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AHARONOV-BOHM OSCILLATIONS WITH FRACTIONAL PERIOD IN A MULTICHANNEL WIGNER CRYSTAL RING

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ABSTRACT

We study the persistent current in a quasi 1D ring with strongly correlated electrons forming a multichannel Wigner crystal (WC). The influence of the Coulomb interaction manifests itself only in the presence of external scatterers that pin the WC. Two regimes of weak and strong pinning are considered. For strong pinning we predict the Aharonov-Bohm oscillations with fractional period. Fractionalization is due to the interchannel coupling in the process of quantum tunneling of the WC. The fractional period depends on the filling of the channels and may serve as an indicator of non-Fermi-liquid behaviour of interacting electrons in quasi 1D rings.

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I. INTRODUCTION

Aharonov–Bohm (AB) flux Φ threading a mesoscopic ring induces equilibrium persistent current.[1] This current oscillates with Φ changing in the interval $[-I_0, I_0]$. For an isolated ring the period of oscillations is $\Phi_0 = hc/e$ and the amplitude is maximal in a ballistic regime. At low temperatures the amplitude is given by the single–electron current on the Fermi level, $I_0 = ev_F/L$, where L is the ring circumference and v_F is the Fermi velocity. These properties predicted in the framework of a quite naive model of one-dimensional (1D) noninteracting electrons are in a good agreement with the recent measurements of persistent current in the ballistic ring formed by the splitting gate technique in a 2D electron gas.[2]

In a semiconductor quantum ring[2] the electron density is low, unlike that in real metal rings.[3,4] It means that the Coulomb interaction is weakly screened and electron–electron correlations may affect essentially the thermodynamics of electrons in the ring. The influence of Coulomb interaction on the persistent current in a 1D ballistic ring has been studied recently in[5–7]. In Ref. 5 the general theorem (analogous to the Kohn theorems[8]) claiming that Coulomb interaction does not affect the persistent current in a perfect (impurity–free) ring was proved.

Any potential barrier created in the ring by either an impurity or an external gate potential leads to the suppression of the persistent current. In the Fermi–gas model the amplitude of oscillations decreases in inverse proportion to the impurity potential.[9,10] The effect of suppression is enhanced tremendously in the case of strongly repulsive interacting electrons.[6,7] At low temperature transport of charged carriers through the barrier is due to the quantum tunneling process which in 1D is extremely sensitive to electron–electron interactions.[11–14] In the case of strong Coulomb interaction, electrons in a 1D ring form the Wigner crystal (WC), which can be considered as a limiting case of the Luttinger liquid with small amplitude of zero–point fluctuations.[12] Pinning of the WC by ring inhomogeneities leads to the suppression of the persistent current. However the period of the AB oscillations remains equal to Φ_0. For a strictly 1D ring the amplitude of the persistent current and its anomalous temperature dependence have been studied in[6,7]. In a real situation the number,

M, of 1D channels (propagating modes) exceeds one ($M = 4$ in the experiment[2]). Therefore the results for the persistent current have to be reconsidered for the multichannel ring.

In the present paper we study the persistent current in a multichannel WC ring. Now the effect of pinning of the WC turns out to be even more dramatic because not only the amplitude but the period of the AB oscillations as well are changed in comparison with those in the Fermi-gas model. The period of oscillations is related to the charge of elementary excitation which tunnels through the barrier. In the 1D ring the charge of the current excitations in the WC coincides with electron charge, e, and the period of oscillations is given by the fundamental quantum flux Φ_0. In a multichannel ring the elementary current excitation turns out to be (due to the interchannel Coulomb coupling) *multielectron* with a charge ke. Then the period of the AB oscillations in a ring with a barrier becomes a fraction, Φ_0/k, of the fundamental flux quantum. The integer k depends on the fillings of the channels. By changing the electron concentration by varying the gate voltage one may observe the AB oscillations with fractional period. Due to this property the persistent current can be used to detect non-Fermi-liquid behavior of correlated electrons in quasi 1D ballistic rings. So far such a behavior has been observed in the unique experiment by Milliken, Umbach, and Webb[15] in the measurement of the tunneling conductance across a barrier between quantum Hall edge states.

II. PERSISTENT CURRENT IN A MULTICHANNEL WIGNER CRYSTAL RING.

To describe the dynamics of strongly-correlated electrons in quasi 1D ring we will use the model of the multicomponent Luttinger liquid proposed in Ref. 13. In this model the density of Lagrangian in continuum limit is written as follows:

$$\mathcal{L}_0 = \frac{m}{8\pi^2}\sum_{j=1}^{M}\frac{1}{n_j}\left[\dot{\varphi}_j^2 - v_j^2(\varphi_j')^2\right] - \qquad (1)$$

$$-\frac{1}{8\pi^2}\sum_{i,j=1}^{M}\oint dy U(x-y)\varphi_i'(x)\varphi_j'(y).$$

Here, $\varphi_j(x,t) \equiv 2\pi u_j(x,t)/a_j$ is the dimensionless scalar field and u_j is the displacement of electrons from the equilibrium positions in the WC chain with period $a_j (j = 1, 2, ..., M)$. The electron density in the channel j is $n_j = a_j^{-1}$ and $v_j = \pi\hbar n_j/m$ is the Fermi velocity for noninteracting electrons with mass m.

The first term in eq. (1) describes in the boson representation a free electron gas with linearized spectrum. The energy of Coulomb correlations is given by the second term which contains electron-electron interactions within the channels ($i = j$) and between different channels ($i \neq j$). In what follows, we will assume the interaction to be short-range, *i.e.* $U(x-y) = V_0\delta(x-y)$. Since in a quantum ring the conducting area and the pinning barrier are formed electrostatically they are smooth on the lattice period scale and therefore we neglect interchannel electron transitions. Thus, for an isolated ring the number $N_j = Ln_j$ of electrons in each channel is conserved.

The topological AB interaction of the ring with magnetic flux Φ can be written in a form of total temporal derivative (see, e.g., review[16]),

$$\mathcal{L}_{AB} = \frac{\hbar}{L}\sum_{j=1}^{M}\dot{\varphi}_j\left(\frac{\Phi}{\Phi_0} + p_j\right). \tag{2}$$

Here $p_j = \frac{1}{2}(0)$ for even (odd) number N_j of particles in the channel and p_j is a fictitious normalized statistical flux $\bar{\Phi}_s = \Phi_0/2$ which appears in a ring with an even number of electrons. For the fixed number of particles in each channel the topological index p_j which describes the parity effect[17] takes on M independent values.

The thermodynamic persistent current is given by the derivative of the free energy F with respect to the magnetic flux Φ,

$$\mathcal{I}(\Phi) = -\frac{1}{c}\frac{\partial F}{\partial \Phi}. \tag{3}$$

We present the free energy of quantum lattice in the form of a path integral over boson fields φ_j,

$$F = -T\ln\left\{\sum_{\{k_j\}=-\infty}^{\infty}\int\prod_{j=1}^{M}D\varphi_{k_j}\exp\left(-S_E/\hbar\right)\right\}. \tag{4}$$

Here S_E is the Euclidean (imaginary time $\tau = it$) action for the Lagrangian $\mathcal{L}_0 + \mathcal{L}_{AB}$,

$$S_E = \oint dx \int_0^{\hbar/T} d\tau \left(\mathcal{L}_0 + \mathcal{L}_{AB}\right). \tag{5}$$

To calculate the flux-dependent part of the free energy one needs to evaluate integral (5) on the "trajectories" $\varphi_{k_j}(x,\tau)$ which minimize the action (5). These trajectories satisfy the classical equations of motion and "twisted" boundary conditions in imaginary time (see, e.g.[16]),

$$\varphi_{k_j}(x, \tau + \hbar/T) - \varphi_{kj}(x,\tau) = 2\pi k_j, \tag{6}$$

where $k_j = 0, \pm1, \pm2, ...$ is the homotopy index which classifies homotopically inequivalent trajectories.

In an impurity-free isolated ring, the trajectories φ_{k_j} are independent on x:

$$\varphi_{k_j}(x,\tau) = 2\pi k_j \frac{T\tau}{\hbar}. \tag{7}$$

The Coulomb energy in eq. (1) vanishes for these trajectories ($\varphi_i'(x) = 0$) and therefore the persistent current in the ideal WC is the same as that of the Fermi-gas model[9],

$$\mathcal{I}(\Phi) = \frac{eT}{\hbar} \sum_{j=1}^{M} \sum_{n=1}^{\infty} (-1)^{nN_j} \frac{\sin(2\pi n \Phi/\Phi_0)}{\sinh(\pi T L n / 2\hbar v_j)} . \tag{8}$$

This obvious result is a manifestation of the general theorem.[5]

Now let us proceed to the calculations of the persistent current in the WC with imperfections. We will consider that the pinning potential $V_p(x)$ is smooth on a scale of the lattice periods $a_j (j = 1, 2, ..., M)$ and local on the mesoscopic scale L. The energy of lattice–impurity interaction is also a 2π–periodic function of the displacement $\varphi_j(x)$. In Ref. 12 the general form for the pinning energy in a one–dimensional WC has been proposed. Generalizing this result for the multichannel ring, we find the following contribution to the Lagrangian,

$$\mathcal{L}_p = V_p \delta(x) \sum_{j=1}^{M} (\cos\varphi_j - 1). \tag{9}$$

The amplitude V_p depends on the distance between the conducting channel and the electrode which produces the pinning potential.

In the presence of pinning potential a free sliding motion of the WC becomes impossible. The charge transport (at $T = 0$) is accomplished by the macroscopic quantum tunneling of a crystal through the barrier. It was shown[7] that the mechanism of tunneling is different for weak ($\alpha V_p \lesssim \Delta_s = \hbar s/L$) and strong ($\alpha V_p >> \Delta_s$) pinning. Here,

$$\alpha = \pi\hbar/msa \equiv v_F/s \tag{10}$$

is a parameter which characterizes the amplitude of quantum fluctuations (correlation exponent, g, in Luttinger liquid approach[11]) and s is the velocity of plasmon (sound-like) excitation in the WC lattice. For a multichannel ring, $\alpha = \alpha_j = v_j/s_j$ and $s_j = (v_j^2 + v_j V_0/\pi\hbar)^{1/2}$. In what follows we consider the limit of stiff crystal, when $\alpha_j << 1$.

First we analyze the tunneling in the regime of weak pinning, assuming however that V_p is larger than the level spacing Δ_j,

$$\Delta_j = \frac{\hbar v_j}{L} << V_p < \frac{s}{v_j}\frac{\hbar s}{L} \,. \tag{11}$$

The right inequality in eq.(11) implies that the energy ms^2/N_j of elastic deformation (per atom) exceeds the pinning energy V_p. Therefore the WC in each channel tunnels through the barrier homogeneously – without any elastic deformations. Homogeneous trajectories φ_j in different channels do not interact with each other and the net current can be written as a sum of single-channel currents. Using for the single-channel current the result obtained in[6,7] and omitting a nonessential pre-exponential factor, we get

$$\mathcal{I}(\Phi) \sim \sum_{j=1}^{M} \frac{ev_j}{L} \sum_{n=1}^{\infty} (-1)^{nN_j} \exp\left(-n\sqrt{2\pi\frac{V_p}{\Delta_j}}\right) \sin\left(2\pi n\frac{\Phi}{\Phi_0}\right) \,. \tag{12}$$

Here $V_p >> \Delta_j$ (see eq. (11)) and hence the higher harmonics (with $n > 1$) are suppressed. However, the second harmonic becomes important if the number of electrons in each channel is random. Then after averaging over the number of particles in each channel, all odd harmonics, and in particular the fundamental one, become exponentially small because of the oscillating factor $(-1)^{nN_j}$. In the real case, $\bar{N} >> 1$ ($\bar{N}$ is the averaged over channels number of electrons) they can be neglected and only the second harmonic with period $\Phi_0/2$ contributes to the net current.

III. REGIME OF STRONG PINNING

Now we calculate the persistent current in the regime of strong pinning,

$$\alpha V_p >> \Delta_s = \hbar s/L \ . \tag{13}$$

We consider a ring with two channels because this makes the analytical calculations feasible.

Strongly pinned WC tunnels inhomogeneously.[6,7] The mechanism of such macroscopic quantum tunneling, as proposed by Larkin and Lee,[18] incorporates two stages: *i*) fast tunneling of a small segment of length $l_0 << L$ and *ii*) slow relaxation of the local elastic deformation associated with the first stage. The relaxation occurs due to the tunneling of plasmon excitations. The action corresponding to the second (plasmon) stage diverges logarithmically if $L/l_0 \to \infty$ and the main contribution to the action comes from this stage. The free relaxation in the WC is described by the Lagrangian $\mathcal{L}_0$, eq. (1). For the two–channel ring the dynamical equations take the following form:

$$\begin{cases} \ddot{\varphi}_1 + s_1^2\varphi_1'' + (s_1^2 - v_1^2)\varphi_2'' = 0 \\ \ddot{\varphi}_2 + s_2^2\varphi_2'' + (s_2^2 - v_2^2)\varphi_1'' = 0 \ . \end{cases} \tag{14}$$

At zero temperature the tunneling is described by the instanton solutions of eqs.(14),

$$\varphi_j(x,\tau) = A_j \arctan(\tau u/ \mid x \mid), \tag{15}$$

where u is the velocity of collective excitations in two coupled channels.

Substitution of eq. (15) into eqs. (14) gives a set of linear homogeneous equations for amplitudes A_1 and A_2. This set has nontrivial solutions if its determinant is zero,

$$u^4 - (s_1^2 + s_2^2)u^2 + s_1^2 s_2^2 - \frac{v_1 v_2}{\pi^2\hbar^2}V_0^2 = 0. \tag{16}$$

Two positive solutions of this equation give rise to two eigenmodes which we designate as "fast" (u_+) and "slow" (u_-). Each eigenmode gives an additive contribution to the tunneling action, eq. (5). For these eigenmodes, the ratio A_1/A_2, obtained from anyone of eqs. (14), is given by

$$\frac{A_1}{A_2} = \frac{s_1^2 - v_1^2}{u^2 - s_1^2} \ . \tag{17}$$

On the other hand, instantons (15), being the vacuum-to-vacuum trajectories, have to satisfy the twisted boundary conditions (6) which at $T = 0$ are reduced to the following form:

$$\Delta\varphi_{k_j} = \varphi_{k_j}(x, +\infty) - \varphi_{k_j}(x, -\infty) = 2\pi k_j \ . \tag{18}$$

Thus, we have three conditions (which are eqs. (17) and (18)) for two unknowns A_1 and A_2. In general, these conditions cannot be satisfied simultaneously at arbitrary fillings of the channels. On the other hand, the Larkin–Lee mechanism of tunneling provides the minimum of action in the case of strong pinning in a 1D ring. In what follows we propose a simple modification of the Larkin-Lee scenario for the multichannel ring.

Without loss of generality one can consider that $n_1 > n_2$ ($v_1 > v_2$). Then the ratio A_2/A_1, calculated from eqs. (15) and (18) is rational and greater than one. After substituting the solution u_+ of eq.(16) in eq.(17) the ratio A_2/A_1 becomes

$$\frac{k_2}{k_1} = \frac{A_2}{A_1} = \frac{n_1 - n_2}{2n_1} \left\{ \sqrt{\left[1 + \frac{\pi^2\hbar^2}{mV_0}(n_1 + n_2)\right]^2 + \frac{4n_1 n_2}{(n_1 + n_2)^2}} - \frac{\pi^2\hbar^2}{mV_0}(n_1 + n_2) - 1 \right\} . \tag{19}$$

It is easy to show that in the limit of a stiff crystal ($\alpha << 1$) the slow mode u_- is uncharged (shifts in the channels have opposite signs: $\varphi_1 + \varphi_2 = 0$) and therefore it does not contribute to the persistent current. In the same limit, eq. (19) is reduced to a simple form,

$$\frac{k_2}{k_1} = \frac{A_2}{A_1} = \frac{N_2}{N_1} = f \ , \tag{20}$$

where f is the filling fraction. Since for a mesoscopic ring f is always rational, the parameters k_1 and k_2 are integers and the twisted boundary conditions, eq. (18), are satisfied. Thus we do have a formal instanton solution given by eq. (15) with amplitudes $A_{1,2} = k_{1,2}$, where $k_2/k_1 = N_2/N_1$ is an irreducible fraction. This solution describes a multielectron tunneling across the barrier and gives rise to the AB oscillations with fractional period $\Phi_0/(k_1 + k_2)$.

In a mesoscopic ring the numbers N_1 and N_2 of electrons in channel are large. This means that the formal solution given by eq. (20) predicts large winding numbers k_j (apart from very unlikely cases when N_2/N_1 is reduced to a fraction with small nominator and denominator). The tunnelling action (which is a quadratic form of $k_{1,2}$) along such trajectories will also be large. Then one should conclude that the inhomogeneous tunneling is not favorable and the Larkin-Lee scenario fails in the multichannel ring. Below we show that this is not the case and propose an instanton solution which, being a trail function, minimizes the total action with respect to the "variational" parameters $k_{1,2}$.

Let us estimate how large the winding numbers can be so that the inhomogeneous tunneling can be realized. The upper limit for $k_{1,2}$ is given by the condition $S_{inh} < S_h$, where the action S_{inh} is calculated on inhomogeneous trajectories, eq. (15), and the action S_h corresponds to the tunneling of two heavy particles with masses mN_1 and mN_2 along the homogeneous trajectories. This gives:

$$\frac{u_+}{2}\left[\frac{k_1^2}{v_1}\left(1+\frac{s_1^2}{u_+^2}\right)+\frac{k_2^2}{v_2}\left(1+\frac{s_2^2}{u_+^2}\right)+\frac{2V_0}{\pi\hbar}\frac{k_1k_2}{u_+^2}\right] ln\left(\frac{L}{2l_0}\right) < \sqrt{\frac{2mLV_p}{\hbar^2}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} \,. \quad (21)$$

To estimate the left-hand side of this inequality it is sufficient to substitute the length

$$l_0 = a\frac{\alpha V_p}{\Delta_s} \quad (22)$$

obtained by Larkin and Lee for 1D case.[18] A numerical estimation of eq. (21) shows that for resonable values of the parameters of the WC, the integers $k_{1,2}$ cannot exceed 3. Therefore the original Larkin–Lee scenario of tunneling is realized in a multichannel ring for very few values of the filling factor, $f^* = 0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1$. At any other fillings, strong coupling between channels (see eqs. (19) and (20)) gives rise to a slightly different scenario of tunneling.

In our new scenario, instead of tunneling along the trajectories φ_{k_j} with large winding numbers $k_j = N_j$, the WC would tunnel along the trajectory $\varphi^*_{k_j}$, with small winding numbers $k_j^* (k_j^* \leq 3)$, the "nearest" to φ_{k_j}. The integers k_j^* are chosen in such a way that the difference

$$f - f^* = \frac{N_2}{N_1} - \frac{k_2^*}{k_1^*} = \epsilon \quad (23)$$

is minimal. Here a small parameter ϵ describes the "deviation" from the exact trajectory φ_{k_j}. To calculate the action associated with the trajectories $\varphi^*_{k_j}$ we need to know the amplitudes A_j of instantons in eq. (15). To satisfy one of the boundary conditions eq. (18) we can take $A_1 = 2k_1^*$. Then the amplitude A_2 is obtained from eqs. (20) and (23),

$$A_2 = 2(k_2^* + \epsilon k_1^*) \,. \quad (24)$$

Because of the second term in eq. (24) the twisted boundary condition eq. (18) is not satisfied and the additional small shift

$$\delta\varphi_2 = 2\pi\epsilon k_1^* \tag{25}$$

appears in the second channel. This means that the quantum states of the WC before and after tunneling are slightly different. On the other hand at $T = 0$ only a ground state-to-ground state tunneling process is allowed which implies the exact equivalence of the quantum states before and after tunneling. To resolve this contradiction one needs to introduce a process which would compensate the small shift $\delta\varphi_2$. This process occurs in the second channel only and it has no affect to the tunneling in the first channel. Then it is clear that the compensation of the shift eq. (25) is due to a homogeneous shift of the WC in the second channel by an angle $-\delta\varphi_2$. This negative shift can be included in the stage of the fast tunneling of a segment l_0. Using the trial function proposed by Larkin and Lee[18] we represent the tunneling trajectories of the first stage as[19]

$$\varphi_1(x,\tau) = 2\pi k_1^* \left(1 - \frac{|x|}{l_0}\right)\frac{\tau}{\tau_0}\,, \tag{26}$$

$$\varphi_2(x,\tau) = 2\pi k_2^* \left(1 - \frac{|x|}{l_0}\right)\frac{\tau}{\tau_0} - 2\pi k_1^*\epsilon\frac{\tau}{\tau_0}\,. \tag{27}$$

Here $|x| \leq l_0$ and the variational parameters l_0 and τ_0 (the length of the tunneling segment and the "duration" of the first stage, correspondingly) are obtained by minimization of the first stage action and the complete action.[18] This gives,

$$\tau_0 = \frac{\hbar u_+}{8V_p}\left[\frac{k_1^{*2}}{v_1}\left(1+\frac{s_1^2}{u_+^2}\right) + \frac{k_2^{*2}}{v_2}\left(1+\frac{s_2^2}{u_+^2}\right) + 2\frac{V_0}{\pi\hbar}\frac{k_1^*k_2^*}{u_+^2}\right] \sim k_1^{*2}\frac{\hbar u_+}{V_p v_1}\,, \tag{28}$$

$$l_0 = v_0\tau_0 \sim k_1^{*2}\frac{V_0}{V_p}\,, \tag{29}$$

$$v_0 = \frac{s_1 f}{k_1^{*2}f + k_2^{*2}}\left[3\frac{V_p\tau_0 v_1}{\pi\hbar s_1} + \sqrt{\left(3\frac{V_p\tau_0 v_1}{\pi\hbar s_1}\right)^2 + \left(k_1^{*2} + \frac{1}{f}k_2^{*2}\right)\left(k_1^{*2} + \frac{1}{f}k_2^{*2}\frac{s_2^2}{s_1^2} + 2k_1^*k_2^*\frac{V_0 v_1}{\pi\hbar s_1^2}\right)}\right]\,. \tag{30}$$

By the order of magnitude, the velocity $v_0 \sim u_+$. Under the condition of strong pinning, eq. (13), it follows that $a_j << l_0 << L$. To calculate the tunneling action we substitute the solutions given by eqs. (26), (27) (first stage) and eq. (15) (second stage) in eq. (5).

The contribution of the second term in eq. (27) describing the homogeneous shift can be neglected ($\epsilon \ll 1$) and for the action on the first stage we get

$$\frac{S_1}{\hbar} = \frac{4\pi}{3}\frac{v_0}{v_1}\left(k_1^{*2} + \frac{1}{f}k_2^{*2}\right) . \tag{31}$$

The action on the relaxation stage (second stage of tunneling) diverges logarithmically with the ring circumference (infrared catastrophe). In the logarithmic approximation

$$\frac{S_2}{\hbar} = \frac{1}{\alpha^*}\ln\left(\frac{L}{2l_0}\right) , \tag{32}$$

where

$$\frac{1}{\alpha^*} = \frac{u_+}{2}\left[\frac{k_1^2}{v_1}\left(1 + \frac{s_1^2}{u_+^2}\right) + \frac{k_2^2}{v_2}\left(1 + \frac{s_2^2}{u_+^2}\right) + \frac{2V_0}{\pi\hbar}\frac{k_1 k_2}{u_+^2}\right] . \tag{33}$$

Just as in a single-channel case,[7] the principle contribution to the action comes from the relaxation stage since $\ln(L/l_0) >> 1$. In a stiff crystal ($v_j << s_j$), the velocity $u_+ \sim s_1$. Therefore the actions in eqn. (31) and (32) are quasiclassical, $S_2 \gg S_1 \gg 2\pi\hbar$. In this approximation instantons behave like a 1D "dilute gas"[20] and for the persistent current we get,

$$\mathcal{I}(\Phi) \sim (-1)^{N_1+N_2}\frac{eu_+}{L}\left(\frac{V_0}{LV_p}\right)^{1/\alpha^*} \sin\left[2\pi\left(k_1^* + k_2^*\right)\frac{\Phi}{\Phi_0}\right] . \tag{34}$$

The large dimensionless parameter $1/\alpha^*$ determines the effective stiffness of the two–channel ring. It follows from eq. (33) that $\alpha^* < \alpha_{1,2}$, hence, coupling of channels increases the stiffness of the ring. Therefore, the amplitude of the AB oscillations is additionally suppressed in a multichannel ring in the regime of strong pinning. The more important effect that appears in this regime is the fractional period, $\Phi_0/(k_1^* + k_2^*)$, of the persistent current. In the multi-channel ring the period is not a fundamental one but it depends on the filling fraction f. When f is varied continuously, the integers k_1^* and k_2^* in eq. (23) take on different values from the interval $0 \leq k_j^* \leq 3$. The smallest period, $\Phi_0/5$, appears if the ratio N_2/N_1 is near to 2/3. If $n_2 << n_1$, the channel with lower concentration does not contribute to the net current and a two–channel ring behaves like a single–channel one. At equal fillings ($n_1 = n_2$) the period halving takes place ($k_1^* = k_2^* = 1$) which is almost evident from qualitative considerations. It is interesting to note that this effect even remains for a "soft" ($\alpha_j \sim 1$) WC.

The fractionalization of the period is accompanied by a fast drop of the amplitude of the AB oscillations because the tunneling action (exponent $1/\alpha^*$) grows quadratically with integers the k_1^* and k_2^*.

IV. CONCLUSIONS

We considered here the AB oscillations of persistent current in a multichannel ring of strongly correlated electrons. The Luttinger liquid approach has been used to describe electron-electron correlations in the ring. In the ring without scatterers the persistent current is the same as in the Fermi-gas model because the Coulomb interaction is rotationally invariant. Any external scatterer (produced, e.g., by the split-gate voltage) breaks the symmetry and pins the WC. At $T = 0$ electron transport in the pinned WC occurs due to the tunneling through the pinning potential.

If the pinning energy is less than the energy of elastic deformation, (weak pinning) the WC tunnels through the barrier without deformation, *i.e.* as a single particle. In the nondeformed WC, electrons in different channels do not interact with each other, therefore the total persistent current is a sum of single-channels currents. A very different picture appears in the regime of strong pinning. Now the tunneling process is acomplished by nonhomogeneous deformation of the WC that leads to the strong coupling between channels. In the coupled channels the elementary charge which tunnels through the barrier is a multiple of e, and then the period of the AB oscillations is a fraction of Φ_0. In a stiff WC the denominator of this fraction depends only on the number of electrons in the channels. Thus, by changing the filling factor one can change the period. Numerical estimations show that the minimal period can be $\Phi_0/5$. Oscillations with smaller period have very small amplitude (which decays rapidly with denominator) to be observable.

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