ON THE POSSIBLE DETECTION OF QUANTUM-MECHANICAL INTERFERENCES BETWEEN GRAVITATIONAL FORCES AND NUCLEUS-NUCLEUS COULOMB FORCES

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Abstract — We report on possible effects of quantum-mechanical interferences between gravitational forces and the nucleus-nucleus Coulomb interaction. We show that, although very small, these effects could be measured on using low energy scattering between identical heavy nuclei. For the system $^{208}Pb+^{208}Pb$ ($E_L = 5$ MeV). The angular precision needed would be about 0.001°.

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In a scattering experiment, the angular distribution is obtained by displacing a beam counter in a plane parallel to the Earth's surface and therefore the effect of the gravitational field on the scattered beam will be the same at all scattering angles. If instead, the detector is displaced in a plane perpendicular to the Earth's surface, the beams observed at different angles travel at different heights and this should give rise to a gravitational phase shift between them [1]. As a result, the combined effect of the Earth's attraction and the mutual interaction between the colliding particles will lead to a specific quantum-mechanical interference. In the case of nuclear Mott-scattering this interference produces a phase shift which can be expressed by a simple formula which juxtaposes the effects of the Newtonian potential, the Coulomb potential and quantum mechanics.

To study the interferences between the gravitational forces and the projectile-target force, we shall consider the scattering of two identical particles in a plane perpendicular to the Earth's surface. This is in fact a simple way to observe the interferences between two beams (direct and recoil) which travel at different heights between the collision point and the detector. So let us consider the scattering of two identical spin zero nuclei of mass m and incident energy in the Lab. system, $E_L = \frac{1}{2}mv^2$. We shall assume that the collision takes place in a quasi-classical regime [2]. Owing to the identity of the particles, the projectile scattered at angle θ interfere with the recoiling target at $\pi - \theta$. The scattering cross-section writes,

$$d\sigma/d\Omega = |f(\theta)|^2 + |f(\pi-\theta)|^2 + 2|f(\theta)||f(\pi-\theta)| \cos\psi(\theta) \qquad (1)$$

where $f(\theta) = (d\sigma^c/d\Omega)^{1/2} \exp\, i\alpha(\theta)$ and $\psi(\theta) = \alpha(\theta) - \alpha(\pi - \theta)$. The classical cross-section $d\sigma^c/d\Omega$ and the phase $\alpha(\theta)$ are determined [2] from the knowledge of the projectile-target interaction. If the scattering plane is perpendicular to the Earth's surface one must include in (1) the additional effect due to the Earth's attraction. It is worth noting that this will yield a cross-section which is no longer symmetric about $\theta = \pi/2$. The Newtonian potential acting on a particle of mass m at a distance r from the centre of the Earth is,

$$V_N = -GMm/r$$

where M is the mass of the Eearth and G the gravitational constant. Since the distances involved in a scattering experiment are negligible compared to the radius R at the Earth, one can write,

$$V_N = V_0 + V_g, \quad V_g = mgh$$

V_0 is the gravitational potential at some reference level above the surface of the Earth, h the vertical height above this level and $g = GM/R^2 \sim 980$ cm sec^{-2} the acceleration of gravity. The Newtonian potential is very small with respect to the kinetic energy of the incident beam, so that its effect on the beam trajectory, over the distances considered here, can be neglected. Therefore the classical cross-section $d\sigma^c/d\Omega$ is still determined by the projectile-target interaction alone. However since the two beams travel at different heights, this will produce a phase shift between them which may change significantly the interference term in (1). So, the corrected scattering cross-section is given by the same formula (1) except that the phase $\psi(\theta)$ is replaced by

$$\phi(\theta) = \psi(\theta) + \delta\phi(\theta)$$

where $\delta\phi(\theta)$ is the gravitational phase shift between the direct and the recoil beams. We shall calculate separately the phase shift arising from the scattered trajectories $\delta\phi_s(\theta)$ and the one arising from the incident trajectories, $\delta\phi_i(\theta)$;

$$\delta\phi(\theta) = \delta\phi_i(\theta) + \delta\phi_s(\theta)$$

Let us consider in a scattering plane perpendicular to the Earth's surface, two trajectories (in the C.M. system) of impact parameters b_θ (direct) and $b_{\pi-\theta}$ (recoil). The incident $\bar{k}$ vector is parallel to the Earth's surface. (See figure 1).

Let $s(\theta)$ be the distance covered by the scattered beam after the collision has taken place until its impact on the detector D. Owing to the Earth's attraction, the phase shift between the scattered trajectories deviated through the angles θ and $\pi - \theta$ is,

$$\delta\phi_s = -s(\theta)\delta p/\hbar$$

The small change δp in the momentum, $p = m(v/2)$, arises from the difference ΔV_g between the gravitational potential on each trajectory. If

$$f_\perp = mg|\cos\theta|$$

is the component of the Earth's force on the axis perpendicular to the beam trajectory, one has,

$$\Delta V_g = \Delta b f_\perp, \quad \Delta b = b_\theta - b_{\pi-\theta} \tag{3}$$

and from energy conservation one obtains

$$\delta(p^2/2m) = \Delta V_g \quad \text{or} \quad \delta p = 2\Delta V_g/v \tag{4}$$

where v is the incident velocity in the Lab. system. Finally from (4) one obtains,

$$\delta\phi_s = -\frac{1}{\hbar}\Delta V_g \Delta t, \quad \text{where} \quad \Delta t = 2s(\theta)/v = L/v\cos\theta/2 \tag{5}$$

L is the target-detector distance fixed by the experimental device and Δt is the time taken by a particle scattered in the Laboratory in the direction $\theta_L = \theta/2$ to cover the distance L. To obtain the total phase shift $\delta\varphi$, one should add to $\delta\phi_s$, the phase $\delta\phi_i$ arising from the height difference between the incident direct and recoil trajectories. The phase $\delta\phi_i$ is similarly given by

$$\delta\phi_i = -\frac{1}{\hbar}\Delta V_g \Delta t, \quad \text{where} \quad \Delta V_g = mg(b_\theta - b_{\pi-\theta})$$

and Δt is the time taken by the projectile to travel between the collimating aperture of the beam source and the target. Now we take advantage from the following facts :

i) $d\sigma^c/d\Omega$ is slowly varying so that the interference term of the scattering cross-section oscillates, in the forward and in the backward hemispheres, as, $\cos[\psi(\theta)+\delta\phi_i(\theta)+\delta\phi_s(\theta)]$ and $\cos[\psi(\pi-\theta)+\delta\phi_i(\pi-\theta)+\delta\phi_s(\pi-\theta)]$, respectively.

ii) Furthermore, both phases $\psi(\theta)$ and $\delta\phi_i(\theta)$ are antisymmetric, i.e., $\psi(\theta)+\delta\phi_i(\theta) = -\psi(\pi-\theta)-\delta\phi_i(\pi-\theta)$ so that the phase difference between the two interference terms reduce to :

$$\Delta\theta = |\delta\phi_s(\theta) + \delta\phi_s(\pi-\theta)| \tag{6}$$

iii) Since $\psi(\theta)$ is rapidly varying, $\delta\phi_s(\theta)$ remains almost unchanged over a period of the oscillations. Therefore, $\Delta\theta$ can be obtained by measuring the angular shift between the maxima (minima) of the oscillations in the forward hemisphere and those in the backward hemisphere.

As an application we shall now calculate $\Delta\theta$ in the case of low energy scattering of $^{208}Pb +^{208} Pb$. Below the Coulomb barrier, the elastic scattering reduces to the Mott-scattering. Furthermore the incident and the recoil beams move on Rutherford trajectories. This allow us to write,

$$\Delta b = b_\theta - b_{\pi-\theta} = d \cot\theta \tag{7}$$

where $d = 2\eta/k$ is the distance of closest approach and $\eta = (ze)^2/\hbar v$ is the Sommerfeld parameter. Inserting (7) in (3) one obtains from (5) and (6),

$$\Delta\theta = \sqrt{8}\,\frac{mgdL}{\hbar v}\cot^2\theta \sin\big(\frac{1}{2}|\theta - \frac{\pi}{2}|\big) \tag{8}$$

It is worth noting that in this expression three fundamental constants are inseparably linked : the gravitational constant G, the elementary charge e and Planck's constant h. As seen from (8) in order to get a sizeable effect of these "Newton-Coulomb interferences", the energy of the incident beam should be sufficently small. Unfortunately this choice turns out to be restricted by the period of the Mott-oscillations which behaves as $\delta_M \sim \frac{\pi}{\eta}\sin\theta$. Remember that $\Delta(\theta)$ is obtained by measuring the angular shift between the extrema of the Mott-oscillations in the forward hemisphere and those in the backward hemisphere. A compromise value could be $E_L = 5$ MeV. Taking for the target-detector distance $L = 10^3$ cm, one obtains for the three angles $\theta = 10°,\ 12°,\ 14°$ ($\pi - \theta = 170°,\ 168°,\ 166°$) respectively,

$$\Delta\theta \equiv |\delta\phi_s(\theta) + \delta\phi_s(\pi-\theta)| = 1.95\ 10^{-3}\ \text{deg},\ 1.32\ 10^{-3}\ \text{deg},\ .94\ 10^{-3}\ \text{deg}.$$

The periods of the corresponding Mott-oscillations are respectively,

$$\delta_M \sim 4.6\ 10^{-3}\ \text{deg},\ 5.5\ 10^{-3}\ \text{deg},\ 6.5\ 10^{-3}\ \text{deg}.$$

So, an angular accuracy somewhat smaller than 0.001° would be needed for $\theta < 14°$ ($\pi - \theta > 166°$). The determination of scattering angles to such an accuracy has already been achieved at GANIL in a measure of Mott-scattering in the $^{208}Pb +^{208} Pb$ system [3]. This has been obtained with a target-detector distance $L = 6\ 10^2$ cm. We therefore believe that a somewhat larger distance L will improve the angular resolution enough to allow an accurate measure of $\Delta\theta$. Therefore, what are the conditions necessary to obtain larger values for both the gravitational shift and the Mott-period. Roughly speaking one can say that both $\Delta\theta$ and δ_M are multiplied by about the same factor x if one devides the mass of the nuclei by this factor and simultaneously the incident energy by x^3. For example, compared with the system $^{208}Pb + ^{208}Pb$ (5 MeV), by choosing the system $^{120}Sn + ^{120}Sn$ (208/120$\sim$ 1.7), $\Delta\theta$ and δ_M are multiplied by about ≈ 1.7 if the incident energy is lowered from 5 MeV to $5/1.7^3 \sim 1.$ MeV. Before ending, we should like to mention that other small effects (electronic screening, color Van der Walls forces, ...) could induce slight deviations of the elastic cross-section from Mott-scattering [3, 4]. But since the corresponding phase shifts are antisymmetric, they have no influence on the evaluation of the gravitational phase shift as given by the formula (6).

FIGURE CAPTIONS

Fig. 1 Two trajectories of impact parameters b_θ (direct) and $b_{\pi-\theta}$ (recoil) in a scattering plane perpendicular to the Earth's surface. $f_\perp$ is the component of the gravitational force on the axis perpendicular to the scattered trajectories. (See text)

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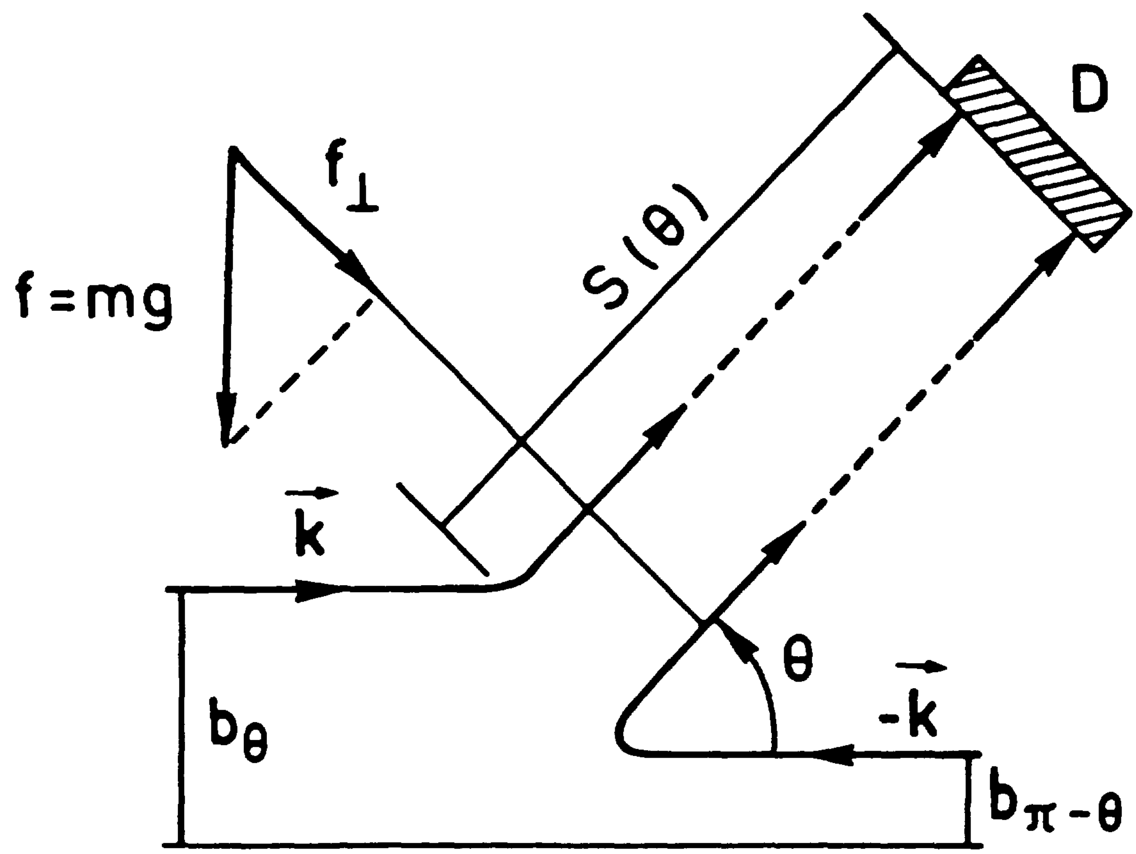

Fig. 1