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INTRODUCTION

During the multifragmentation of atomic nuclei it seems that identical (or almost identical) initial conditions are leading to very different partitions of the system in interaction. In such a case it is necessary to develop approaches that are able to describe the observed diversity of the final channels. On the other hand the multifragmentation being characterised by the formation of relatively large fragments, one may think that the mean field plays an important role to organise the system in nuclei. Indeed, the mean field (ie, the long range part of the bare nucleon-nucleon interaction) is at the origin of the cohesion of the clusters. Moreover, it has been shown that extentions of mean-field approaches including a Boltzmann-like collision term were providing excellent descriptions of many aspects of heavy ion reactions around the Fermi energy (see for example ref.[1] and references therein).

The problem with mean-field approaches is that they are unable to break spontaneously symmetries. Therefore, they cannot describe phenomena where bifurcations, instabilities or chaos occurs. However, since few years, many tests and studies have been reported showing that the stochastic extensions of mean-field approaches were good candidates for the description of such catastrophic processes. Indeed, the presence of a source of stochasticity allows to explore a large variety of evolutions. Therefore, such approaches may provide valuable descriptions of the dynamics of phase transitions (at least in the case of first-order phase transitions for which the mean-field is known to give a reasonable description of equilibrium properties).

We will see that this description of the dynamics of first order phase transitions in infinite and finite system is now partially achieved. An important conclusion is that in some specific cases we have shown that well-defined collective motions were initiating the self-organisation of the unstable matter in fragments. We will see that in the case of finite systems the possible signals kept from this early fragmentation stage can inform us on the possible occurrence of a liquid-gas phase transition in nuclei.

1

PHASE DIAGRAM AND SPINODAL INSTABILITIES

Let us first recall that nuclei are understood as drops of a Fermi liquid [2, 3, 4] and, since we can also observe free nucleon gas, we expect the existence of at least one liquid-gas phase transition.

As matter of fact, the nuclear forces are known to have a long-range attractive tail and a short-range repulsive hard-core and so to be analogous to a Van der Waals interaction. Therefore, we expect the same phenomenology as far as phase transitions are concerned.
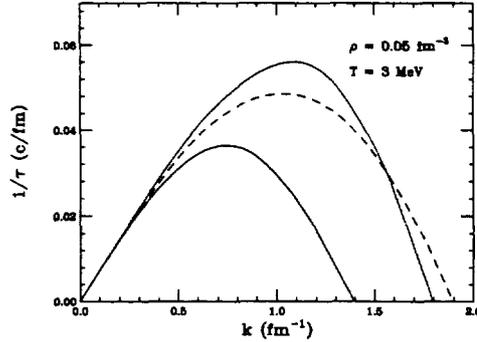
Figure 1. RPA dispersion relation obtained using a realistic effective force (solid line) compared with the semi-classical result (dashed line). The RPA prediction computed for an equilvalent zero range interaction is also shown (dotted-dash line). (Extracted from ref.[7])

The phase diagram of nuclear matter is still partially unknown because it is very difficult to extract unambiguous information from the experimental observation. Indeed, it is only possible to create in laboratory tiny short-lived fragments of excited matter. However, it is generally believed that during a nuclear collision the system may explore a large portion of the nuclear phase diagram and that the observed copious fragment production might be related to a liquid-gas phase transition.

In particular, it is generally believed and shown by one-body approaches that the composite system formed during a collision may enter deep in the liquid-gas coexistence region and even in the spinodal zone that is the region where the system is mechanically unstable against infinitesimal density fluctuations. Considering the involved size and time scales this is certainly a region adequate for the nuclear multifragmentation.

In the spinodal region the density fluctuations are spontaneously amplified. This region can be investigated by studying the evolution of small density undulations. In particular, we can study the linear response of the considered system. In the RPA approach, instabilities corespond to modes with an imaginary eigenfrequency (see for example references.[5, 6, 7]) This frequency is the inverse of the instability growth time.

Figure 1 presents an example of RPA result extracted from reference.[7] This calculation is performed for the infinite nuclear matter at 3 MeV temperature and at about one third of the saturation density. This figure presents the imaginary RPA frequencies as a function of the wave number k of the considered perturbation. The slope at the origin is the imaginary zero sound velocity as predicted by the Landau theory of Fermi liquids.[8, 9, 10] The most important characteristic of this dispersion relation is the fact that it presents a strong maximum at a given wave number followed by an ultraviolet

2

cutoff after which the modes are stables. These properties are directly related to the range of the nuclear force and to the nucleon zero-point motion (see ref. [7]). This can be easily understood: when the wave length of the oscillations is shorter than the range of the attractive part of the potential or than the zero point motion of the nucleons it will be washed out in the mean-field and so the corresponding fluctuations will not be amplified.

The most unstable wave length appears to be around 10 fm with an instability time around 30 fm/c. It should be noticed that this wave length does not evolve much in the spinodal region except nearby the spinodal boarder where it rapidly diverges. In fact the limit of the spinodal region corresponds to infinite instability time associated with an infinite wave length. From this point of view, this limit has no real meaning for evolution of a small system over a finite time. Indeed, such a small system cannot wait an infinite time for the fluctuations to develop and cannot afford infinite wave length because of its limited dimensions.

## SPINODAL INSTABILITIES IN FINITE NUCLEI

We can now study the fragmentation of a hot and diluted nucleus lying deep inside the spinodal zone of instabilities. We have considered masses, charges, densities, temperatures, spins, expansion,... as predicted by one-body dynamic approaches.[11] To describe the spontaneous symmetry breaking associated with the fragmentation of hot spherical sources, we have used the recently developed stochastic approaches.[12]
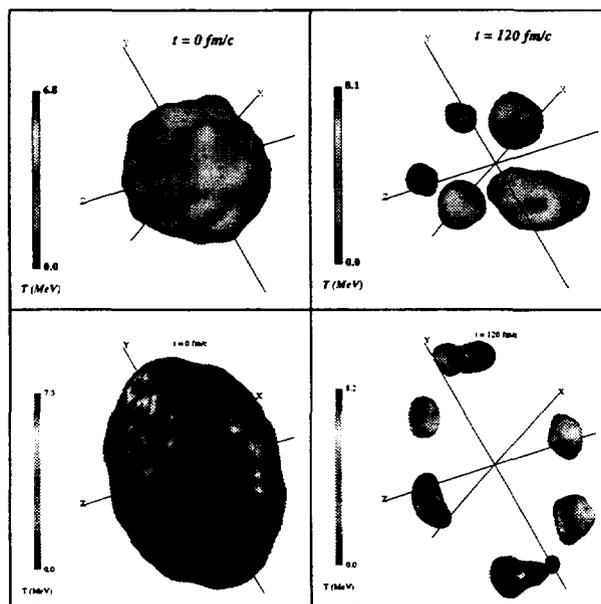
Figure 2. Stochastic mean-field evolutions of two different sources: Part a) (top) Spinodal decomposition of a spherical source; Part b) (bottom) of a disk (from ref.[13]).

Figure 2.a presents one of the many predicted partitions of a large hot and diluted nucleus containing 210 nucleons that have been fragmented in 5 pieces under the influence of spinodal instabilities. This is a rather typical event (the average multiplicity computed over 400 simulation being sharply peaked on the production of 5 fragments).

This dominant multiplicity is clearly linked to the occurrence of spinodal instabilities since this is topologically the optimum way to have undulations, in a finite system, with an average distance between density lobes compatible with the most unstable wave length of 10 fm.
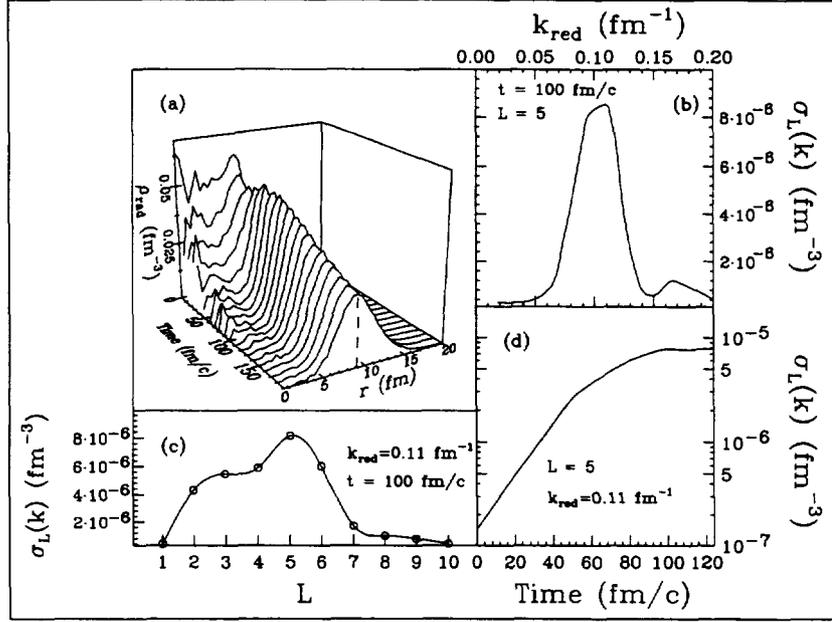
Figure 3. Analysis of the multifragmentation events analogous to the one presented in figure 2a). The part a) presents the radial projection of the densty, part b) multipole expansion computed at 100 fm/c, part c) analysis in terms of Bessel functions for the multipole L=5; part d) time evolution of the L=5 fluctuations.

This characteristic is better seen on the analyses presented in figures 3. On the first hand, we observe in figure 3.a radial oscillations associated with a wave length around 10 fm. In average the matter gets concentrated close to the surface and a hole is produced at the centre of the nucleus.

In order to get a deeper insight into the characteristic of the unstable modes in finite nuclei we have performed an analysis in terms of spherical harmonics projecting the fluctuations on the modes defined by $j_L(kr)Y_{LM}(\hat{r})$. This analysis shows that these projections are strongly peaked at wave numbers that, taking into account the L dependence of the Bessel function, correspond to a distance between two radial maxima around 10 fm. Moreover, this strong peaking of the k-dependent projection demonstrates that the actual fluctuation resembles to a Bessel function, i.e., to the multipole expansion of a plane wave with 10 fm wave length.

On the other hand, if for this k_L of maximum instability we draw the L-dependence of the measured fluctuation (figure 3.c) we can observe that the multipolarity 5 dominates the instabilities. This multipolarity L=5 corresponds to a distance of about 10 fm between two maxima of density fluctuation. As a matter of fact, this multipolarity L=5 induces the fragmentation of the system in 5 equal pieces. For this multipolarity the most unstable radial k corresponds to $k = 0.6 fm^{-1}$ that is the most unstable wave number of the infinite matter system at the considered temperature and density. Finally, on figure 3.d) we can clearly see the exponential amplification of unstable modes

4

with a characteristic growth time $\tau \approx 35 fm/c$ in agreement with the nuclear matter calculation.

In conclusion we have seen that a large enough finite system is developing instabilities very close to the one predicted for the equivalent infinite nuclear matter. Therefore, one may hope that studying the spinodal decomposition of finite system may directly provide information on the nuclear equation of state.

SPINODAL INSTABILITIES AND THE GENESIS OF NON-COMPACT GEOMETRIES.

It might be important to study the role of the initial shape of the source on the final topology of the fragment partition. On figure 2 we show the comparison of the predicted disassembly of two different initial sources; the first one being spherical and the second one prolate with an aspect ratio 2:1. In both cases the system develops radially a spinodal instability. Making a hole at the centre is the only possible way to develop the instabilities considering the limited size of the system. Therefore, the spinodal decomposition of both systems is generating non compact geometries with a hole in the centre. These fragmentations resemble to bubble-like or torus-like topologies but it should be notice that in the present simulations they only arise from spinodal instabilities. Such non-compact topologies might have been observed experimentally by the MSU group and the Nautilus group in GANIL.

PARTITIONS OF NUCLEI DUE TO SPINODAL INSTABILITIES AND COMPARISON WITH EXPERIMENTS.

Using the stochastic mean field approaches we can now directly compute the various characteristics of the multifagmentation events such as the various partitions and the fragment velocities. For example figure 4 shows the primary charge distribution computed right after the multifragmentation of the system shown on figure 3.
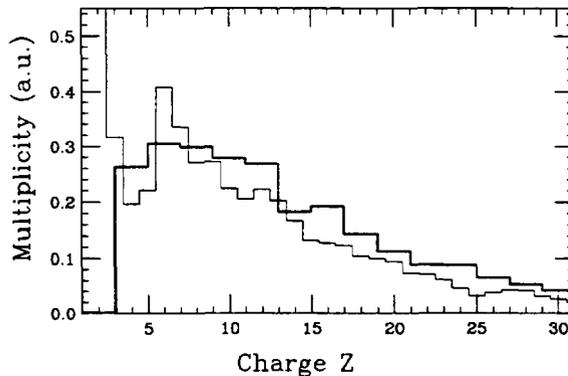
Figure 4. Fragment mass distribution associated with the simulations shown on figure 2 and 3. Both the primary (thick solid line) and the secondary (thin line) distributions are shown.

One can observe that the primary distribution is strongly influenced by the properties of spinodal instabilities. Indeed, the small wave length being stable the spinodal

5

instabilities are unable to produce small clusters. Conversely, the dynamics is dominated by the most unstable wave length around 10 fm and consequently the primary charge distribution is peaked around the Ne-like nuclei. On the other hand, the large scale behaviour is not affected by the existence of an intrinsic range and one can observe a large mass tail coming from i) the beating of different modes ii) the presence of large wave length instabilities iii) and the coalescence effects due to the residual interaction between fragments before the complete disassembly of the system.

In this approach, the fragments are formed excited. Starting with a 3MeV initial temperature the fragment temperature appears to be around 4MeV. This temperature may seem rather low compared with initial available energy in the reactions leading to the formation of the dilute system we have considered. However, it should be noticed that this initial energy has been spend in pre-equilibrium particle emision, in surface energy, in expansion velocity and in residual excitation energy... Since the fragments are still hot, the desexcitation phase must be taken into account prior to any comparison with experimental observations. We will see that the characteristics of the initial fragments are not completely washed out by the secondary decay process. This is because the fragments are decaying through the emission of few particles leaving behind a residue with a mass close to original one (see the final fragment distribution on figure 4).

In conclusion we can observe that the spinodal decomposition is characterised, in a finite system, by the occurrence of typical scales associated with the wave length and the growth time of the most unstable modes in nuclear matter. An expected consequence is the existence of a bias in the fragmentation patterns favouring partition in approximately equal mass pieces with a lack of primary small fragments. Moreover, our simulations of the spinodal decomposition seem to indicate that the above signals are partly preserved by the secondary decay step.

An other characteristic is given by the time needed to form fragments from an already diluted source (to have the total multifragmentation time one must then add the time needed to reach the spinodal zone). The instability times are predicted to be around 30 to 50 fm/c therefore about 100 fm/c are needed to get separated fragments. These short time scales are also an indication of the presence of instabilities.

PARTITIONS OF NUCLEI DUE TO SPINODAL INSTABILITIES AND COMPARISON WITH EXPERIMENTS.

Some experimental data are already pleading in favour of the spinodal decomposition scenario for the time scales, for the favoured partition in equal mass fragments and even for the quenching of small fragment production. However, before entering this discussion, we would like to stress that it might be premature to conclude about the fact that some multifragmentation reaction might be related to a spinodal decomposition because both the experimental results might still present some ambiguities and because it may be that modified versions of other theoretical models describing the multifragmentation without involving spinodal instabilities, might also explain the experimental data. However, the fact that the composite system should enter the spinodal zone is predicted by almost all the one-body approaches and we will see that our stochastic mean-field simulations of subsequent spinodal decomposition are able to describe correctly many aspects of multifragmentation in central events.

6

First, let us discuss the results obtained for the Xe + Cu reaction at 45 MeV per nucleon.[14] Using kinematical constraints the authors of these articles were able to select central reactions for which they observed that 3 fold events are dominated by partitions of the composite system in close mass fragments. For example, figure 5 presents a Dalitz plot obtained in the experiment which shows that the three emitted intermediate mass fragments have approximately identical masses. The total charge distribution of the three fragments is also plotted in the same figure. It is strongly peaked around Z=35 demonstrating that the 3 fragments have individual charges around 12 in average.
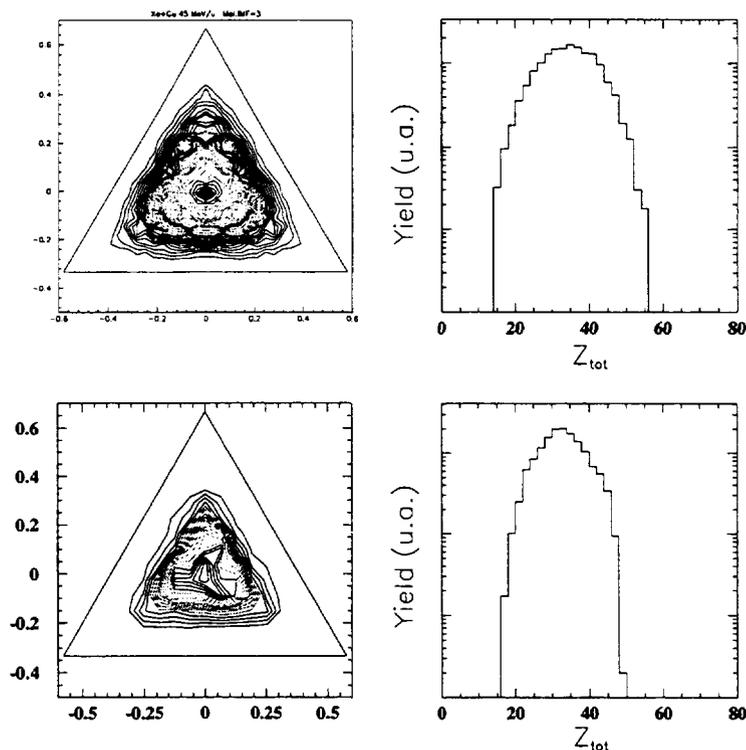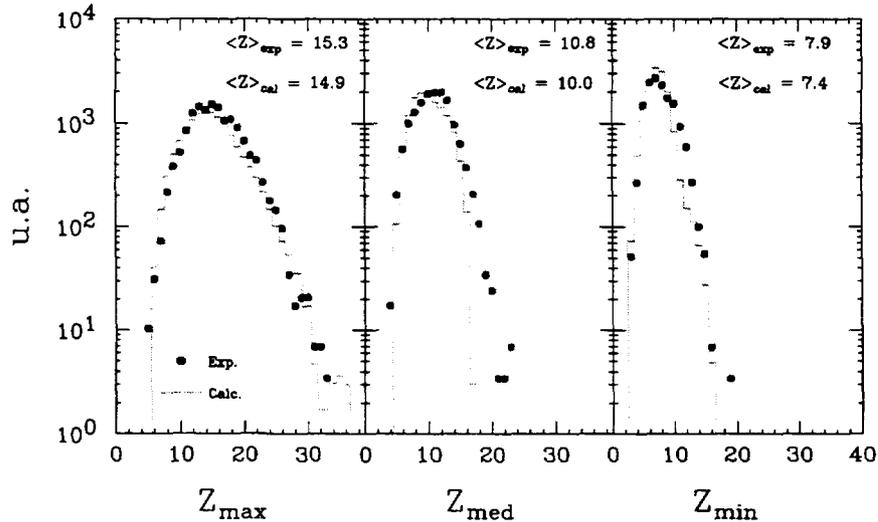

Figure 5. Comparison between the experimental results of reference[14] and our stochastic mean-field simulation for the three-fold events. Left part Dalitz representation of the charge partition. The distance to the side of the equilateral triagle are proportionnal to the relative charges of the 3 fragments; right part, distribution of the summed charge of the three fragments.

These features are well reproduced by our theoretical calculation performed as follow: i) the reaction is first treated within a standard one-body approach using the BUU code based on a lattice hamiltonien method as described in ref.[15]; This calculation is performed until the system runs across instabilities; From this point and on the mean field loose its validity and one should take into account correlations and fluctuations; It should be noticed that during the first stage of the reaction the inclusion of the fluctuations was not crucial because the mean field dynamics complemented with a collision term was representing a reasonable ensemble average; ii) The unstable dynamics is simulated using a stochastic mean-field approach as described in reference[12] which corresponds to the addition of specific noise to the BUU dynamics; This simulation is followed until fragments are formed; iii) Finally, when the fragments are formed they are still hot and their decay may take a very long time; this slow process is well described by statistical decay approaches; Therefore, instead of simulating this decay

7

within the mean-field approach, which does not predict correctly the particle and fragments emission, we prefer to use a statistical model. This part that includes both the fragments classical trajectory and the evaporation process is simulated using the code SIMON developed by D. Durand.[17]

Figure 5 also presents the theoretical prediction for the charge partition and the charge distribution. We can see that the theory explain the production of equal mass fragments with a total mass close to the experimental one. Only the width of the total mass distribution is underestimated by about 20 %. This might be related to fluctuations in temperature and size of the initial composite source that have not been taken into account in the actual theoretical calculation.


Figure 6. Chargedistributions of the three measured fragments. The dots are the experimental results from ref.[14] and the solid line the theoretical predictions.

The comparison can be performed in more details looking at the charge distribution of the ordered 3 fragments (see fig. 6). One can see that the theoretical predictions nicely reproduce the experimental distributions. The theoretical simulations are also able to reproduce the kinematic observables in particular the observed peaking at 120 degrees of the fragment relative angle that was understood as a confirmation of the partition of the system in 3 equal-mass fragments.

It should be noticed that all the other models of multifragmentation were unable to describe these experimental data (see ref.[14] for more details). Therefore, the success of our calculation with no fitting parameter can be seen as a strong indication about the validity of the proposed scenario.

We have also performed a comparison with the recent results of the INDRA collaboration[16] concerning events with the formation of a composite source in the Xe+Sn reaction at 50 MeV per nucleon. Indeed, also in this case our one body approaches are predicting the formation of a composite system diving deep in the spinodal region. Figure 7 presents the fragment charge distribution associated with these events while Figure 8 displays the individual charge distributions of the 3 largest fragments. One can see a rather good agreement between experiment and theory. In particular the tail at large Z is well reproduced by the theory. We would like to recall that this

8

tail is coming from both the mode beating and the final state interaction between fragments. On the other hand, the charge distributions of the 3 largest fragments are well reproduced both in centre position and in global shape (and width).
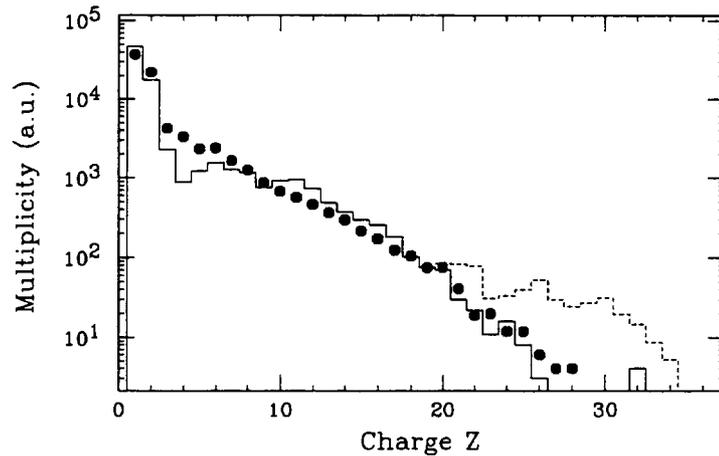
Figure 7. Fragment size distribution. The dots are the experimental results from ref.[16] and the solid line the theoretical predictions (filtered using the experimental selection, the dashed line being the result before the application of these cuts).
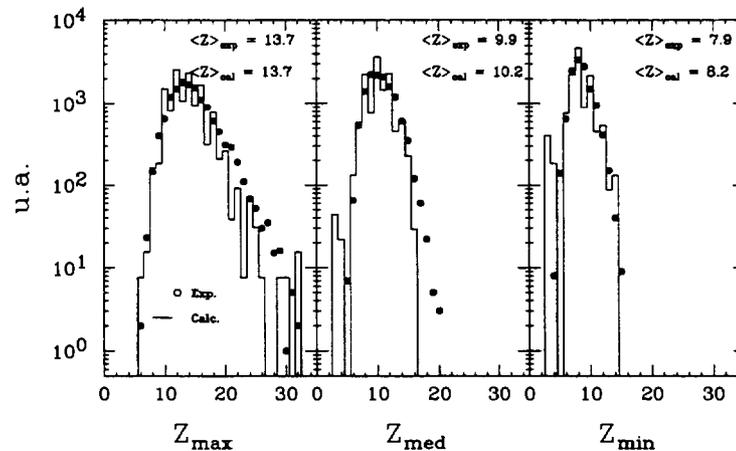

Figure 8. Same as figure 6 for the INDRA results.

In conclusion, while more studies are certainly needed to compare detailed characteristics of the multifragmention events with the spinodal decomposition scenario, the presented results are very encouraging. Stochastic mean-field approaches can be now applied for realistic simulation in 3D. These dynamic approaches are now able to compete with multifragmentation models and can be directly compared with experiments.

9

PRELIMINARY RESULTS OBTAINED USING A FAST VERSION OF THE FERMIONIC MOLECULAR DYNAMICS

An alternative way to address the problem of the multifragmentation of Fermi liquids is to consider molecular dynamics in which the antisymmetrisation of the wave function is explicitly taken into account.[18, 19, 20, 21] These approaches are based on a variational formulation of quantum mechanics complemented with the definition of an ensemble of parameterised trial wave functions. Often, these trial wave functions are nothing but Slater determinants built from gaussian wave packets. The parameters of these gaussians can be treated as classical degrees of freedom. These approaches are very appealing since they treat in an elegant way the problem of the antisymmetrisation and many applications have already been reported in the literature. However, these applications have been limited to small systems because of the numerical difficulties in the calculation of the two-body interaction.

We have developed a much faster approach based on the remark that, since the trial functions are a sub-set of the Slater determinants, i.e., of the independent-particle many-body wave function, the fermionic molecular dynamics can be seen as an approximate solution of the mean-field equations. Therefore, one can start directly with the variational formulation of the TDHF approximation using an effective force. In such a way, without any additional approximation, the numerical efforts are strongly reduced because of the introduction of the mean-field potential. In particular, the computation time just increases quadratically in the number of particles.
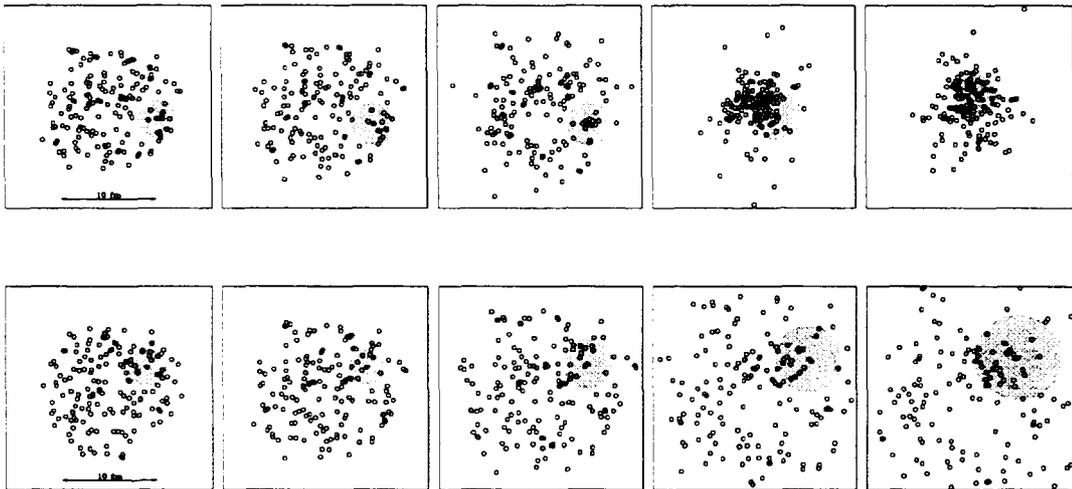
Figure 9. Fermionic molecular dynamics of large systems (A=160): Bottom part, excited at a total enegy of +2 MeV per nucleon, one can observe a total vaporisation of the system; Top part, with a total energy of -2 MeV per nucleon, in such a case a residue is formed evaporating particles. The center of each individual gaussian is represented. For one Gaussian also its width is shown. The time is evolving by steps of 25 fm/c from 0 to 100 fm/c going from the left to the right.

Figure 9 presents the first fermionic molecular dynamics simulations involving 160 particles.[21] In these simulations we have studied the evolution of a hot and diluted spherical system looking for a possible spinodal decomposition. However, we have only observed two types of behaviour i) either the global system is bound and the system will try to go back to the saturation density slowly evaporating particles; ii) either the

10

system is unbound and it will be soon vaporised.

The key of this amazing behaviour is found in the evolution of the width of the gaussians (see fig 9) that in our calculations are considered as dynamic variables. This width appears to increase when the system gets diluted so that it introduces an additional smoothing of the mean-field, washing out the spinodal instabilities and reducing the formation of fragments. In particular, the increase of the width reduces the interactions between particles and quenches the fragment formation. In the present stage of our understanding it seems that fermionic molecular dynamics without the width as a dynamic variable (i.e., with a fix width) might be a better approximation in order to treat fragments correlations. In particular, such a fix width calculation correctly converges towards classical molecular dynamics while because of the additional width parameter the full molecular dynamics seems to lead to a different phenomenology. This peculiar role of the width is now under investigation.

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