ALTERNATING PHASE FOCUSSING INCLUDING SPACE CHARGE*

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Introduction

It is well known that longitudinal stability can be obtained in a non-relativistic drift tube accelerator by traversing each gap as the rf accelerating field rises. However, the rising accelerating field leads to a transverse defocussing force which is usually overcome by the use of magnetic focussing (solenoidal or quadrupole) inside the drift tubes. The development of the radio frequency quadrupole, which is now widely used, is one way to provide for simultaneous longitudinal and transverse focussing without the use of magnets.

With the advent of strong focussing, it was recognized that one could avoid the use of magnets by traversing alternate gaps between drift tubes as the field is rising and falling, thus providing an alternation of focussing and defocussing forces in both the longitudinal and transverse directions. Exploration of this idea[1, 2, 3] shows that the stable longitudinal phase space area is quite small. However, it is not clear that the parameter space has been explored fully, and recent efforts suggest that alternating phase focussing (APF) may permit low velocity acceleration of currents in the 100-300 m.a. range[4]. The present paper is an effort to study the parameter space and to test crude analytic predictions by adapting the code PARMILA, which includes space charge, to APF.

We assume a synchronous phase configuration of period $N\beta\lambda$ where the synchronous phase pattern is

$$\phi_s^{(i)} = -\phi_0 - \phi_1, \quad i = 1, 2, \cdots N/2, \tag{1}$$

$$\phi_s^{(i)} = -\phi_0 + \phi_1, \quad i = N/2+1, \cdots N.$$

Here ϕ_0 and ϕ_1 are positive, with ϕ_0 representing a small asymmetric offset to accompany the large alternating ϕ_1. Our initial exploration is with $\phi_0 = 0$, but it is quickly apparent that the longitudinal phase stable area can be increased for $\phi_0 > 0$. The desired synchronous phase configuration can be obtained by choosing drift tube lengths which alternate as well, and the analysis becomes complex quickly for values of N greater than 2. In an earlier work we approximated the equations for zero space charge and related the current carrying capacity of the linac to the size of the phase stable area[5]. In that work we also found that it was necessary to increase N (abruptly, of course) as β increased, in order to keep gradient requirements within practical bounds.

In the present work we explore the matching requirement when N changes abruptly and conclude that such changes produce serious mismatch of either or both the transverse and longitudinal motion. We therefore explore what may be possible for a single value of N and save for later the possibility of devising some method to change N adiabatically, or of constructing some alternate method of phase space matching.

Analysis

If we neglect acceleration and space charge and expand the equation for longitudinal motion to terms quadratic in $\psi = \phi - \phi_s$, we obtain

$$\frac{d^2\psi}{dw^2} + k\psi\sin(\phi_0 \pm \phi_1) - \frac{k\psi^2}{2}\cos(\phi_0 \pm \phi_1) \cong 0, \tag{2}$$

where $w = s/\beta\lambda$ is a continuous variable similar to cell number, and where the average accelerating field E is represented by the dimensionless parameter $k = 2\pi eE\lambda/mc^2\beta$. For $\phi_0 \ll 1$ we can separate ψ into a slowly varying and a rapidly oscillating part, and obtain the smoothed version of Eq. (2)

$$\frac{d^2\psi}{dw^2} + k\psi[\phi_0\cos\phi_1 + \frac{kN^2}{48}\sin^2\phi_1] - \frac{k\psi^2}{2}\cos\phi_1 \cong 0, \tag{3}$$

which suggests a phase stable area extending from $-\psi_u/2 \le \psi \le \psi_u$, where

$$\psi_u = 2\left[\phi_0 + \frac{kN^2\sin^2\phi_1}{48\cos\phi_1}\right] \tag{4}$$

Any abrupt change in parameters will lead to mismatch of the longitudinal phase space unless

$$\frac{|d\psi/dw|_{max}}{|\psi|_{max}} \sim [k\phi_0\cos\phi_1 + k^2N^2\sin^2\phi_1/48]^{1/2} \tag{5}$$

is continuous across the transition, as estimated from the small oscillation frequency. A similar analysis leads to the

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equation for the transverse motion

$$\frac{d^2x}{dw^2} + kx\left[-\frac{\phi_0\cos\phi_1}{2} + \frac{kN^2\sin^2\phi_1}{192}\right] = 0, \qquad (6)$$

where k has been replaced by $-k/2$ in the linear term in Eq. (2) because there are two transverse coordinates. Once again an abrupt change in parameters will lead to a mismatch of the transverse phase space unless

$$\frac{|dx/dw|_{max}}{|x|_{max}} \sim [-k\phi_0\cos\phi_1 + k^2N^2\sin^2\phi_1/192]^{1/2} \quad (7)$$

is continuous across the transition.

It is clear that continuity of the oscillation frequencies in Eq. (5) and (7) requires that both $k\phi_0\cos\phi_1$ and $kN\sin\phi_1$ be continuous across any transition. If we also seek to maximize the phase stable region related to Eq. (4), we must separately keep $\phi_0, k\cos\phi_1$ and $kN\sin\phi_1$ continuous across an abrupt transition. We earlier[5] made a crude estimate of the current carrying capacity of the linac and found, from an analysis of the longitudinal motion only, that $I_{max} \sim \beta^2 k\psi_u^3$. Thus, any discontinuity in k will lead to a discontinuity in ψ_u and/or I_{max} which we are trying to avoid. For this reason the present numerical investigation is carried out with constant k, that is, with the gradient E increasing proportional to β, as well as constant ϕ_1, ϕ_0, and N.

Numerical Results

The analytic results for the current limit of an APF represent only an approximate guideline for the actual results for several reasons: 1) The space charge forces are assumed to arise from a uniformly charge ellipsoid. Such a distribution is known not to be self consistent. 2) The focussing power of the alternating synchronous phase configuration is evaluated in the smoothed approximation only. 3) The size of the longitudinal phase stable area is estimated from the smoothed approximation for the focussing. 4) Acceleration is ignored in determining the stable phase space configuration. For these reasons, we use the program PARMILA[6], adapted for an APF configuration, to follow a collection of charged particles through a drift tube accelerator with idealized gaps, that is, where the r.f. forces are assumed to occur only at the center of a gap. The space charge force is included as an increment to the transverse and longitudinal momenta of the particles, applied once each cell at the gap center.

The numerical investigation is complicated by the fact that, because the cell lengths vary according to the APF synchronous phase configuration, there is no point in a focussing period where the focussing is symmetric with the interchange of $z \rightarrow -z$. Thus, the orientation of the equivalent space charge ellipsoid requires the identification of two parameters in both the longitudinal and transverse coordinates (for example, the Courant-Synder $\beta_\ell, \alpha_\ell, \beta_t, \alpha_t$ strong focussing parameters). Proper matching of the input phase space distribution to the APF linac therefore requires a search for the optimum values of the four parameters. Moreover, the matched parameters will change as the APF parameters and/or the level of space charge change. We have devised an iterative method to determine the matched values of $\beta_\ell, \alpha_\ell, \beta_t, \alpha_t$ which involves following the border particles in the transverse and longitudinal phase spaces for one focusing period. We then determine the least square fit to the appropriate linearized matrix elements and from these determine $\beta_\ell, \alpha_\ell, \beta_t, \alpha_t$.

We use a frequency 800 MHz and inject the protons at 2 Mev. Most of the numerical results have been obtained for $N = 4$ with the synchronous phase pattern corresponding to $\phi_s^{(n)} = +80°, -80°, -80°, +80°, \cdots$, with $\phi_0 = 0$. For each value of k, we choose input emittances corresponding to a space charge detuning of both the longitudinal and transverse particle motions which runs from 5% to 25%. And for each selection of emittances, we search in $\beta_\ell, \alpha_\ell, \beta_t, \alpha_t$ to optimize the match of the input beam to the APF linac.

Preliminary results indicate that, for $\phi_0 = 0$, the current which can be carried by the APF linac increases with increasing k, until non-linear effects cause the current to saturate and eventually decrease. Furthermore, the maximum current seems to occur when the tunes with space charge are approximately 0.85 of those without space charge. Our criterion for satisfactory beam containment is that the transverse and longitudinal beam size do not grow by more than 60% in the first 100 cells. We show three typical PARMILA runs for the cases with 5000 macroparticles in a 6-D uniform distribution, and with the emittances $\epsilon_x = \epsilon_y = 0.01\pi - cm - rad$, $\epsilon_z = 0.0011\pi - cm - rad$, for the input beam. The values of k and the corresponding maximum currents obtained are

Case I $k = 0.564$, $I = 215$ mA

Case II $k = 0.602$, $I = 320$ mA

Case III $k = 0.638$, $I = 290$ mA

The figures show the phase spaces of injection beam, beam profiles and phase spaces of last cell of the transverse (x) and longitudinal (ψ) motion. Note the strong focussing oscillations with a period of four cells, the period of the focussing force corresponding to $N = 4$. The non-linear distortion of the longitudinal ellipse at the last cell can also be observed.

Work is now under way to explore the effect of increasing k with β on the beam size and emittance growth. Work is also now under way to demonstrate the effect of introducing a small value of ϕ_0, which is expected to increase the APF linac current. Results will also be explored for different values of ϕ_1 and N. Finally, we intend to explore methods of changing N continuously in order to maintain matched parameters as β increases, without exceeding the maximum obtainable accelerating gradients.

References

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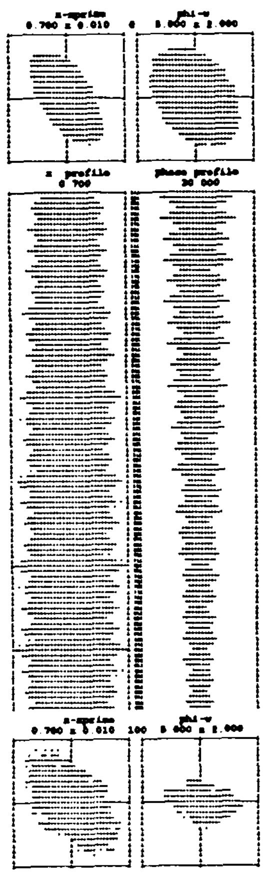

Figure 1: Transverse and Longitudinal beam profiles and phase space distributions for k=0.564, I=215 mA

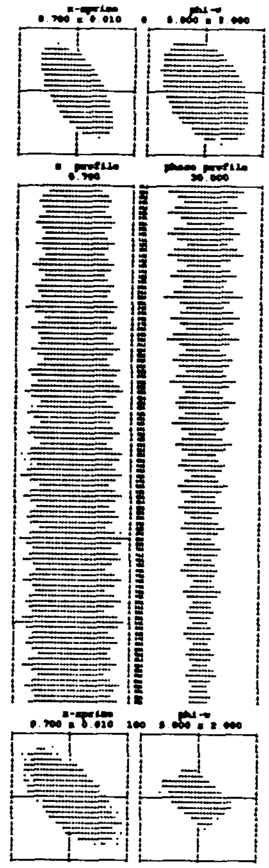

Figure 2: Transverse and Longitudinal beam profiles and phase space distributions for k=0.602, I=320 mA

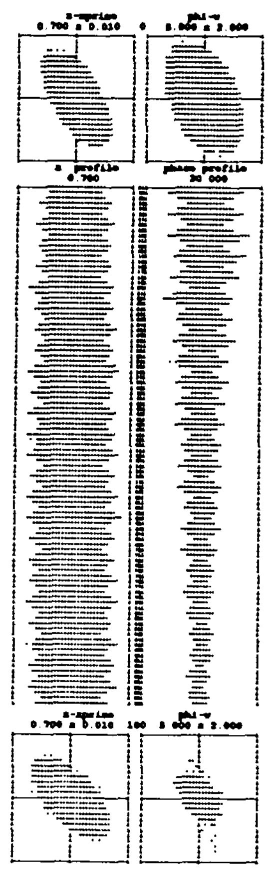

Figure 3: Transverse and Longitudinal beam profiles and phase space distributions for k=0.638, I=290 mA