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STRONG MAGNETIC FIELD GENERATION IN LASER PLASMA

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ABSTRACT

An attempt has been made to solve the magnetic field evolution equation by using Green function and taking convective, diffusion and $\nabla n \times \nabla T$ as a dominant source term. The maximum magnetic field is obtained to be an order of megagauss.

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1 Introduction

Study of generation of magnetic fields, so called "spontaneous" magnetic fields in plasma produced by laser has been started with great interests in many laboratories since 1970's. In connection with laser fusion studies, first measurement of such fields were conducted by Stamper et al. [1] in experiments with solid targets irradiated with powerful laser beams. It has been reported [2,3] that the magnetic field of megagauss order has been detected in the measurement of such field by Faraday rotation technique. In the work of Diado et al. [4] an experimental result has been reported for the generation of magnetic field of the order of 600 kilogauss. Self-generated magnetic fields have profound influence on transport characteristics, parametric instabilities as well as absorption of laser light in inertial confinement fusion. Soon after the experimental report of Stamper et al. [2] on the measurement of magnetic field, thermoelectric mechanism has been suggested [5,6] for the generation of magnetic field. The development of ultra intense short pulse laser, a new parameter regime [7] has been opened up in the study of nonlinear laser plasma interaction. Recent numerical simulations by Wilkes et al. [8] of the interaction of ultra intense laser pulse with and over dense plasma target have revealed extremely high self-generated magnetic fields of the order of 250 MG in the overdense plasma. Sudan [9] has argued that the extremely high magnetic fields observed in the numerical simulation of Wilkes et al. cannot be explained on the basis of physical mechanisms. Rather, he suggested an alternative mechanism for the magnetic field generation in terms of dc currents driven by special gradients and temporal variation of the pondermotive force of the laser light on the plasma electrons. The dc magnetic field is found to be of the same order as the laser oscillating magnetic field. On the other hand Tsintsadze et al. [10] presented a very attractive mechanism for the generation of high magnetic field by non uniform intense laser beam in a non uniform collisionless plasma having equilibrium electron temperature and density gradients. Their theory holds in the limits when interparticle collision are negligible and laser beams are arbitrary large amplitude.

In this paper the magnetic field evolution equation has been solved by using Green function and taking convective, diffusion and crossed gradients of density and temperature as a dominant source term.

2 Evolution Equation for B

The evolution equation for magnetic field can be obtained from the generalized Ohm's law. The equation reads [11,12]:

$$\begin{aligned}\frac{\partial \vec{B}}{\partial t} &= \nabla \times (\vec{U} \times \vec{B}) - \frac{c^2}{4\pi}\nabla \times \vec{\eta} \cdot (\nabla \times \vec{B}) - \frac{kc}{en_0}(\nabla n \times \nabla T) \\ &+ \frac{c}{4\pi e}\nabla \times \frac{1}{n_0}\left\{\nabla\left(\frac{B^2}{2}\right) - (\vec{B}\cdot\nabla)\vec{B}\right\} - \frac{mc}{e^2}\left(\nabla \times \frac{1}{n_0}\frac{\partial \vec{J}}{\partial t}\right) \\ &+ \frac{c}{e}\left(\nabla \times \frac{1}{n_0}\nabla p_r\right) - \frac{c}{e}\nabla \times (\overleftrightarrow{\beta}\,\nabla kT - \overleftrightarrow{a}\,\hat{b} \times \nabla kT) \qquad (1)\end{aligned}$$

where symbols are defined in Refs.[11,12].

This equation for self generated magnetic field in Laser Produced Plasma is a non linear equation. Several attempts have been made to solve the equation. However, in these attempts only a few terms in the above equation are retained because of non-linearity nature of the equation. Here, we try to solve the equation considering only the following few terms so that the magnitude of B-field due to respective terms may be evaluated.

The dominant contributive terms are convective, diffusive and source ($\nabla n \times \nabla T$) term. Hall term, thermal force term and radiation pressure term has been neglected as they have negligible effect on the magnitude of magnetic field. With this approximation, the evolution equation reads

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{U} \times \vec{B}) + \frac{c^2}{4\pi\sigma}\nabla \times (\nabla \times \vec{B}) - \frac{ck}{en}(\nabla n_e \times \nabla T_e) \tag{2}$$

The gradient in number density of electron is taken along laser axis which is taken to be z-axis, and its variation taken as

$$\frac{1}{n_e}\frac{\partial n_e}{\partial z} = \left|\frac{\Delta n_e}{n_e}\right| \frac{1}{l_n} \exp\left[-\frac{z}{l_n}\right] \tag{3}$$

where Δn is the change in n_e over the scale length l_n.

Similarly, the temperature profile is assumed along the radial direction with the scale length,

$$\frac{1}{l_T} = \frac{1}{T}\frac{\partial T}{\partial r} \tag{4}$$

Such geometry is set due to the cross gradients of density and temperature in source term. It can be noted that

$$\begin{aligned} \nabla \times (\vec{u} \times \vec{B}) &= (\nabla \cdot \vec{B})\vec{u} - \nabla \cdot \vec{u}\vec{B} \\ &= -(\nabla \cdot \bar{u})\vec{B} \end{aligned} \tag{5}$$

and

$$\begin{aligned} \nabla \times (\nabla \times \vec{B}) &= \nabla(\nabla \cdot B) = -\nabla^2 \vec{B} \\ &= -\nabla^2 B \end{aligned} \tag{6}$$

Here $\nabla \cdot \vec{u}$ has dimension time^{-1}, and we let

$$\nabla \cdot u = \frac{1}{\tau_B} \tag{7}$$

τ_B is the characteristic time. Finally, it is supposed that B has only variation along z-axis. With these considerations the evolution equation reduces to

$$\frac{\partial B}{\partial t} = \frac{1}{\tau_B}B + \frac{c^2}{4\pi\sigma}\frac{\partial^2 B}{\partial z^2} - \frac{ckT}{el_n l_T}\left|\frac{\Delta n}{n}\right| \exp\left[-\frac{z}{l_n}\right] \tag{8}$$

3 Derivation and Calculation of B

Let $\alpha_1 = c^2/4\pi\sigma$ and $\alpha_2 = -\frac{ckT}{el_n l_T}\left|\frac{\Delta n}{n}\right|$. Then, Eq.(8) becomes

$$-\frac{\partial^2 B}{\partial z^2} + \frac{1}{\alpha_1}\frac{\partial B}{\partial t} + \frac{1}{\alpha_1 \tau_B} B = \frac{\alpha_2}{\alpha_1} \exp\left[-\frac{z}{l_n}\right] \tag{9}$$

Using the Green function as

$$G(\xi - z, t) = \tau_B l_T \sqrt{\frac{\alpha_1}{4\pi t}} \exp\left\{-\frac{(\xi - z)^2}{4\alpha t}\right\} \exp\left\{-\frac{1}{\tau_B}\right\}$$

the solution of Eq.(9) may be obtained as

$$B(z,t) = \tau_B l_T \int_{-\infty}^{\infty} G(\xi - z, t)\frac{\alpha_2}{\alpha_1} \exp\left\{-\frac{\xi}{l_n}\right\} d\xi$$

$$= \tau_B l_T \sqrt{\frac{\alpha_1}{4\pi t}} \cdot \frac{\alpha_2}{\alpha_1} \exp\left\{-\frac{z}{l_n}\right\} \exp\left\{-\frac{t}{t_B}\right\} \int_{-\infty}^{\infty} \exp\left\{-\frac{(\xi - z)^2}{4\alpha_1 t}\right\} \cdot \exp\left\{-\frac{\xi - z}{l_n}\right\} d\xi \tag{10}$$

On evaluating the integral [13] we get

$$B = \tau_B l_T \alpha_2 \exp\left\{-\left(\frac{1}{\tau_B} - \frac{\alpha_1}{l_n^2}\right) t - \frac{z}{l_n}\right\} \tag{11}$$

Substituting for α_1 and α_2, we get

$$B = \frac{ckT}{el_n}\tau_B \left|\frac{\Delta n}{n}\right| \exp\left\{-\left(\frac{1}{\tau_B} - \frac{c^2}{4\pi\sigma l_n^2}\right) t - \frac{z}{l_n}\right\} \tag{12}$$

Here, the hydrodynamic characteristic time, τ_B is given by

$$\tau_B = \frac{\pi\sigma l_n^2}{c^2} \tag{13}$$

where σ is plasma conductivity and is given by

$$\sigma = 7.2 \times 10^7 \left(\frac{T^{3/2}}{Z ln\Lambda}\right) Sec^{-1} \tag{14}$$

Z is atomic number, T is temperature in Kelvin. The Coulomb logarithm Λ, is given by

$$\Lambda = 1.24 \times 10^7 \left(\frac{T^{3/2}}{n}\right)^{1/2} \tag{15}$$

where, number density is in m^{-3}

For $T = 9$ keV, $l_n = 10\mu m$ and $n = 10^{27} m^{-3}$ [14] we have

$$\Lambda = 4.17 \times 10^5$$

$$\sigma = \frac{5.9 \times 10^{18}}{Z} \sec^{-1}$$

and

$$\tau_B = \frac{2.069 \times 10^{-8}}{Z} \sec = \frac{2.06 \times 10^4}{Z} \ ps$$

Finally, taking $|\Delta n/n| = 0.276$, we have

$$\frac{ckT\tau_B}{el_n}\left|\frac{\Delta n}{n}\right| = \frac{5.14 \times 10^6}{Z} \ Gauss = \frac{5.14}{Z} \ MGauss \tag{16}$$

so that the magnetic field, B, is given by

$$B = \frac{5.14}{Z} \exp\left\{-\frac{3}{4}\frac{t}{\tau_B} - \frac{z}{l_n}\right\} \ MGauss \tag{17}$$

4 Results

The variation of B with z/ln for three values of t/τ, namely, 0.1, 0.5 and 5 are illustrated in Fig. 1. The maximum value of B is found to be 0.37 MG at $t/\tau = 0.1$ and $z/ln = 0$. This value of the field is below the value given by Raven et al. [3] because a number of terms has been neglected and density profile is taken as exponential. Further work on this line with new density profile will be performed to explain experimental results.

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VARIATION OF B-FIELD

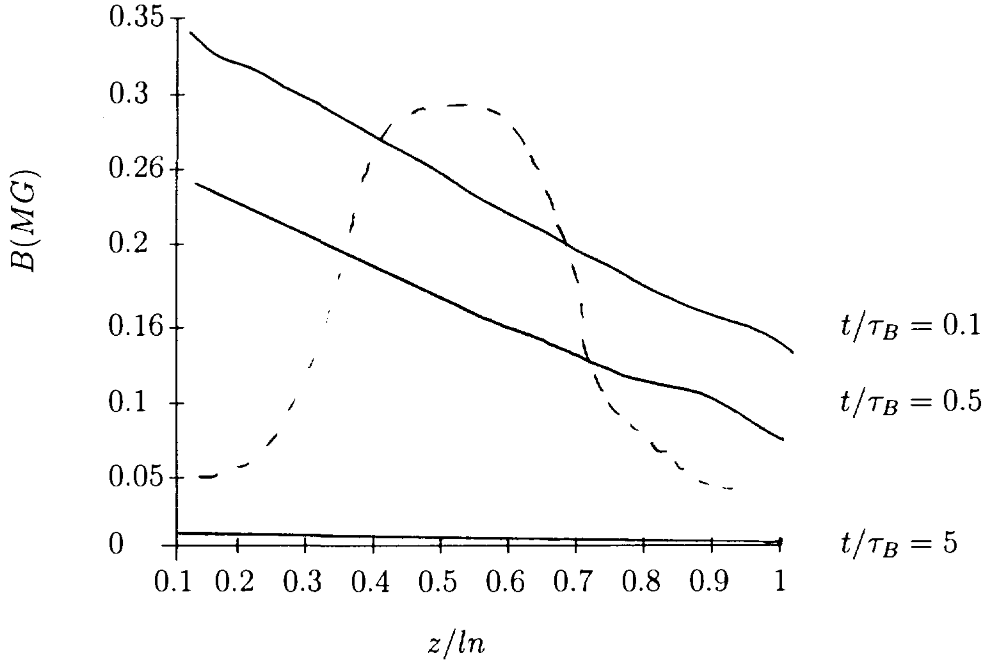

Fig. 1 - Variation of $B(MG)$ vs z/ln.
Solid line (–) from Eq.(17) and broken line (- - -) from Ref.[14].