, while it might allow for the development of a neutrino-antineutrino plasma, will not allow for the coherent action required to drive cloud formation. Thus, we only treat scalar exchange. Even so, there remains the rich possibilities of different couplings to different generations. Aside from a few comments, we leave that to further work, concentrating our discussion on the simpler system of one flavor of neutrino.

The paper is organized as follows. In section 2 we review the treatment of infinite matter in QHD and apply that to the problem at hand. In section 3 we do the same for finite clouds of neutrinos. In section 4, we describe the consequences such clouds would have on the the evolution of structures in the Universe and, in section 5, we confront this picture with what data exists, extracting limits on the parameters. In section 6 we discuss the changes in the analysis of Tritium beta decay experiments within such neutrino clouds, and comment on the effects of such clouds on stellar dynamics in section 7. In section 8 we discuss some aspects of the extension to include more than one generation, illustrating the remarks with special cases applied to two generations. We offer our conclusions in section 9.

2 The Infinite Problem

The Lagrangian for a Dirac Field interacting with a scalar field is well known:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_\nu)\psi + \frac{1}{2}\left[\phi(\partial^2 - m_s^2)\phi\right] + g\bar{\psi}\psi\phi \tag{1}$$

which gives as the equations of motion

$$\left[\partial^2 + m_s^2\right]\phi = g\bar{\psi}\psi \tag{2}$$

$$\left[i\partial\!\!\!/ - m_\nu\right]\psi = -g\phi\psi. \tag{3}$$

We omit nonlinear scalar selfcouplings here, even though they are required to exist by field theoretic selfconsistency, as they may consistently be assumed to be sufficiently weak as to be totally irrelevant.

We look for solutions of these equations in infinite matter which are static and translationally invariant. Equation (2) then gives

$$\phi = \frac{g}{m_s^2}\bar{\psi}\psi, \tag{4}$$

which, when substituted in (3) gives an effective mass for the neutrino of

$$m_\nu^* = m_\nu^{(0)} - \frac{g^2}{m_s^2}\bar{\psi}\psi. \tag{5}$$

where $m_\nu^{(0)}$ is the renormalized vacuum mass that the neutrino would have in the absence of other physical neutrinos.

These equations are simply the equations of Quantum Hadrodynamics [3], and we will be using them in a small coupling regime where there is no question of the validity of neglecting higher order processes.

These equations are operator equations. We next act with each of these equations on a state $|\Omega\rangle$ defined as a filled Fermi sea of neutrinos, with a number density ρ, and Fermi momentum k_F, related as usual by $\rho = {k_F}^3/(6\pi^2)$. The operator $\bar{\psi}\psi$ acting on this state gives

$$\bar{\psi}\psi|\Omega\rangle = \frac{w}{(2\pi)^3}\int_{|\vec{k}|<k_F} d^3k \frac{m_\nu^*}{\sqrt{(m_\nu^*)^2 + k^2}}|\Omega\rangle, \tag{6}$$

where w is the number of neutrino states which contribute — $w = 2$ for Majorana neutrinos and $w = 4$ for Dirac neutrinos. Thus the effective mass is determined from the integral equation

$$m_\nu^* = m_\nu^{(0)} - \frac{g^2 w}{2\pi^2 m_s^2} \int_0^{k_F} k^2\, dk \frac{m_\nu^*}{\sqrt{(m_\nu^*)^2 + k^2}}. \tag{7}$$

To discuss the solutions of this equation we reduce it to dimensionless form, dividing by $m_\nu^{(0)}$, and introducing the parameter $K_0 = \frac{g^2 w (m_\nu^{(0)})^2}{2\pi^2 m_s^2}$, and the variables $y = \frac{m_\nu^*}{m_\nu^{(0)}}, x = \frac{k}{m_\nu^{(0)}}, x_F = \frac{k_F}{m_\nu^{(0)}}$. Then equation (7) becomes

$$y = 1 - yK_0 \int_0^{x_F} \frac{x^2\, dx}{\sqrt{y^2 + x^2}} \tag{8}$$

$$= 1 - \frac{yK_0}{2}\left[e_F x_F - y^2 \ln\left(\frac{e_F + x_F}{y}\right)\right], \tag{9}$$

with $e_F = \sqrt{x_F^2 + y^2}$. One can regard equation (9) as a non-linear equation for y as a function of either e_F or x_F. As a function of e_F, y is multiple valued (when a solution exists at all), whereas y is a single valued function of x_F.

The total energy of the system is a sum of the energy of the neutrinos, $E_\nu = e_\nu\, m_\nu^{(0)}\, wN$, and the energy in the scalar field, $E_s = e_s\, m_\nu^{(0)}\, wN$, where N is the total number of neutrinos in each contributing state. These expressions serve to define the per neutrino quantities e_ν and e_s. Also, $E_s = \mathcal{E}_s\, V$, where $\mathcal{E}_s = \frac{1}{2} m_s^2 \phi^2$ is the energy density of the (here uniform) scalar field.

One finds that

$$e_\nu = \frac{3}{x_F^3}\int_0^{x_F} x^2\, dx \sqrt{x^2 + y^2}$$

$$= \frac{3}{x_F^3}\left\{\frac{x_F^3 e_F}{4} + \frac{x_F y^2 e_F}{8} - \frac{y^4}{8}\ln\left(\frac{e_F + x_F}{y}\right)\right\} \tag{10}$$

and

$$e_s = \frac{K_0}{2}\frac{3}{x_F^3} y^2 \left(\int_0^{x_F} \frac{x^2\, dx}{\sqrt{x^2 + y^2}}\right)^2$$

$$= \frac{1}{2K_0}\frac{3}{x_F^3}(1 - y)^2. \tag{11}$$

Notice that for large values of x_F,

$$y \rightarrow \frac{2}{K_0 x_F^2}$$

$$e_\nu \rightarrow \frac{3 e_F}{4}$$

$$
\begin{aligned}
&\rightarrow \frac{3x_F}{4} \\
e_s &\rightarrow \frac{3}{2K_0}\frac{1}{x_F^3}.
\end{aligned} \tag{12}
$$

It is also useful to note that, for small x_F,

$$
\begin{aligned}
y &\rightarrow 1 - \frac{K_0 x_F^3}{3} \\
e_\nu &\rightarrow 1 + \frac{3x_F^2}{10} \\
e_s &\rightarrow \frac{K_0}{2}\frac{x_F^3}{3}.
\end{aligned} \tag{13}
$$

For the neutrino system to be bound, the minimum of $e = e_\nu + e_s$ as a function of density (or x_F) must be less than 1, its value in the zero density limit.

These points are illustrated in Figures 1 and 2, for the case $K_0 = 20$. In Figure 1, the average energy per neutrino, e, and the Fermi energy, e_F, are plotted, in units of the vacuum mass m_0, against the Fermi momentum in the same units, x_F. In Figure 2, the same information is displayed as e vs e_F, demonstrating the multivaluedness of the solution viewed this way. The general shape of Figure 1 will be exhibited for any value of K_0, but only if K_0 is large enough will there be a minimum with $e < 1$. This is displayed in Figure 3, from which we may deduce that K_0 must be greater than ≈ 3.3 for binding to occur. Note also that the local minimum disappears for $K_0 = 2.67$, showing that there is no metastable state for smaller values of the effective coupling strength.

3 Neutrino Clouds

Note from equation (13) that the energy varies as $1 + \text{const.} \times k_F^2$ near $x_F = 0$, where *const.* is positive. Even with K_0 large enough to produce binding, at low density e will be greater than 1, then will decrease with increasing x_F to its minimum value before increasing as $x_F \rightarrow \infty$. Thus, for low densities, it may be energetically favourable for the neutrinos to form finite size clouds.

To investigate this possibility we use the Thomas-Fermi approximation to obtain the scalar field through the equation

$$\nabla^2\phi + m_s^2\phi = g\langle\bar{\psi}\psi\rangle, \tag{14}$$

substituting the local density value for $\langle\bar{\psi}\psi\rangle$.

Noting that

$$y = 1 - \frac{g}{m_\nu^{(0)}}\phi, \tag{15}$$

we convert the local density equation (14) for ϕ to a local density equation for y:

$$\frac{d^2y}{dz^2} + \frac{2}{z}\frac{dy}{dz} = G(y, e_F, K_0) \tag{16}$$

with

$$G(y, e_F, K_0) = -1 + y\left(1 + K_0 \int_0^{x_F} \frac{x^2\,dx}{\sqrt{x^2+y^2}}\right), \tag{17}$$

and $z = r\,m_s$, where r is the radial distance from the center of the cloud and we have assumed spherical symmetry. In this equation, e_F (which is a chemical potential for the neutrinos) is constant with position, but y and thus $x_F = \sqrt{e_F^2 - y^2}$ are functions of position. Clearly the surface of the cloud is defined by the condition that $x_F = 0$, as there the neutrino density drops to zero, at the (scaled) radius $z = z_0$. For $z > z_0$, x_F remains zero, although the scalar field does not immediately vanish. Instead, outside the cloud the solution of the differential equation for the effective mass y becomes

$$y = 1 - (1 - e_F)\frac{e^{-(z-z_0)}}{z/z_0} \tag{18}$$

which satisfies the condition $y(z_0) = e_F$ at the surface, and the condition that $y \rightarrow 1$ as $z \rightarrow \infty$. From this exterior solution we compute the values of y and $\frac{dy}{dz}$ at $z = z_0$, to which the interior solution must be matched:

$$\begin{aligned} y(z_0) &= e_F \\ \left.\frac{dy}{dz}\right|_{z=z_0} &= (1 - e_F)\left(1 + \frac{1}{z_0}\right). \end{aligned} \tag{19}$$

In the usual way one solves the interior equation numerically starting from an assumed central value, $y(0)$, which is adjusted to satisfy the matching conditions. While this technique works in a straightforward way for small clouds, as the cloud increases in size one needs to know $y(0)$ to greater and greater precision, and needs to adopt a special technique to handle such cases numerically.

For large values of z_0 the central region will approximate the conditions of the infinite system, and so we expand the function G of equation (16) as a Taylor series in y about y_0, the solution of $G(y, e_F, K_0) = 0$, which is the effective mass in the infinite system. Explicitly

$$\begin{aligned} G(y, e_F, K_0) &= \left\{-2 + \frac{3}{y_0} - K_0 e_F x_0\right\}(y - y_0) \\ &+ \left\{\frac{3}{y_0^2} - \frac{3}{y_0} + \frac{K_0 e_F}{2}\left(\frac{y_0}{x_0} - 3\frac{x_0}{y_0}\right)\right\}(y - y_0)^2 + \ldots. \end{aligned} \tag{20}$$

Keeping just the first term in equation (20), the equation (16) has the solution

$$y - y_0 = A\frac{\sinh(\kappa z)}{\kappa z}, \tag{21}$$

where

$$\kappa^2 = \left\{-2 + \frac{3}{y_0} - K_0 e_F x_0\right\}. \tag{22}$$

(It is straightforward to show that $\kappa^2 > 0$ for the cases of interest, which are those for which e is close to the lower branch in Figure 2.)

The numerical solution of equation (16) is started at $z = z_1 > 0$, with an assumed value $y(z_1) = y_0 + \Delta y$, where Δy is small enough that the higher order terms in equation (20) can be safely neglected. The value of z_1 is adjusted to satisfy the outer matching conditions. One can then use equation (21) to extrapolate $y(z)$ from $z = z_1$ to $z = 0$, obtaining

$$y(0) = y_0 + \frac{\kappa z_1}{\sinh(\kappa z_1)}\Delta y. \tag{23}$$

In this way we can set $y(0)$ at the extraordinary level of precision required to satisfy the matching conditions.

The general form of the solutions does not depend on the particular value of K_0. An example of solutions with different numbers of neutrinos is presented in Figure 4 for $K_0 = 20$. To construct the Figure, values of e_F were chosen and a solution obtained, giving the density as a function of the radial distance from the center of the cloud. That density was then integrated to obtain the total number N displayed with the appropriate curves. The units are such that N=1 would correspond to $(m_\nu^0/m_s)^3$ total neutrinos. For the Thomas-Fermi approximation to work well, that number should be large compared to 1, which, as will be seen below, is the case. For later reference, we display the radial form of the gravitational force, f_g, due to such a distribution in Figure 5. This force is presented in arbitrary units, since the masses have been scaled out, and is calculated as $f_g = \frac{1}{z^2} \int_0^z e\, z^2\, dz$. Since, for finite clouds, the scalar field is no longer always constant, there is an additional contribution[3] to the scalar field energy density, $\frac{1}{2}(\nabla\phi)^2$, which has been included in our calculation.

To understand the distribution of cloud sizes to be expected, one needs to know the average energy per neutrino as a function of size or of total number. For $K_0 = 200$, Figure 6 displays both the logarithm of the cloud radius (i.e. the radius at which the density goes to zero) and the logarithm of the difference between the average energy in the cloud and the average energy in infinite neutrino matter at its minimum for the same value of K_0 versus the logarithm of the total number. The points are the results of calculations corresponding to the indicated values of e_F. The line through the radius values is a straight line with slope $\frac{1}{3}$, that through the energy differences has slope $\frac{-1}{3}$. Evidently, for large N, the radius scales as $N^{1/3}$ and the energy difference scales as $N^{-1/3}$, appropriate to a surface tension, indicating a preference for large clouds. The line through the e_F values is to guide the eye.

4 Early Universe

Consider the effects of such clustering on the evolution of structures in the early Universe. Throughout the following discussion, we assume that K_0 is large enough to produce bound systems. At an early enough epoch the density will be sufficiently high that the effective mass is negligible. At that epoch, there is no difference between the interacting neutrinos and the relativistic, non-interacting gas that is usually assumed. Consequently, these neutrinos will expand and decrease in density according to the standard scenario until the increase in the effective mass begins to make a difference, which will occur at about the value of the density (x_F) corresponding to the minimum energy per particle for infinite matter.

Were it possible to remove energy from the neutrino gas and entirely from the Universe, so that the gas could be viewed as having zero temperature, that would be the end of the discussion. We would, however, be left with a conundrum. The neutrinos could tolerate no further expansion but the Universe, being driven by all sources of energy density, would be required to continue to expand. This would result in one neutrino cloud located somewhere (defining a "center" ?).

That, of course, is not the situation. The neutrinos will have a temperature comparable to that obtained for an expanding, non-interacting gas. As the expansion continues, that temperature will be converted into (effective) mass and the gas will become supercooled, followed by fragmentation into clouds. Note that no additional dissipation is required, unlike the case where clouds coagulate from free particles. The point here is that the neutrinos were born within a cloud and never achieve a state in which the effective mass rises to its vacuum value.

Many neutrinos have been born at later times through normal stellar burning, supernova explosions or other processes. When they encounter a neutrino cloud, the coherent forward scattering is easily large enough, even for very small values of the coupling to the scalar field so that individual scatterings are small, to cause the neutrinos to lose energy through the Bremsstrahlung of scalars, providing additional dissipation.

Two factors drive the size distribution of these clouds. The first is the distribution of fluctuations, which we assume follows Harrison-Zel'dovich [11]. The second is the increase in energy per neutrino with decreasing cloud size discussed in section 3. The latter effect provides for a mechanism to cut off the distribution of cloud sizes below some smallest value, the actual efficacy of which depends on the detailed parameter values. The general form of the distribution is

$$P(N) \propto N^{-2}\exp(-C/N^{1/3}).$$

Should this process occur before recombination, which, as we shall see in Section 5, is not beyond reason, then the existence of neutrino clouds would have a profound effect on the evolution of small size structures. (By small, in this context, we refer to structures of the size of solar systems, stars or a bit smaller.) At recombination, when matter decouples from the background photon gas, there will be a pre-existing collection of gravitational sources. The longer the time between cloud formation and recombination, the more these will appear to be point sources, but that does not strongly affect the following argument. Whatever the spectrum of fluctuations in the baryon distribution, these pre-existing sources will nucleate baryon condensation with a distribution that more or less follows the size distribution of the clouds.

Many of these collections of baryons will be large enough to initiate nuclear burning and become stars; others will not. Of the latter, some will attract more baryonic matter from the ejecta of exploding stars to form later generation stars, while others will remain too small to evolve into stars and can provide cold, massive objects. Note that, even if a given cloud does not attract a compliment of baryonic matter, it will still function as a gravitational source. In either case, the increase in the energy per neutrino with decreasing cloud radius, discussed above, will provide a lower limit to the distribution of system sizes. Thus, the existence of neutrino clouds can serve as a seed mechanism for stars and could provide a similar seed for (or be themselves) smaller objects such as MACHOs[12].

This scenario suggests that all stars will have their associated neutrino cloud, not because stars attract neutrinos but, rather, because stars form within the gravitational well provided by pre-existing neutrino clouds. One may then ask if, during subsequent evolution, the star and its cloud remain together or if the star, buffeted by forces which ignore the neutrinos, is stripped away leaving the cloud to catalyze another object. This is a quantitative question which we discuss in section 7.

5 Numerical Considerations

Following the spirit outlined in the introduction, we shall consider here the constraints that may be placed on the parameters of the theory under the assumption of one surviving neutrino species. We shall take up the question of other generations in section 8.

First, to impact terrestrial Tritium beta decay experiments, the density of electron neutrinos needs to be about $10^{15}/cm^3$. Robertson et. al. [4] reported that their data could be fit by assuming an additional branch, 10^{-9} of the normal decay, which fed each final state in the same proportion as the normal decay. Applying standard formulas [13] for inverse beta decay, this rate requires a density of $6 \times 10^{15}/cm^3$. A recent paper by the Moscow group [9] indicates a preference for a 6×10^{-11} branch, which would imply a density of $3.6 \times 10^{14}/cm^3$.

For the first case,

$$k_F = (6\pi^2 \rho)^{1/3} = 14eV/c$$

while the second would imply

$$k_F = 5.5eV/c$$

Throughout this section we shall use 10 eV/c, since we are arguing here for the scale of the parameters. (An actual fit requires a more complete treatment of the modification of the spectrum, which we discuss further in section 6). This value of k_F corresponds to a density of $2 \times 10^{15}/cm^3$, which is some thirteen orders of magnitude above that obtained in standard cosmology, assuming that neutrinos are uniformly distributed in space. If we assume that all neutrinos created when the weak interaction was sufficiently strong to keep thermal contact between neutrinos and photons survive to the present epoch, this implies that only 10^{-13} of space is occupied by neutrinos.

As a consequence, neutrinos propagating over galactic or inter-galactic distances will, in the main, be free of the influence of condensed neutrinos and will propagate with their vacuum mass. Therefore, mass limits obtained from the spread of arrival times for neutrinos from SN1987a will apply to the vacuum mass [14]. While lower limits have been obtained by some authors, we shall use a limit of 50 eV here. Coupled with $k_F = 10eV$, this implies

$$x_F > 0.2$$

or

$$K_0 < 4000 \quad .$$

From the requirement that the system be bound, we obtain the lower limit

$$K_0 > 3.3 \quad .$$

For one generation, particle and antiparticle (chiral left and right, if Majorana), K_0 is related to the basic parameters by

$$K_0 = 4\tilde{\alpha}/\pi\mu^2 \tag{24}$$

where

$$\mu = m_s/m_\nu^{(0)} \tag{25}$$

and

$$\tilde{\alpha} = g^2/4\pi \quad . \tag{26}$$

To obtain limits on $\tilde{\alpha}$ directly, we consider those processes involving either the scattering of neutrinos through scalar exchange or processes involving the Bremsstrahlung of scalars. For the purpose of these estimates, we use only leading terms, and it does not matter whether the neutrino is a Dirac or Majorana particle. Consequently, many existing limits on the coupling of a Majoron to neutrinos can be taken over directly.

In a study of the limits on such a coupling in a number of elementary particle processes [15], it was found that the most stringent limit was that given by Barger, Leung and Pakvasa [16] from a study of (K_{l2}) decay. This gave

$$\tilde{\alpha} < 10^{-6} \quad .$$

A more stringent limit can be obtained from recent studies of nuclear double beta decay [17] looking for the spectral signature which would accompany the emission of one extra particle.

$$\tilde{\alpha} < 10^{-9} \quad .$$

Note that this limit only applies if neutrinos are Majorana particles.

For $\sqrt{s} \gg m_\nu$ the total neutrino-neutrino scattering cross-section is given by

$$\sigma \approx 5\pi\tilde{\alpha}^2/s \quad .$$

The strongest constraint from scattering comes from the neutrinos produced by SN1987a. If we assume that all neutrinos born in the Big Bang survive and are distributed roughly proportionally to the baryons, then there are 10^{66} associated with the solar system. At $2 \times 10^{15}/cm^3$, that implies a radius of the cloud of $\approx 5 \times 10^{16}$cm. On this scale, the Earth is essentially at the center of the cloud. Thus the mean free path for 20 MeV neutrinos coming in from the supernova must be greater than 5×10^{17}cm or the cross section must be less than 10^{-34}cm^2. For a mean neutrino energy of 7.5 eV $(3k_F/4)$ and a beam neutrino of 20 MeV, $s \approx 1.5 \times 10^{-10}\text{GeV}^2$, giving $\sigma \approx 10^{-17}\text{cm}^2\tilde{\alpha}^2$ This then requires that

$$\tilde{\alpha} < 3 \times 10^{-9} \quad .$$

Other processes, such as the observation of solar neutrinos or the observation of neutrinos from high energy accelerators lead to less stringent upper bounds.

A more interesting bound comes from the consideration of the survival of Big Bang neutrinos to the present epoch. The dominant disappearance mechanism is for a neutrino and an antineutrino to annihilate into a pair of scalars. For Majorana neutrinos, this simply means that a chiral left and a chiral right neutrino transmute into a pair of scalars. The rate is

$$\omega \approx \frac{3}{8}\tilde{\alpha}^2 k_F \ln \frac{k_F}{m_s}.$$

The total disappearance probability is the integral of this rate to the present from whatever initial time is appropriate. The time before neutrinos condense into clouds is irrelevant to this question as, throughout that time, the scalar field and the neutrinos are both expanding relativistic gases which were once in thermal contact. All that a large rate for this process can do is maintain that thermal contact since, under these conditions, the back rate is the same. Consequently, we consider the integral of the rate from a time when the temperature

Table 1: Infinite Matter Values

K_0	$< e >$	x_F	y	x_F/y
20	.704	.675	.2	.3
200	.410	.405	.06	.15
2000	.233	.232	.0186	.08

corresponds to about 10 eV, or $t = 10^{13}s$. The bound is only weakly dependent on the assumption of radiation or matter dominance for the expansion. Requiring a decrease of less than 0.1 in $\ln(N/N_0)$ yields

$$\tilde{\alpha} < 3 \times 10^{-20} \text{(radiation-dominated)}$$

or

$$\tilde{\alpha} < 10^{-18} \text{(matter-dominated)}.$$

It is tempting to consider scenarios with $\tilde{\alpha}$ above this limit but still below the limits imposed by scattering. In that case, one might argue that neutrinos are Dirac particles and that there is a fractional particle excess as for any other fermion. It is, in fact, such a scenario that would give only an excess of counts at the end points of beta decay spectra (see the discussion in the next section). However, the limits on neutrino generations from nucleosynthesis calculations are based on the assumption of only left handed neutrinos and right handed antineutrinos. The scalar interaction considered here will equilibrate left and right handed neutrinos while they are in equilibrium through the weak interaction, leading to a factor of two too many states per generation. As the remarks above regarding the limits of the time integration for disappearance point out, it does not seem possible to try to argue for the disappearance of some states at an early enough epoch that it could affect the neutron-proton equilibrium and, hence, nuclear abundances.

At this juncture, one may question one's ability to accommodate the scalar field in light of the same argument. This question was addressed several years ago by Kolb, Turner and Walker [18]. The results of that paper indicate that a very light scalar (the case here) would behave as an extra half a generation, which can be accommodated.

Returning to the issue of Majorana neutrinos, the search for the evidence for their existence in nuclear double beta decay is a thriving industry [17, 19]. If such decays were mediated by a mass term, modern experiments limit that mass to less than 1-2 eV. When taken with our standard value of $k_F = 10$ eV, this implies that $y/x_F < 0.2$ independent of the vacuum mass. Table 5.1 lists, for some representative values of K_0, values of x_F, y and the ratio y/x_F for infinite matter at its minimum energy per neutrino. This suggests a preference for larger values of K_0 within the range discussed above.

Another possible source of constraint is the gravitational attraction that such a neutrino cloud would create due to its energy density (note that the static scalar field contributes here). In section 3 the gravitational acceleration due to various clouds was displayed. Nieto, et al [20] have recently discussed an anomalous acceleration observed on the Pioneer spacecraft, essentially constant from 10 to 50 AU with a value of $10^{-9} m/s^2$. While Figure 5 raises the possibility of a nearly constant acceleration over a wide range of distances, the magnitude would require an average energy, at a density of $2 \times 10^{15}/cm^3$, of $\approx 50eV$, far in excess of the values considered here. For this discussion (one generation only), this implies that no useful constraints are likely from gravity. On the other hand, such considerations add strength to the argument that the range of the interaction ought to be of the order of several AU, since the extent of the surface is given by that range. Even though the clouds could be much larger than the scalar range, such clouds would have a relatively sharp surface and would not produce a radial dependence that was gentle enough to appear constant.

Therefore, assuming the range of the interaction to be of the order of 1 AU give or take a few orders of magnitude, since

$$1AU \approx 1.5 \times 10^{13} cm$$

we have

$$m_s(1AU) \approx 1.3 \times 10^{-18} eV$$

which suggests that we take

$$10^{-21} < \mu < 10^{-17} \quad .$$

Thus, for a given K_0 , we have a range of allowed $\tilde{\alpha}$, as may be deduced from Figure 7. On that figure, the solid vertical line represents the μ appropriate to a range of $1AU$ and $m_\nu^0 = 10\ eV$, the dotted lines on either side representing a change of 2 orders of magnitude. The diagonal lines are lines of constant K_0, the right hand one for $K_0 = 4$ and the left hand one for $K_0 = 40,000$. The other constraints already discussed appear as horizontal lines representing the various upper limits on $\tilde{\alpha}$. For μ and K_0 in the ranges above, we would obtain

$$10^{-42} < \tilde{\alpha} < 10^{-31}$$

Such small values of $\tilde{\alpha}$ trivially satisfy all other operative constraints.

6 Tritium beta decay

As remarked in section 5, a scenario which led to a (smaller) cloud of Dirac neutrinos would provide for a more natural explanation of additional counts at the end point of a beta decay spectrum than would a scenario involving a larger number of Majorana neutrinos. The reason, which is well known [21], is summarized here.

The endpoint of the spectrum, calculated from the energy release in the decay, is not affected by the presence of the background neutrinos. Electrons emitted following the absorption of a neutrino will have an energy beyond the endpoint equal to the total energy of the neutrino. Thus, for a cold gas, the spectrum will reflect the Fermi-Dirac distribution extending from $E_0 + m^*$ to $E_0 + e_F$ where E_0 is the true endpoint. (One may safely neglect the change in the field energy due to the disappearance of one neutrino.) If the neutrinos are Majorana, there will be an equal distribution of anti-neutrinos which will block the emission of the lowest energy anti-neutrinos from the normal beta decay. Since the distributions are equal, the blocking will exactly balance the additional emission, leading to no net increase in the rate.

In either case (Dirac or Majorana), the change in the spectrum is more complex than the simple addition of a spike. The density required to affect the experiments produces a Fermi momentum of at least several electron volts, and the distribution of additional events must reflect that. It may well be that fitting with the correct spectral shape will destroy the feature, reported in [4], that a fit, equally acceptable to that with a negative mass squared, can be achieved. It is also possible that the correct shape will not change or improve the fit. That question can only be resolved by fitting actual data with all experimental effects included. Note that the analysis described here, as envisaged by [21], is for one neutrino, assumed to be the electron neutrino. We return to this in section 8.

7 Cloud Dynamics

To analyze the dynamics of the system of cloud plus star in a galactic environment, and, in particular, to determine if a star stays within the cloud that seeded its formation, would require a modified Fokker-Planck treatment [22]. While a complete treatment would require the complications of at least three generations of neutrinos, we consider only one generation, to illustrate some of the issues.

The primary mechanism for altering a star's trajectory is the gravitational scattering between two stars that pass relatively near each other and, to lowest order, the clouds simply follow along. Since the cloud-star system is polarizable, there will be an induced dipole-dipole interaction, analogous to atomic scattering, which will produce a Van der Waals like interaction. While this may alter the specifics of the velocity distribution slightly, it should not have a major impact on the issue of the cloud remaining with its star.

A more serious question involves the interaction between two clouds when they touch. If we assume that the cloud contains an energy equivalent to about $1 M_\odot$, uniformly distributed to 10^{17} cm., then the gravitational binding energy of the star to the cloud is $\approx 10^{39}$ eV or about 10^{-27} eV/ν. According to Figure 6 for $K_0 = 200$, the surface contribution to the energy per neutrino is $\approx 10^{-3} m_0/N^{1/3}$. Thus, for the cases of interest here, the surface tension of the clouds overwhelms the gravitational interaction with the stars and the stable final configuration would have one star denuded and the other dressed with twice as many neutrinos. Binney and Tremaine [22] present the estimate that 2 stars, with $R \approx R_\odot$, would actually collide once every 10^{19} yr. If, however, clouds extend to 10^{17} cm. the ratio of geometric cross sections is $\approx 2 \times 10^{12}$, so the encounter rate would be about 2 in 10^7 yr., which is relatively fast on Galactic timescales.

Note, however, that the cloud stays with one star or the other. The evolutionary result of such collisions would be that neutrino clouds would be found only with a fraction of the stars and that that fraction would be smaller in more densely populated regions. Furthermore, the simple argument presented above takes no account of other neutrino generations. The possibility that the Sun has remained with an attendant cloud remains viable.

8 More Generations

The preceeding discussion has been confined to the artificial case of one generation for pedagogical purposes, but, as the last sections illustrate, it is not reasonable to expect a good description of nature without allowing for at least three generations of light neutrinos. Necessary as this is, the unknown vacuum masses and couplings to the scalar field provide a large number of free parameters. The general arguments setting allowed ranges will not change, but the actual limits may depend on the number of participating neutrino species and the detailed nature of the coupling of the scalar to those species, consequently those limits may change by a few orders of magnitude. On the positive side, the existence of three mass eigenstates allows for structures on different scales and can avoid the possibilities of apparent inconsistencies described above.

To illustrate some of the complexity that will arise when all three generations of neutrinos are considered, it is instructive to examine the case of two generations. In principle, there could be several scalars coupling to the various generations with arbitrary strengths, but that problem is too unconstrained to be instructive. We shall consider two simple examples, one in which the coupling is proportional to the vacuum mass and one in which the coupling has the same strength to both generations.

We first consider the case where the coupling of a neutrino to the scalar field is proportional to the vacuum mass. Label the two neutrinos by h (for heavy) and l (for light). Then, if g denotes the coupling constant for h,

$$g_l = \frac{m_l^{(0)}}{m_h^{(0)}} \times g \tag{27}$$

This gives equations for the effective masses

$$m_h^* = m_h^{(0)} - g\phi \tag{28}$$

and

$$m_l^* = m_l^{(0)} - g_l\varphi \tag{29}$$

If we now define y_h and y_l as in section 2, we obtain

$$y_h = (1 - \frac{g}{m_h^{(0)}}\phi) \tag{30}$$

and

$$y_l = (1 - \frac{g_l}{m_l^{(0)}}\phi) = (1 - \frac{g}{m_h^{(0)}}\phi) \tag{31}$$

or

$$y_l = y_h \tag{32}$$

That is, there is only one function y for both species. We then define the quantities x_h, x_l, e_h and e_l in analogous fashion. Also define the ratio

$$r = \frac{m_l^{(0)}}{m_h^{(0)}} \tag{33}$$

With these definitions, the differential equation for y becomes

$$\frac{d^2y}{dz^2} + \frac{dy}{dz} = -1 + y(1 + \frac{K_0}{2}F) \tag{34}$$

where

$$F = e_{F_h}x_{F_h} - y^2 \ln\frac{e_{F_h} + x_{F_h}}{y} + r^4\left[e_{F_l}x_{F_l} - y^2 \ln\frac{e_{F_l} + x_{F_l}}{y}\right] \tag{35}$$

The factor of r^4 comes from one power for scaling the energy and three powers for scaling the Fermi momentum x_{F_l}.

The solution of the differential equation is carried out in much the same manner as discussed in section 3, with e_{F_h} and e_{F_l} playing the roles of Lagrange multipliers fixing the total numbers of heavy and light neutrinos respectively.

For this system, the density distributions are similar with the species with the larger e_F extending out to larger z. Of course, the factor r^3 in the density and r^4 in both the energy and the driving term of the differential equation mean that the heavy neutrino dominates the system. For the special case where $e_{F_l} = e_{F_h}$, the densities track exactly and the system is equivalent to one generation with $K_0' = (1 + r^4)\, K_0$. The possibility of different values for the e_{F_i} allows for the composite distribution to fall more slowly at the surface than it would for one generation and, if r is not very different from 1, the gravitational force will also fall more slowly.

An example of this is given in Figure 8, in which we have chosen the ratio of light to heavy vacuum masses, r, to be 1/2 and the remaining parameters to give $K_0 = 20$ for the light neutrino. We have kept the number of heavy neutrinos,

N_h, approximately constant and varied the number of light neutrinos, N_l, from 0 to a value slightly in excess of N_h. Since both neutrino densities are involved in the driving term given in Eq (35), e_{F_h} must be varied as well as e_{F_l}, hence the approximate constancy of N_h. For each case we have plotted the individual number densities as well as the gravitational forces, calculated as in Figure 5, as a function of the scaled radius.

There is a particularly interesting set of configurations in which e_{F_h} is less than the value of y corresponding to the infinite system of light neutrinos with a given e_{F_l}. In these cases, which can occur as the Universe expands, outside the cloud of heavy neutrinos y approaches this asymptotic value from below. These configurations correspond to the formation of clouds of heavy neutrinos in the infinite sea of light neutrinos at an epoch in which the light neutrinos are still very relativistic.

The case in which the scalar couples to all generations with the same strength allows for more radical changes in the density distributions, and we now examine that situation for two generations. Here,

$$m^* = m^{(0)} - g\phi \tag{36}$$

for both generations, i.e. the magnitude of the shift is the same for both. Hence, if the shift is significant for the heavy neutrino it may cause the effective mass of the light neutrino to change sign. This is like the case of a massless fermion gaining mass from the presence of a scalar condensate in field theory; the density of heavy neutrinos serves as the source of an external scalar field for the light neutrinos (and vice versa, although with little effect). Care must be taken in the assignment of quantum numbers, but the energy, given by

$$E = (m^{*2} + k^2)^{1/2}$$

is well behaved. If y denotes $m_h^*/m_h^{(0)}$, then

$$y_l = 1 - (1 - y)/r \tag{37}$$

$$e_l = (y_l^2 + x_l^2)^{1/2} \tag{38}$$

The solution of the differential equation proceeds as before, with one subsidiary condition. Since $e_l > 1$ implies that it is energetically favorable for the light neutrino to be at infinity, light neutrinos will be excluded from such a region and x_{F_l} is set to 0 unless the condition

$$-e_{F_l} < y_l < e_{F_l} \tag{39}$$

is met. For r sufficiently small, one may obtain a solution in which the heavy neutrinos occupy a sphere with radius a; from a to some larger radius, b, there are no neutrinos and ϕ is a linear combination of modified spherical Bessel functions of the third kind of order 0; then light neutrinos occupy a spherical shell from b to some large radius c. An example of such a distribution is plotted in Figure 9, along with the gravitational force, as a function of the scaled radius. Here the vacuum mass ratio was $r = 0.1$, $K_0 = 400$ for the heavy neutrino and $N_l \approx \frac{1}{2} N_h$.

In principle, if the shell of light neutrinos occurs at a large enough radius, such a distribution could radically affect the radial dependence of the gravitational force produced by the neutrinos. In practice, that will require a very delicate matching in the solution of the differential equation (troublesome for theorists, not for nature) which requires the development of techniques like those reported is section 2 for the treatment of very large clouds for one generation only.

Since this scalar field may well be a contributor to, but not the only contributor to, the neutrino mass matrix, the real condition is probably more complicated than either of these examples. The general feature that the heaviest mass eigenstate should be most dense and of smallest extent, common in both special scenarios, ought, however, to persist. If the dominant component of a neutrino cloud in the region of one to several AU from the Sun were one of the heavier mass states, say the medium mass eigenstate, with a small component to be the interaction eigenstate associated with the electron, it could be possible to provide enough energy density to affect satellite behavior without violating the bounds from Supernova 1987a or conflicting with Tritium beta decay experiments. In the latter case, while the effect could extend for several tens of electron volts to either side of the end point, a small mixing angle would provide only a small Pauli blocking below and a concomitant small count rate above.

Whatever the detailed couplings might be, the effective mass of each of the mass eigenstates will be affected by the distributions of all three. This could, therefore, alter the usual analyses of the MSW effect in the sun and, since the neutrino densities associated with supernovae is so very large, could lead to very different effects there. Recent analyses of the abundances produced in a neutron rich exterior of supernovae [23] seems to favor an unusual spectrum of masses, which might be achieved, for effective masses, by the scalar field discussed here.

9 Conclusions

We have applied the techniques of Quantum Hadrodynamics to the study of a system of neutrinos interacting through a light, weakly coupled scalar boson. We have shown that, for a wide range of parameters, neutrinos will tend to condense into clouds, with dimensions the scale of the inverse boson mass. In fact, for parameters which cause no conflict with laboratory measurements, such clouds could easily be the right size and density to affect experiments on and around the earth.

We have shown that it is likely that any such condensation would have occurred before recombination and that the formation of neutrino clouds could form a natural seeding mechanism for the formation of hadronic objects on the scale of stars. Neutrino cloud formation, being a phase change, occurs very quickly, so these seeds are available at the earliest possible epoch for star formation.

The extension of this work to more than one neutrino flavor depends on the mass hierarchies of the neutrinos and the scalars as well as the details of the coupling of each scalar to different generations. For the case of two generations of neutrinos and one scalar field, we have looked at two simple choices for the couplings. While different in detail, both generate concentric spherical (assumed) distributions with the lighter neutrinos (as determined by vacuum mass) extending farther out. We would expect this general feature to survive for three generations, raising the possibility of the heaviest species being essentially within the star, the other two occupying different regions of space out to a distance, depending on the detailed history of the system, of a fraction of a parsec.

If the density of the electron component of the neutrinos and antineutrinos around the Sun is high enough, there could be observable effects on very sensitive experiments such as the study of Tritium beta decay to search for antineutrino mass effects or double beta decay measurements seeking evidence that neutrinos are Majorana particles.

One consequence of the existence of such an interaction would be that all such measurements would have to be interpreted in terms of effective masses. rather than the vacuum masses that are relevant to model building.

Whether terrestrial effects are observed or not, evidence for or against the existence of such a scalar interaction is most likely to come from astronomy and astrophysics. The implications of the existence of neutrino clouds, with respect to both the time scale and the distribution of sizes, should be amenable to testing through modelling and observation. The gravitational effects within our own Solar system, while subtle, could be observable in very high accuracy satellite tracking data. Depending on the precise model for several generations. one may be able to observe the modifications of oscillation and propagation in the extremely dense neutrino fluxes associated with supernovae.

Whatever the experimental outcome of such tests may be, this problem remains as a fascinating extension of the theoretical techniques of QHD, developed for the study of atomic nuclei, to vastly different regions of parameter space.

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Figure Captions

Figure 1. Scaled energies vs. scaled Fermi momentum for infinite matter at zero temperature, $K_0 = 20$. The energies and the Fermi momentum are given in units of the neutrino vacuum mass, $m_\nu^{(0)}$, K_0 is defined following Equation (7). The dashed curve is the Fermi energy and the solid curve is the average total energy per neutrino, which includes the contribution of the static scalar field, $e = e_\nu + e_s$.

Figure 2. Scaled total energy per neutrino vs. scaled Fermi energy for infinite matter at zero temperature, $K_0 = 20$.

Figure 3. Scaled total energy per neutrino vs. scaled Fermi energy for infinite matter at zero temperature for several small values of K_0. For values of $K_0 > 3.27$, the system is self-bound. For $2.67 < K_0 < 3.27$ there is a local minimum above $e = 1$, allowing for the possibility of metastable states.

Figure 4. Number density ρ vs. scaled radius for three different sized clouds for $K_0 = 20$. In each case e_F was chosen, a solution was obtained as described in the text and the total number N was obtained by integrating the density. The radius is given in units of $1/m_s$; the number in units of μ^{-3}. For the solid curve, $e_F = .74$ and $N = 2.881$; for the dashed curve, $e_F = .76$ and $N = .851$; and for the dot-dashed curve, $e_F = .78$ and $N = .373$.

Figure 5. Gravitational force, in arbitrary units, for the three cases shown in Fig. 4. f_g at z is given by $\frac{1}{z^2} \int_0^z e\, z^2\, dz$.

Figure 6. Dependence of the average energy per neutrino and of the radius on the number of neutrinos in the cloud, shown for $K_0 = 200.$ on a log-log plot. The open squares are the values obtained for the logarithm of the difference between e for the cloud and its value for infinite matter; the solid line has slope $-1/3$. The stars are values of the logarithm of the radius at which the neutrino density goes to zero; the dot-dash line has slope 1/3. The filled circles are the values of the e_F associated with each calculation, the dotted curve is just a guide for the eye.

Figure 7. Allowed regions of the μ - $\tilde{\alpha}$ plane. The horizontal lines indicate upper limits on $\tilde{\alpha}$ obtained from the labeled processes as discussed in the text. The solid vertical line corresponds to the value of μ for $m_s \simeq 1.3 \times 10^{-18}\, eV$, appropriate to a range of $1AU$, and $m_\nu^0 = 10eV$. The dashed vertical lines correspond to changing μ by two orders a magnitude each way. The diagonal lines represent constant K_0, the lower for $K_0 = 4$ and the upper for $K_0 = 40,000$.

Figure 8. Densities and gravitational force for two neutrino mass eigenstates with $g \propto m$. For this example, $\frac{m_l^0}{m_h^0} = \frac{1}{2}$ and $K_0 = 20$ for the light neutrino. The four cases keep N_h approximately constant and vary N_l. The coupling of both densities to the scalar field requires e_{F_h} to vary, hence the approximate equality. Case A: $N_h = .040$, $N_l = 0$, and $e_{F_h} = .75$. Case B: $N_h = .042$, $N_l = .011$, $e_{F_h} = .725$ and $e_{F_l} = .825$. Case C: $N_h = .040$, $N_l = .027$, $e_{F_h} = .7$ and $e_{F_l} = .93$. Case D: $N_h = .041$, $N_l = .051$, $e_{F_h} = .675$ and $e_{F_l} = .975$. For each case, the long-dashed line gives the density for the heavy neutrino, the dot-dashed line gives the density of the light neutrino and the solid line gives the gravitational force due to the total distribution calculated as for Fig. 5. While the units of the gravitational force are arbitrary, the relative strengths scale properly.

Figure 9. Densities and gravitational force for two neutrino mass eigenstates with constant coupling to the scalar field. $K_0 = 400$ for the heavy neutrino and $\frac{m_h^0}{m_l^0} = .1$ The curves have the same meaning as in Fig. 8. $e_{F_h} = .7$, $e_{F_l} = .85$, $N_h = .000339$ and $N_l = .00162$.

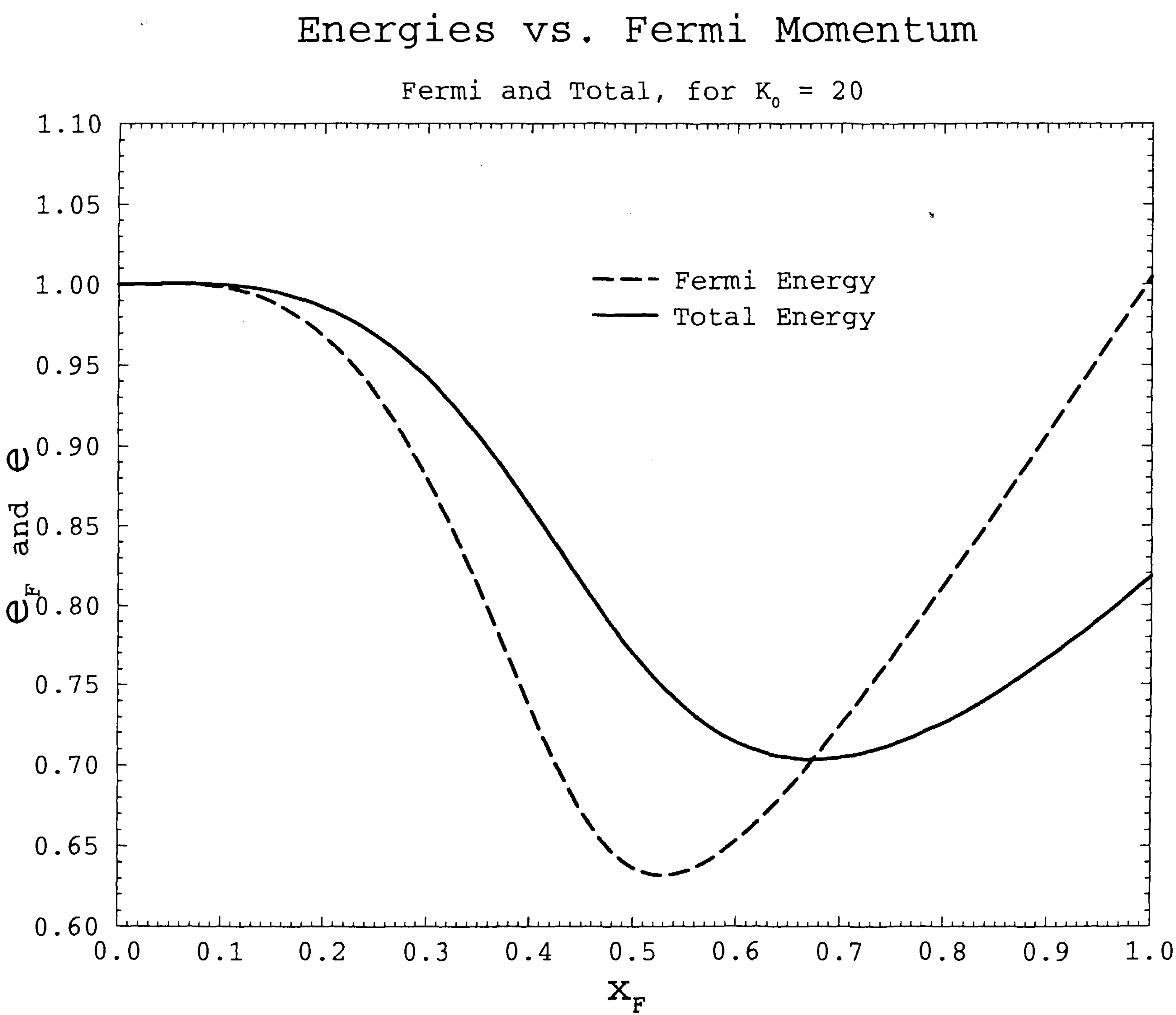
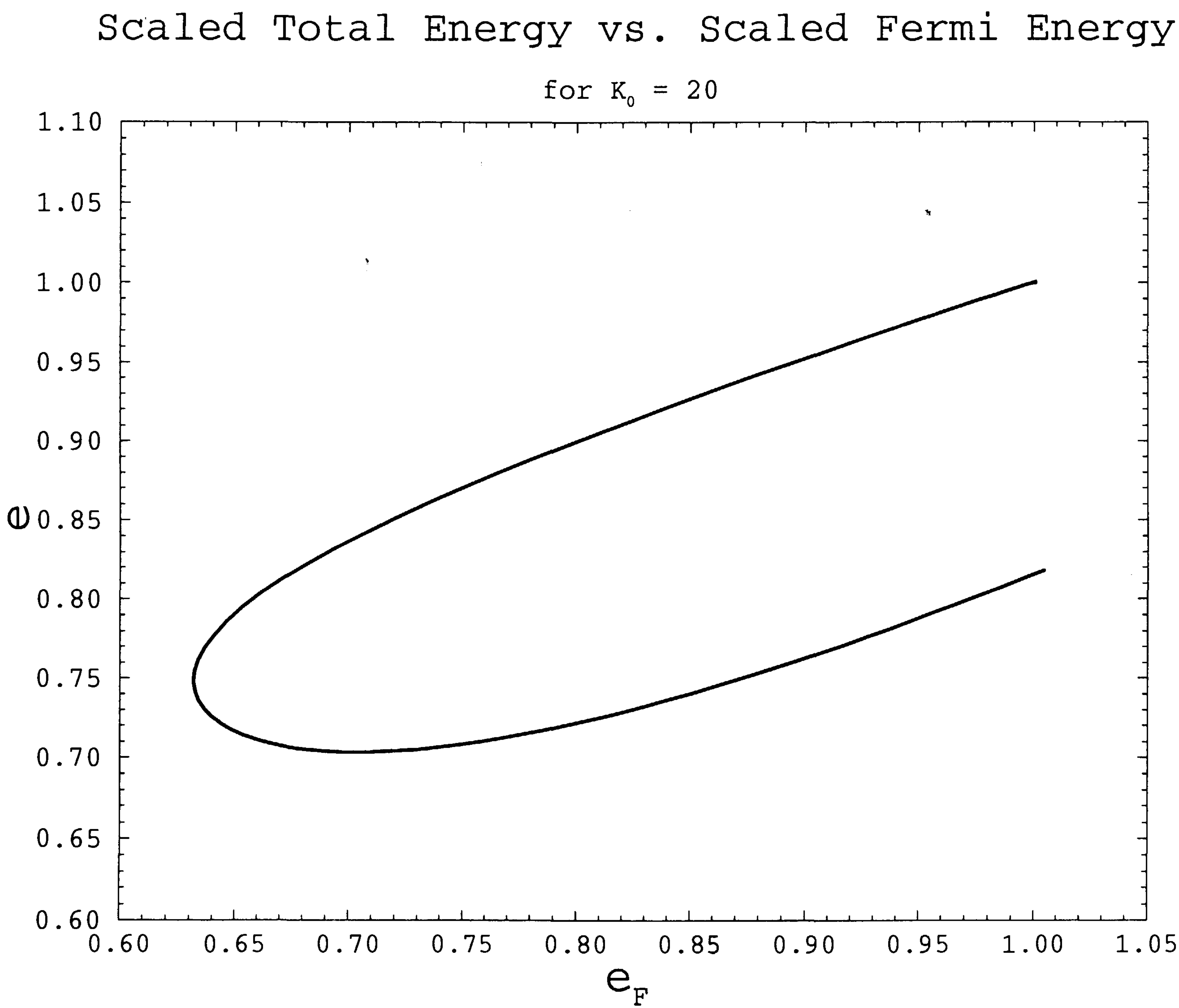
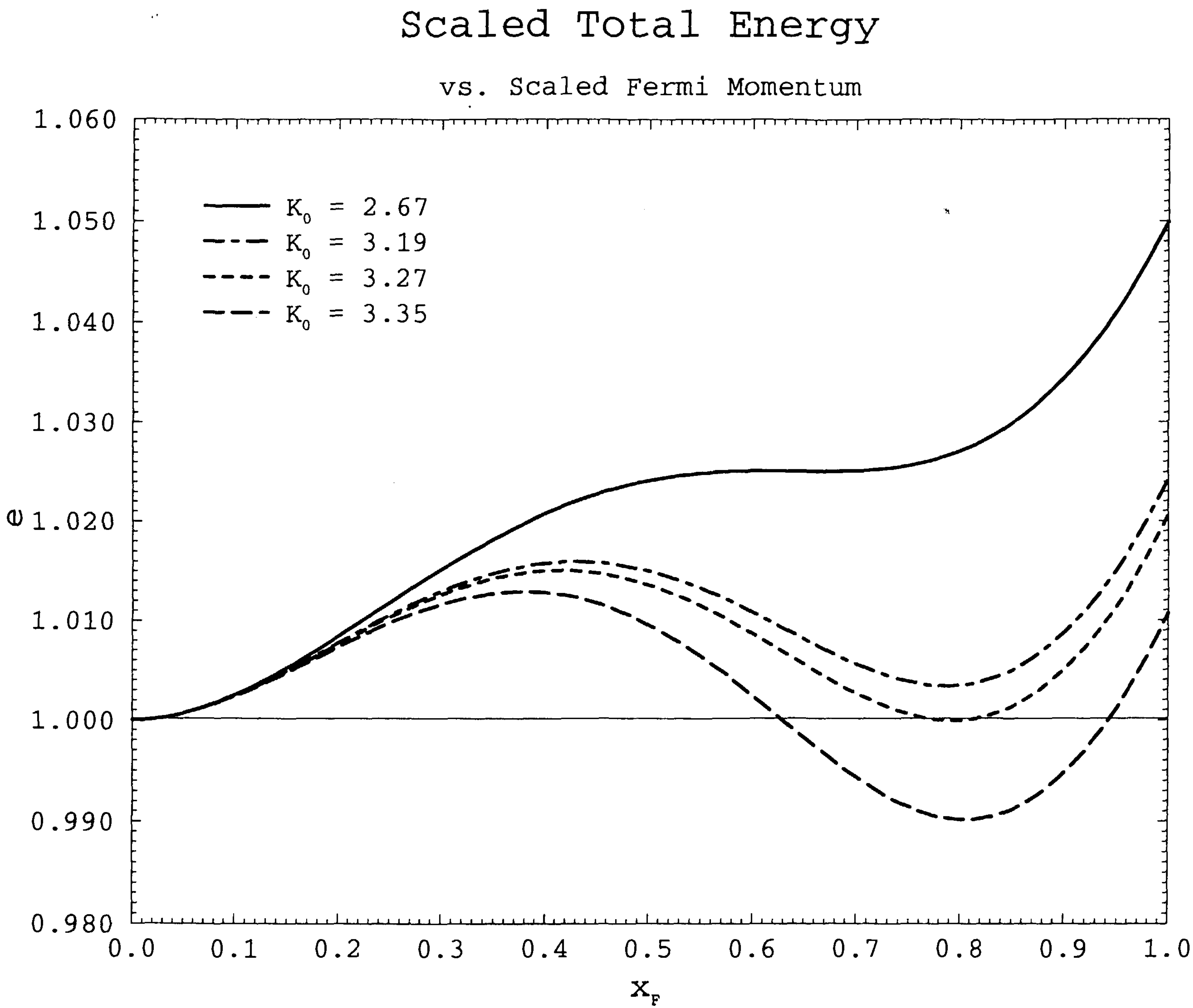

Energies vs. Fermi Momentum
Fermi and Total, for K_0 = 20
Fermi Energy
Total Energy
e_F and e
x_F
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 1



Scaled Total Energy vs. Scaled Fermi Energy
for K_0 = 20
1.10
1.05
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
e
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
1.05
e_F

Figure 2



Scaled Total Energy
vs. Scaled Fermi Momentum
K0 = 2.67
K0 = 3.19
K0 = 3.27
K0 = 3.35
e
1.060
1.050
1.040
1.030
1.020
1.010
1.000
0.990
0.980
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
xF

Figure 3

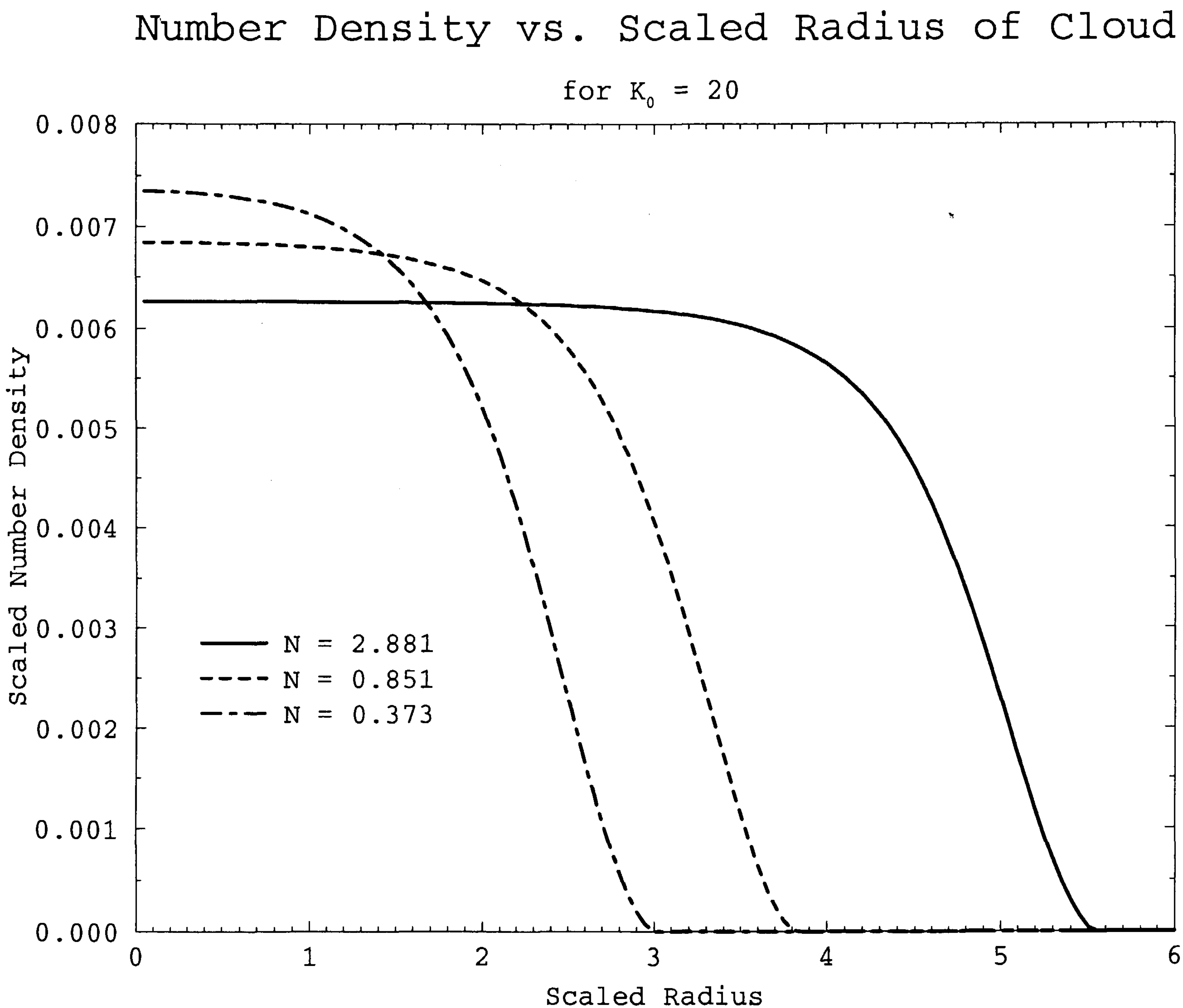

Number Density vs. Scaled Radius of Cloud
for K_0 = 20
Scaled Number Density
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
N = 2.881
N = 0.851
N = 0.373
0
1
2
3
4
5
6
Scaled Radius

Figure 4

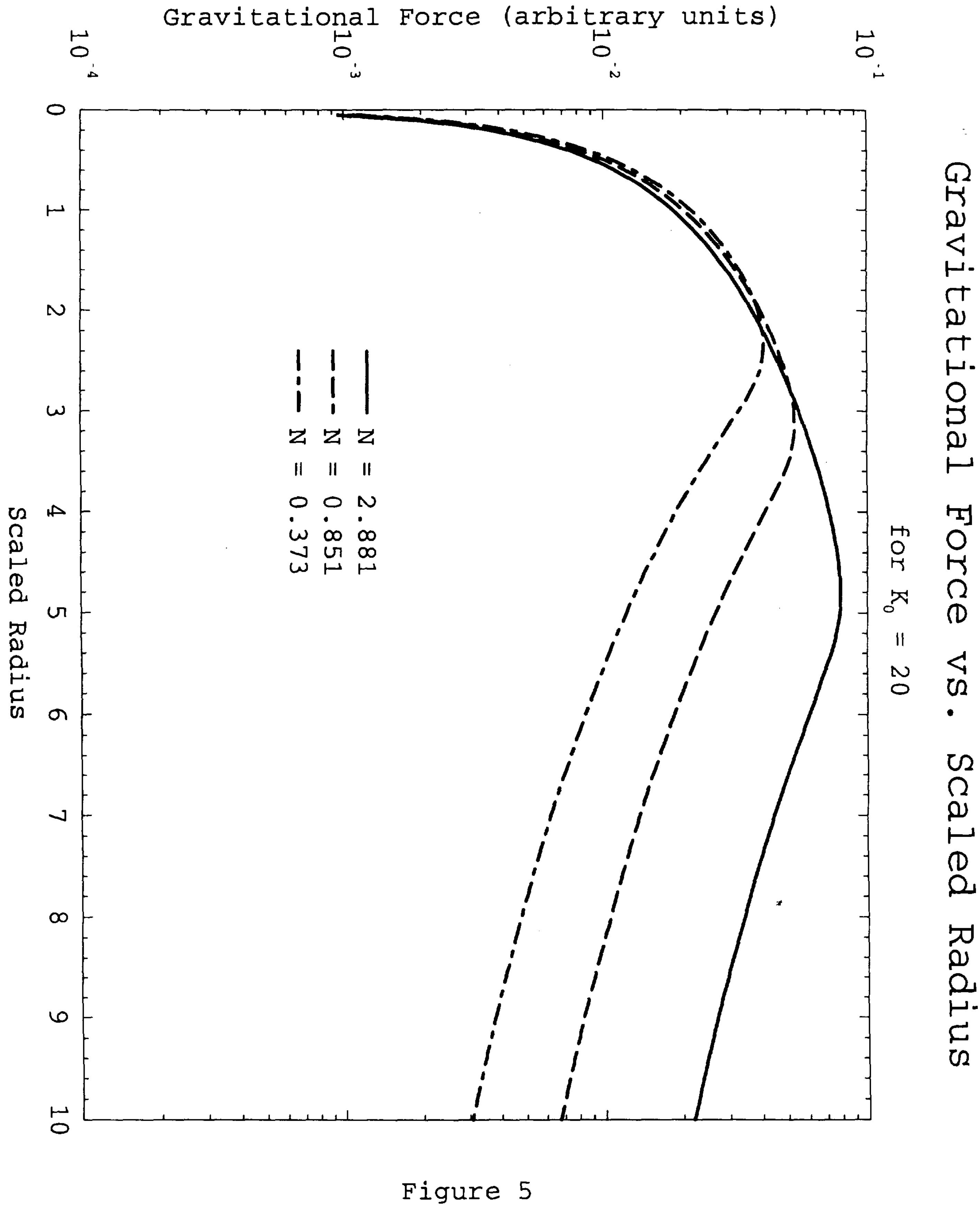

Gravitational Force vs. Scaled Radius
for K0 = 20
Gravitational Force (arbitrary units)
10-4
10-3
10-2
10-1
Scaled Radius
0
1
2
3
4
5
6
7
8
9
10
N = 2.881
N = 0.851
N = 0.373

Figure 5

Size and Neutrino Energy for Finite Clouds
vs. Total Number of Neutrinos (for $K_0 = 200$)

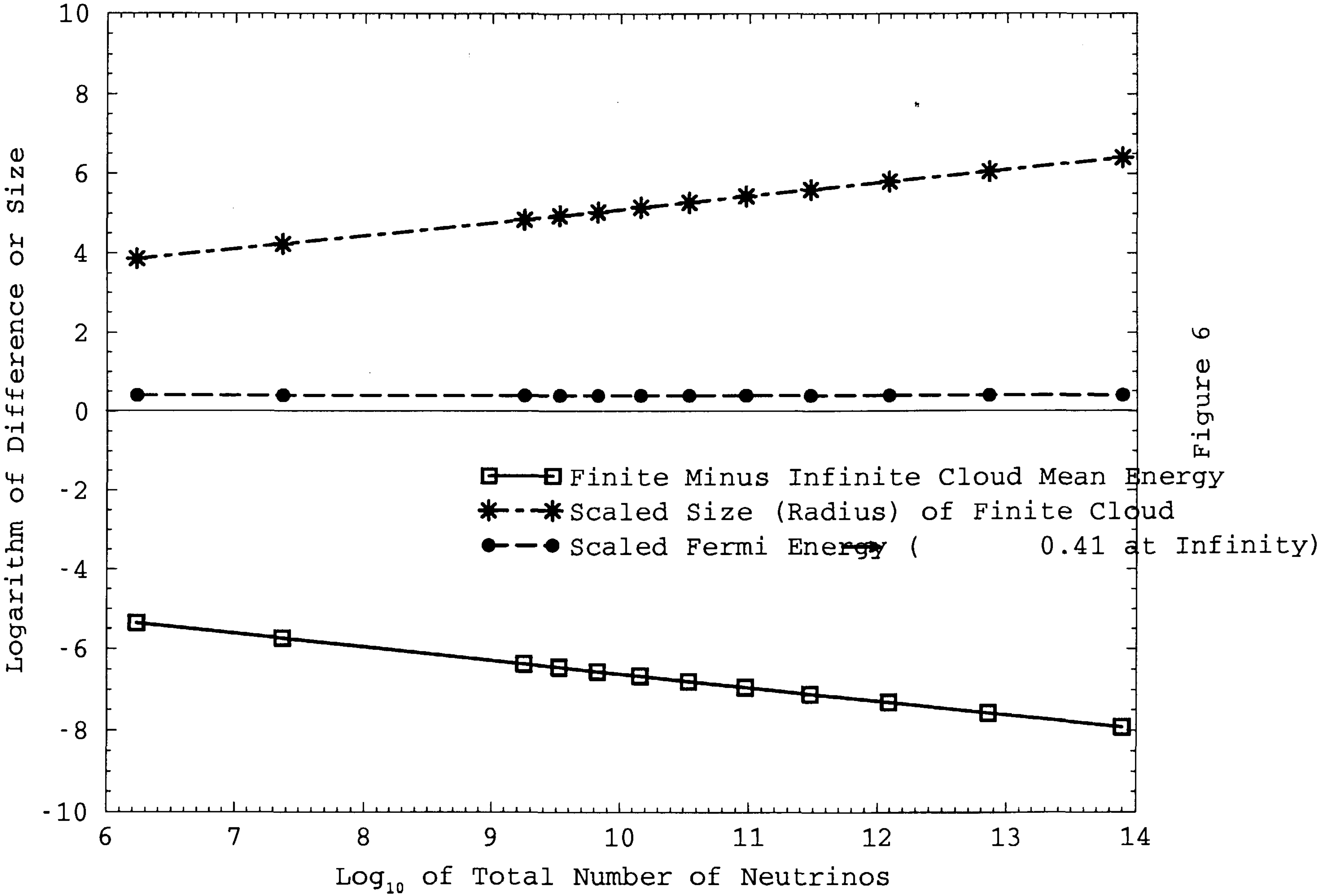


Figure 6

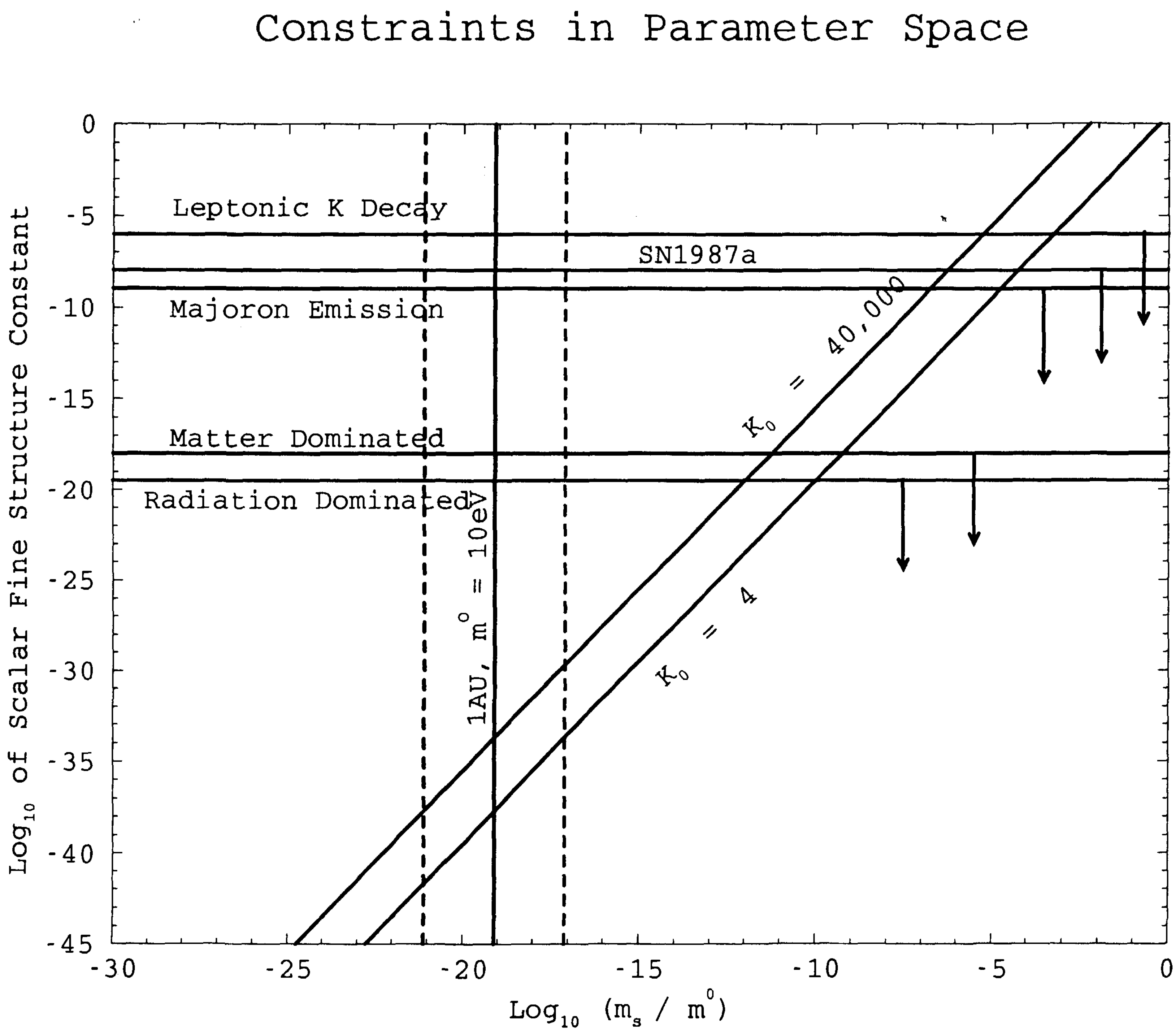
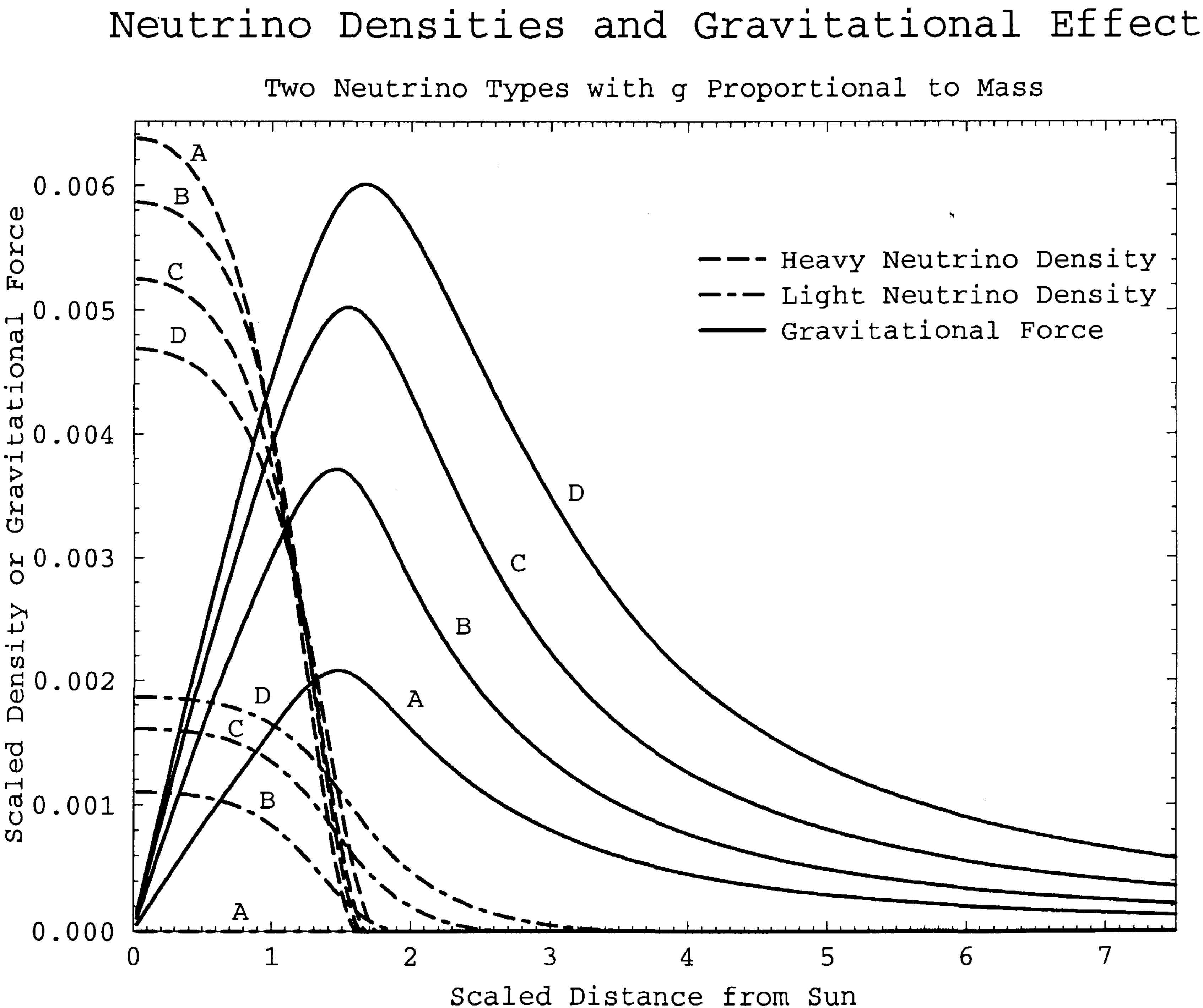

Constraints in Parameter Space
Leptonic K Decay
SN1987a
Majoron Emission
Matter Dominated
Radiation Dominated
1AU, m° = 10eV
K₀ = 40,000
K₀ = 4
Log₁₀ of Scalar Fine Structure Constant
Log₁₀ (mₛ / m⁰)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-30
-25
-20
-15
-10
-5
0

Figure 7



Neutrino Densities and Gravitational Effect
Two Neutrino Types with g Proportional to Mass
Heavy Neutrino Density
Light Neutrino Density
Gravitational Force
Scaled Density or Gravitational Force
Scaled Distance from Sun
0.000
0.001
0.002
0.003
0.004
0.005
0.006
0
1
2
3
4
5
6
7
A
B
C
D

Figure 8

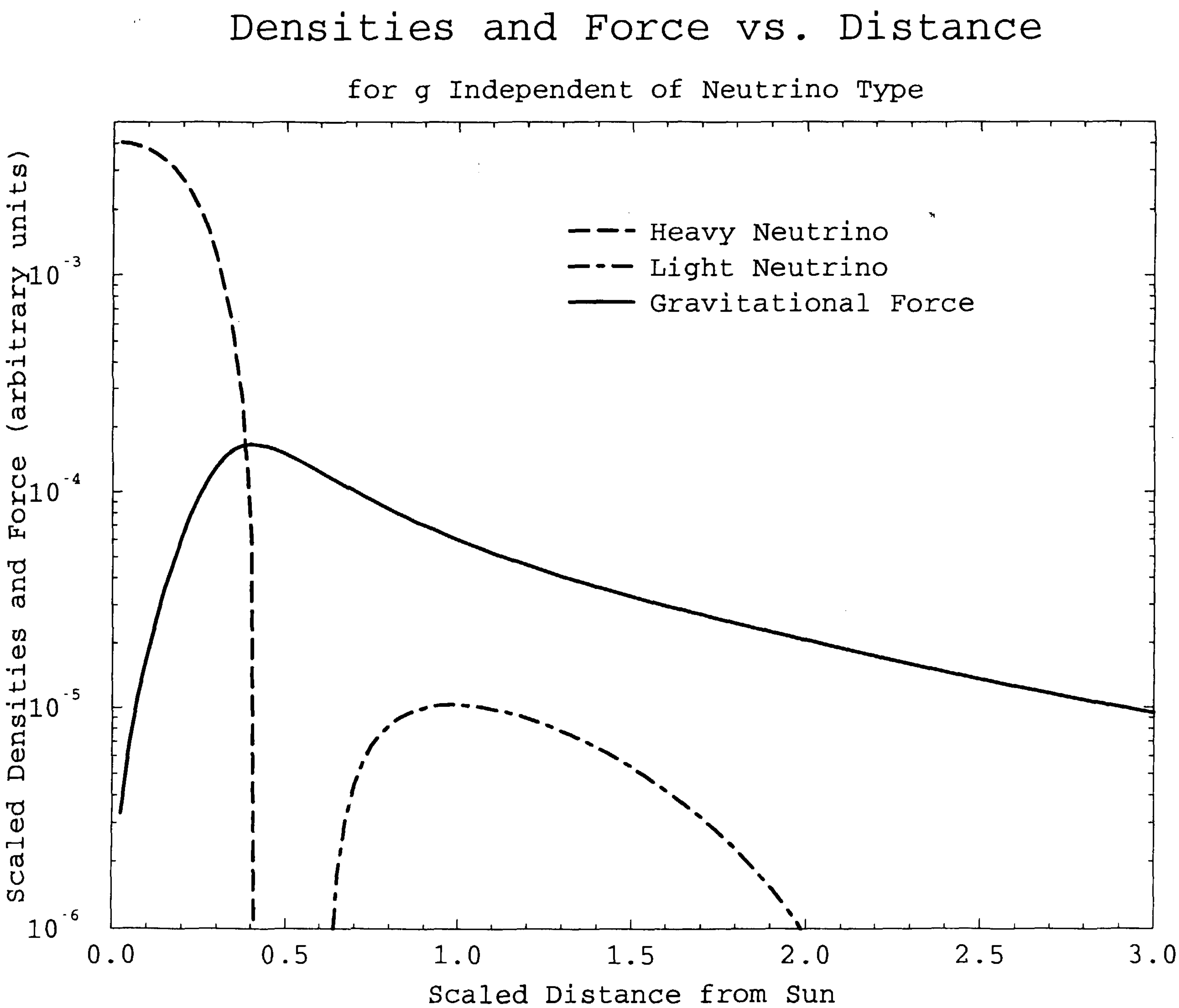

Densities and Force vs. Distance
for g Independent of Neutrino Type
Heavy Neutrino
Light Neutrino
Gravitational Force
Scaled Densities and Force (arbitrary units)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Scaled Distance from Sun

Figure 9