Modelling of Attenuation and Scattering of Ultrasound in Polycrystalline Copper

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Modelling of Attenuation and Scattering of Ultrasound in Polycrystalline Copper

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January 1997

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This report concerns a study which has been conducted for the Swedish Nuclear Power Inspectorate (SKI). The conclusions and viewpoints presented in the report are those of the authors and do not necessarily coincide with those of the SKI.
Summary

This is the third in a series of three\textsuperscript{1,2} reports concerned with modelling the interaction of ultrasound with polycrystalline copper. Together they describe a research programme which was designed to select the best available models to describe the attenuation and scattering of ultrasound by polycrystalline copper and to compare the predictions of these models with observations made on a series of copper specimens.

The first report was a literature review which identified the Rose\textsuperscript{4} model to describe backscattering of ultrasound as a materials property (the Figure of Merit or FOM) and the ISMBB\textsuperscript{6} (Independent Scatterer Model for Broadband) model to account for the effects of the measuring system on the backscattered ultrasound. The second report described a data collection exercise in which fourteen copper specimens were characterised metallurgically and ultrasonically examined at three frequencies. This report describes the results of the analysis of the data reported in 2 using the models described in 1.

It is demonstrated that when the ISMBB model is used to determine a Figure Of Merit for fine grained equiaxed specimens it yields results which are in close agreement with predictions of the Rose model. Since the ISMBB is a model which is independent of material properties it is also suitable for correction of data from non-ideal specimens and it may therefore be used in the process of optimising testing conditions for copper materials which are of interest to SKI and SKB.

The Rose model agrees with experimental results for fine grained material tested at 5.0 MHz and it is clearly apparent from experimental data (A scans) that it is in qualitative agreement with experimental data across a very wide grain size range. Insufficient data was available for a rigorous quantitative check on the Rose model but insertion of a single adjustable parameter in the ISMBB model for all specimens at each test frequency enabled the experimentally determined values for FOM to be fitted convincingly to the Rose predictions across the full grain size range for 3.7 and 5.0 MHz tests. The fit of data from 2.1 MHz tests was unconvincing, probably because backscattered noise at this frequency was low compared with equipment-induced effects. The fit at the two higher frequencies is achieved by taking a simple average grain size, even when the grain size within specimens is highly variable. The role of
crystallographic twins in backscattering is explored and it is concluded that they do not act as scatterers in the same way as grain boundaries and when the adjustable parameter referred to above is used it not necessary to account for twins in the prediction of noise. Methods exist to make the analysis quantitatively rigorous and they are identified, this could be useful in future if it becomes necessary to predict absolute values of rms noise levels.

Rms noise varies with depth in specimens of variable grain size and the variability has been linked to the grain size variability in some cases. In order to fit experimental data to the model it is necessary to use mean grain size measurements and mean rms values measured over a representative depth.

Peak noise is generally higher than rms noise by a factor of five but isolated peaks are observed at substantially higher levels. These are believed to be the result of reflections from favourably oriented twins and they appear as false positive defect indications.
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13 APPENDIX B IMPLEMENTATION OF THE ISMBB MODEL

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1. Introduction

This is the third in a series of three\textsuperscript{1,2} reports concerned with modelling the interaction of ultrasound with polycrystalline copper. Together they describe a research programme which was designed to select the best available models to describe the attenuation and scattering of ultrasound by polycrystalline copper and to compare the predictions of these models with observations made on a series of copper specimens.

The motivation for the work was a recognition by SKI that they are likely to be required to approve an ultrasonic test scheme for large copper vessels to be used for nuclear waste disposal. It is well known that copper is difficult to inspect ultrasonically when its microstructure is anything but ideal and it has been suggested on the basis of practical tests\textsuperscript{3} that, 1.) a suitable ultrasonic frequency might be 2.0 MHz, and 2.) that this would require a grain size of 150\,μm or less if acceptable signal to noise ratios are to be achieved. The large size of the proposed canister and the choice of almost pure copper for its manufacture dictate that such a grain size can not be achieved reliably. Whilst the grain size which will eventually be achieved in production is not clear at this stage, it is clear that if ultrasonic testing is necessary there will be a strong requirement to optimise the choice of equipment, the test procedure and the analysis of results. This optimisation process would be greatly assisted if a mathematical model capable of predicting attenuation and scattering of ultrasound by the copper as a function of its microstructure were available.

The first report\textsuperscript{1} presents a review which summarises the literature related to modelling of interactions between metal microstructures and ultrasound with particular reference to attenuation and backscattering (or noise) in copper.
It concludes that a number of models for prediction of ultrasonic attenuation exist but that no experimental verification has been found for any. A common feature of all the models is that they divide the frequency spectrum into three regions as follows;

a) The Rayleigh region, where \( \lambda >> <d> \) and \( \alpha \propto u^2 f^4 <d^6> / <d^3> \)

b) The intermediate region, where \( \lambda \approx <d> \) and \( \alpha \propto u^2 f^4 <d> \alpha \) and

c) The geometric or diffusion region, where \( \lambda << <d> \) and \( \alpha \propto 1/<d> \)

Where \( \lambda \) is wavelength, \( \alpha \) is attenuation in the selected units (Nepers/M or Decibels/M), \( u \) is a measure of mechanical anisotropy in the grains, \( f \) is the ultrasound frequency and \( d \) is the grain diameter, the brackets \(< >\) refer to material averages. If the grain size in a specimen is uniform the expression for the Rayleigh region can be simplified to \(<d^3>\) as the two expressions are equivalent. However, where the grain size is not uniform the expression should not be simplified and the attenuation is greater because the \(<d^6>\) term places greater emphasis on the larger grains.

The copper of interest in this work could have grain sizes in the range 50 to 2000 microns. Using a typical value of 4720 ms for longitudinal ultrasonic velocity in copper, the wavelength of 2 MHz ultrasound will be 2.36 mm, and that of 5 MHz ultrasound 0.944 mm. Thus the attenuation could be characterised mainly by the Rayleigh region with the Intermediate region applying for extreme cases.

The first report also points out that the measured noise in any experimental situation derives both from the backscatter processes and the equipment used to measure it. Thus if any model treatment of the backscatter process is to be compared with practical observations it is necessary to have a procedure which accounts for the measurement method and removes its effects. The understanding of a procedure to account for the effects of the measurement method would enable optimisation of the measurement process.
A model proposed by Rose\textsuperscript{4} to predict the level of ultrasonic backscattering (noise) as a function of grain size was described. A second model (the ISMBB model) proposed by Thompson\textsuperscript{4} which predicts the effects of the measurement system on backscattering, was also described. Both models had been used by Thompson\textsuperscript{5} in experiments on copper. A backscatter coefficient derived by application of the second model to practical observations made on copper with a uniform grain size of 100\textmu m was in good agreement with a backscatter coefficient calculated as a material property using the Rose model. It was concluded that if this good agreement could be reproduced across the range of grain sizes of interest then it would provide the means of optimising the test procedures which are being sought and also a means of determining the minimum defect size which might reliably be detected by any specific procedure.

The second report\textsuperscript{2} describes the preparation and characterisation of a series of 14 specimens having grain structures varying from uniform fine grains to mixed coarse grains such as might occur in the disposal canisters. (Details of the origin, size and nominal grain size of all specimens are given in Table 1). It also describes the procedures for volume scanning the specimens using longitudinal waves at nominal 2.1, 3.7 and 5.0 MHz frequencies. Attenuation coefficients, mean rms noise levels and peak noise levels for each specimen tested at each frequency were recorded. The results are reproduced in Table 5 and the procedures are reproduced from reference 2 in Appendix A. It was concluded that, contrary to observations made by Foister\textsuperscript{7} on other systems, there was no systematic relationship between peak noise and rms noise levels in copper. The likely reason suggested for this was that the large number of crystallographic twins present in copper each act as reflectors to ultrasound to a greater or lesser degree. The peak noise signals in individual specimens may arise from reflection by large twins oriented perpendicular to the beam and there was some metallographic evidence to support this.

Attempts to obtain matching data using shear waves failed owing to extreme noise levels which masked the back face signals. The same broad band unfocussed probes were used in this work as were used in the longitudinal wave work. They were initially selected in order to examine the full specimen thickness at a single shot. There was some evidence that the use of focused probes and band-pass filters may have enabled the rms noise levels and attenuation coefficients...
to be measured using shear waves but this was not checked for all specimens since such a method using shear waves would not have provided results from shear wave work which would have been strictly compatible with the longitudinal wave programme.

This report describes the Rose model and the Thompson (ISMBB) model (with certain simplifications suggested by Foister for single phase materials) in sections 2 and 3. The implementations of the models are described in section 4 and the input data required for the model treatments is detailed in section 5.

### 2 The Rose Model

Rose has developed a rigorous analysis of ultrasonic backscatter from polycrystalline materials. The following description is taken loosely from reference 4 with acknowledgements to Professor Rose.

Ultrasound scatters from the microscopic single crystals that constitute polycrystalline solids. The scattering originates from the crystallite-crystallite variations in the density and elastic constants. For single phase materials, each crystallite has the same density and the same crystallite symmetry. Hence in single-phase materials scattering arises from the variation in velocity, which in turn is due to the anisotropy of the elastic constants and the more or less random orientation of the crystallites.

The goal of the model is "Given a sufficiently detailed description of the microstructure of a material, predict the backscattered ultrasound".

The motivation is twofold. First, engineers attempt to control the microstructure in order to optimise mechanical properties such as strength and fracture toughness when designing alloys. A calculable theory of backscatter will, in principle, allow them also to consider the acoustic quietness, (and it impact on inspectability), as a goal in the design process. Second, changes in the microstructure of alloys can be ascertained from backscatter measurements.

An exact calculable theory of ultrasonic backscatter is not possible due to the complexity of the ultrasound/microstructure interaction. Two limiting approximations are made in order to achieve compatible formulas. First assume that the acoustic contrast is weak; i.e. the deviations of density and velocity are small compared to the average density and velocity,
(known as the Born approximation). The second approximation is that the material is, on the average, homogeneous and isotropic.

The model also assumes that the material property deviations vary randomly and in a statistically independent manner from crystallite to crystallite. That is the material property deviations of one crystallite give no information about the material property variations of any other crystallite.

The model is developed by considering first the microstructure of the material and then the ultrasound scattered by a single grain embedded in a material whose density and elastic constants are determined by the Voigt average over all the grains.

Rose describes the Backscatter Coefficient $\eta$ as the power backscattered per unit volume and expresses it as:

$$\eta = \sum n \langle |A|^2 \rangle$$

for polycrystalline solids, given the limiting assumptions described above. Hence $n$ denotes the number of grains per unit volume and $A$ denotes the Born scattering amplitude for an imaginary system that consists of a single grain embedded in an otherwise uniform effective medium as described above.

The equation has the satisfying interpretation that the total power backscattered is equal to the sum of the power backscattered from each grain independently.

In the principal equation of the model the "Backscatter Coefficient" $\eta$ is given by:

$$\eta(k) = \frac{Q}{C_{11,0}} \frac{k^4 \langle a^4 \rangle}{2\pi \langle 1 + (2ka)^2 \rangle^2 \langle a^3 \rangle}$$

where $\eta$ is the Backscatter Coefficient, $k$ is the wave number, $a$ is the grain radius and

$$Q = \frac{16u^2}{525} \quad \text{and} \quad C_{11,0} = \left(11C_{11} + 4C_{12} + 8C_{44}\right) / 15$$
where $C_{ij}$ are the elastic constants

and where $u$ is the elastic anisotropy and is given by

$$u = C_{11} - C_{12} - C_{44}$$

**This is the Scattering Model.** It gives a prediction of the Backscatter Coefficient which is derived only from material characteristics and an assumed ultrasonic frequency.

In discussion with Professor Rose it was agreed that the assumptions in the model may reasonably be made for copper of uniform grain size and a random crystallographic texture.

3 **The ISMBB Model**

This description of the Independent Scatterer Model for Broad Band (ISMBB) is taken loosely from, and with acknowledgements to, a paper by Professor Margetan.

The ISMBB assumes that the observed ultrasonic noise is an incoherent superposition of the singly scattered echoes from individual crystallites (grains) in the metal specimen. The model expresses the root mean square (rms) level of the backscattered grain noise in terms of a factor which describes the metal microstructure, and other factors which describe the ultrasonic measurement system. The microstructural factor is referred to as the Figure-of-Merit (FOM) for inherent noise severity, and is frequency dependent. The FOM is equal to the square root of the Backscatter Coefficient of the Rose formulation. The FOM for a material is independent of the measurement system.

The model takes into account the various parameters associated with both the measuring equipment and the experimental set up by referring all measurements from within the volume of the material to a reference signal, normally taken to be the front face reflection from the specimen. This neatly removes the need to establish the acoustic efficiency of the transducer.

The effects of attenuation both in the water, (for an immersion inspection), and in the material are incorporated, as is the effect of diffraction which are determined by the physical characteristics of the transducer.

The terms in the principal equation of the model have been rearranged by Foister (to simplify implementation of a solution) with the result given below:
\[ \frac{\Gamma_{\text{med}(f)}}{\Gamma_{\text{ref}(f)}} = \eta^2 \frac{4\rho_0v_0\rho_1v_1^2}{\left( (\rho_0v_0)^2 - (\rho_1v_1)^2 \right) b^2}\pi f\left| D(f) \right| \left( \int_{v_0/v_1}^{1} G(z) e^{-\alpha z} dz \right)^2 \]  

where \( \Gamma_{\text{med}(f)} \) and \( \Gamma_{\text{ref}(f)} \) are frequency components of the backscattered noise from the material and a reference reflector respectively. In this presentation of the model the reference reflector is the front face reflection, an alternative approach (used by Thompson et al.) is to use a reflection from an optically flat fused quartz reflector situated at the same distance from the transducer as the specimen. This requires some modification to equation 3.1 which will not be discussed further.

The model accounts for the beam profile by the term \( G(z) \) where \( b \) is the probe radius and \( F \) is the focal length:

\[ G(z) = \frac{4.356b^2}{\left( \frac{z}{F} \right)^2 \left( 1 + \frac{0.56325v_1F}{b^2f} \right)} - 2\frac{z}{F} + 1 \]

and a diffraction correction for the probe is given by the term:

\[ D(f) = \left| 1 - e^{-\pi f} \left( J_0[s] + iJ_1[s] \right) \right|, \quad s = \frac{\pi fb^2}{v_1z_0}, \quad z = z_0 + z_1 \frac{v_1}{v_0} \]

Subscripts 0 and 1 refer to water and to metal. The exponential term in the integral accounts for attenuation in the material where \( \alpha \) is the attenuation coefficient and \( z \) is the axial

\[ Margetan^9 \text{ has pointed out that a mathematical error is present in equation 3.2 as follows. The term} \]

\[ \left( 1 + \frac{0.56325v_1F}{b^2f} \right) \text{ in the denominator should be} \left( 1 + \left( \frac{0.56325v_1F}{b^2f} \right)^2 \right). \text{ This correction will be used from this point on.} \]

\[ \text{ndtfinalr1} \]
distance from the probe. $z_u$ is the water path and $z_i$ is the distance in the specimen. $\rho$ and $v$ are the densities and velocities of sound respectively.

This model can be used to analyse backscattered noise to obtain the Backscatter Coefficient, or conversely if the Backscatter Coefficient is known the model can be used to predict the average noise spectral characteristics and average noise levels for various experimental set-ups.

4 Implementation of The Models

4.1 The Rose Model

The Rose model can conveniently be executed in Excell, (The Microsoft spreadsheet).

4.2 The ISMBB Model

In a first attempt at solution of the ISMBB model an implementation prepared by Rolls Royce plc and coded in "C" was used. This implementation included assumptions that the transducer was single frequency and that rms noise could be represented by a single value which corresponded to the rms of the mean value of each successive point on A scans taken over a defined area for 256 successive digitisation steps on the A scans (corresponding to a distance of 21 mm in the specimen). The implementation was prepared after Foister who justifies these assumptions on the basis that 1,) “the effect of frequency is so small that consideration of the centre frequency alone leads to no more than a ten percent error which is a small effect in ultrasonic terms, and 2) the limits of integration in the beam profile correction were set to enclose the thickness in the material (21mm) corresponding to the 256 points on the A scans which were used for the rms determination. These assumptions have been agreed by others and are not in question.

In view of disappointing results using the Rolls-Royce implementation Professor Thompson who originated the ISMBB model and used it successfully with copper was asked take two specimens from the present work and to carry out determinations of Figures of Merit (FOM = $\eta^{1/2}$) on his in-house experimental system. The reason for this was that this implementation (prepared by Professor Margetan) is integrated with the experimental system and this makes
it very difficult to input data from other sources. The Margetan implementation is different in
that it selects Fourier Components in the backscattered signal for processing and outputs
relationships between frequency and FOM and frequency and frequency and attenuation. It
takes the average rms value in a similar way to that described for the Rolls Royce
implementation but over 1024 digitisation steps in the A scans corresponding to a defined area
of the specimen. 1024 digitisation steps correspond to a fixed distance (24 mm at their
digitisation frequency) in the specimen and the limits of integration in the beam profile
correction are set to the same limits. It also sets the attenuation coefficient (\(\alpha\)) to zero.

The reason for this choice of value for \(\alpha\) is given by Thompson et al\(^6\), they argue that the
appropriate value to use is not the value transferred from pulse echo measurements but a value
related to the rate of decay of noise. They further argue that the two are different since the
pulse echo measurement is influenced by fluctuations in phase whilst noise is not. No
measurement technique is available for attenuation of noise defined in these terms. To
overcome this difficulty they used an inductive method. They calculated figure of merit for
copper specimens as a function of distance from the surface of the specimen using their
experimentally determined noise data and a range of assumed values of \(\alpha\). They showed that
for values of 0.2 Nepers per cm. or more for \(\alpha\) the computed value for FOM increased with
distance from the surface of the specimen. Using a value of 0 for \(\alpha\) led to a very slight
decrease in FOM with distance from the surface of the specimen. Since by definition FOM is
constant they concluded that the appropriate value to use is very close to zero. The values of
FOM were very small (because a fine grained material was used) but they differed
approximately by a factor of two for the zero and 0.2 assumptions for \(\alpha\) and they elected to
use a value of zero.

Following the trials by Professor Thompson the writers prepared an implementation of the
ISMBB in MathCad using the assumptions made by Foister that the centre frequency of the
transducer could be used with no modification for bandwidth and correcting a mathematical
error in the Foister version of equation 3.1 referred to in the footnote on page 13. Details of
this implementation are given in Appendix B.
5 Inputs to the Models and Experimental Results

5.1 Material properties

Standard reference values of material properties taken from reference 1 are given in Table 4, they have been used to solve the Rose model for a range of grain sizes at the three frequencies of interest and the results are given in Figure 1.

5.2 Grain size

A knowledge of specimen grain size is needed to measure the Rose value of backscatter coefficient in order to compare it with value derived from the ISMBB model. Interpretation of grain size in this context presents several difficulties. The first is the treatment of crystallographic twins. Annealed copper is always heavily twinned and since the crystal orientation changes across twin boundaries, so does acoustic impedance and twins should therefore reflect ultrasound. Rose and Thompson accept that twins are likely to act as reflectors but point out that, since there are precise orientation relationships between parts of the metal crystal on either side of a twin boundary a special treatment is needed to allow for them. As no special treatment is available at this time they neglect them.

In view of the uncertainty raised by the effects of twins, for the purpose of this work, measurements of grain size were made both treating twins as grain boundaries and neglecting twins altogether. This has allowed comparison of the two approaches.

The Thompson method of measuring grain size uses a statistical method applied to a photomicrograph. In this method a line of fixed length is projected onto a photomicrograph in different locations a very large number of times (typically 10,000) and the probability of it cutting a grain boundary is measured. This is repeated for lines of varying length so that the probability function for a line of given length cutting a grain boundary may be constructed. The function is of the type:

\[ P = e^{\frac{-l}{b}} \]

where \( P \) is probability, \( l \) is line length and \( b \) is mean grain radius.
This method was used by Thompson et al on two uniform fine grained specimens (specimens 13 and 14) when they carried out independent checks referred to in 4.2 above. It only measures grain boundaries and twins are therefore excluded from the measurements by this route.

Owing to the variation in grain size across some specimens used in this work it was decided to use a mean linear intercept method (as in ASTM Designation: E112-88) in which the interception of every grain (or twin and grain) boundary along a line across the full thickness of the specimen was measured. The line selected was the centre line of the specimen which was also the centre line of the volume of material from which data would be used for examination of the model treatment.

In order to make some allowance for non-uniform grain sizes when attenuation is considered the parameter,

\[
\text{MEGD} = \sqrt[3]{\frac{\langle d^6 \rangle}{\langle d^3 \rangle}}
\]

was calculated.

Results of grain size measurements are given in Table 2. and mean effective grain sizes are given in Table 3. Grain sizes measured counting twins as grain boundaries have been reported elsewhere².

5.3 Transducer Characteristics

The ISMBB Model requires information relating to the transducer used to obtain ultrasonic backscatter signals. The parameters required include frequency, effective diameter, focal length and the water path between the transducer and specimen. The techniques used to obtain this information are described in Appendix A. and the information appears in Table 4.
5.4 Inspection Results

5.4.1 Experimental Observations

5.4.1.1 Attenuation Coefficients

Attenuation coefficients for the frequencies examined are presented as a function of grain size and mean effective grain size in Figures 2 and 3. The same information is presented in an alternative form in Table 3.

5.4.1.2 rms Amplitude

Figures 4 to 17 show the rms amplitude as a function of depth into the specimen at the three frequencies used in the study. The traces start after the uninspectable dead zone arising from the front face signal.

5.4.2 Derived Data for The Implementation of the ISMBB model

When attenuation of noise is assumed to be zero two groups of experimental data are required for implementation of the ISMBB model. The first group, i.e. the average value of the backscatter signal (known as the "root mean square" or rms level), is given together with the peak values of noise for each specimen at each frequency in Table 5 where they are tabulated in order of grain size. The procedures to extract these data from the raw experimental results are described in the Appendix A.

The magnitude of the ultrasonic reflection from the front, (or entry), surface of the specimen is the second data set which is required for implementation of the ISMBB model. As all specimens were prepared to the same surface finish and were situated at the same distance from the transducer, it should be a constant for each transducer (see section 3).

Examination of the experimental records revealed that the front face reflections recorded from the individual specimens for each transducer were far from constant. The cause of this was traced to an experimental recording error. The values required in the attenuation experiments were not and should not be constant since the gain on the instrument was adjusted for each reading and the difference in gain required to equalise the signal from front and back faces was used in the process of assessing attenuation. These were the only front face signals measured and the front face signals corresponding to the noise measurements are not available.
In order to overcome this difficulty and recognising that the front face signal should be constant, it has been set as an adjustable parameter so that in the implementation a value is selected which gives the best fit of the experimental data to the Rose prediction for FOM.

6 Comparison of Experimental and Theoretical Results

The table below gives the FOM determinations made by Professor Thompson and his team on specimens 13 and 14 compared with predictions from the Rose model.

Figures 19, 20 and 21 present the curves relating Figure of Merit (FOM) and Grain Size for each frequency calculated from the Rose model. The experimental data have been fitted by choosing the single value of $\Gamma_{ref}$ which gives the best fit for each curve. The value of $\alpha$ has been set at zero since the choice of any other value would lead to a factorial change in the right hand side of equation 3.1 and the reciprocal factorial change in value of the adjustable parameter $\Gamma_{ref}$.

7 Discussion

7.1 Grain Size Measurement

The Rose model predicts backscatter coefficients ($FOM^2$) as a function of grain size at any particular frequency and it is based on an assumption of uniform grain size. Any test of the model therefore requires an accurate measure of grain size and possibly an allowance for the distribution of grain sizes if the uniform grain size assumption is in doubt.

Comparison of values in Tables 1 and 3 shows that serious discrepancies arise between the standard method of estimating grain size and the mean linear intercept method which was used in this work, both of which are based on mathematics which assume spherical grains. It is considered that the discrepancies arise as a result of sampling problems when a comparative method is used for a structure having mixed grain sizes and from estimating errors. Both the comparative method and the mean linear intercept method used here are based on assumptions.

<table>
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<tr>
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<th>Specimen 13</th>
<th>Specimen 14</th>
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<tr>
<td>Measured FOM (ISMIB)</td>
<td>0.091</td>
<td>0.352</td>
</tr>
<tr>
<td>Theoretical FOM (Rose)</td>
<td>0.091</td>
<td>0.15</td>
</tr>
</tbody>
</table>
of equiaxied grains and a uniform grain size but sampling across the full specimen width allows
the variation in grain size to be taken into account. For this reason the mean linear intercept
version has been accepted rather than the comparative method.

The "Thompson" method (see section 5.2) has its attractions in that it takes a very large
number of readings in order to get a statistically reliable picture, however in working from a
photomicrograph it samples only a small part of the section and it is therefore prone to the
sampling problems of the ASTM comparative method when non uniform structures are
considered. The mean linear intercepts quoted for specimens 13 and 14 in Table 6 are from the
Thompson work and they agree very well with the values in Table 1. It must be recognised
however that these were the two ideal specimens and the same fit can not be expected in the
non uniform materials.

In view of these points it has to be accepted that there is no accurate method for measurement
of grain size and the fit of experimental data to any model treatment will be degraded by
inaccuracies arising from this cause.

The above discussion makes no mention of crystallographic twins. It is considered by the
present authors, and agreed by Thompson and co-workers, that twins have the right
characteristics to be reflectors of ultrasound but so far there is no method for taking reflection
of twins into account. The earlier work of Thompson as well as Thompson's contribution to
this work ( see section 6) indicated that good agreement between theory and experiment is
achieved when twins are neglected. However in view of the uncertainty on this point figure of
merit results have been compared with Rose predictions, both on the basis of grain size
measurements made neglecting twins and made treating twins as grain boundaries, in section
7.3.

Grain size measurements (see section 7.2) indicated that the requirement for uniform grain size
in the specimens has not been met. In order to study the effect of the non uniform grain sizes
observed figures of merit have been compared with the Rose predictions on the basis of mean
grain size and mean effective grain size as reported in table 3.
7.2 Relationships between Microstructure and Peak noise and rms noise

Table 1 presents the origin, the mode of preparation and the nominal mean grain size for each specimen used. The grain sizes were measured using the ASTM comparator charts and converted to mean linear intercept using the ASTM method.

As a first step in an examination of relationships between structure and ultrasonic response we examine the rms, peak recorded amplitude and attenuation as a function of microstructure and grain size. Average rms noise for each specimen is plotted against depth in the specimen in order of increasing grain size in Figures 4 to 17 so that they may be viewed sequentially. The values of mean rms noise are given in Table 6. The following notes are taken from earlier work.

Specimens 13 and 14 (Figures 4 and 5) were the two specimens produced by extrusion and had the smallest grain size in the series at 20-30 and 40-45 microns (ASTM method, Table 1, subsequently measured at 30 and 44 microns using the Thompson method, Table 2). Specimen 13 had a very fine grain size through the depth of the specimen with little variation and no coarse grains. The rms shows a low and consistent value through the depth examined. Specimen 13 produced the lowest peak amplitude of all the specimens, which is consistent with fine grain size and uniform structure and an absence of isolated coarse grains. This is regarded as the ideal structure as far as the assumptions in the Rose model is concerned.

Specimen 14 was similar in structure to specimen 13 but with a population of coarse grains in the centre. This appears to be reflected in the increased rms level. There were also isolated peaks in the reflections which may be attributable to the coarse grains.

Specimen 8 was produced by hot rolling. The grain size was uniform through the thickness but there was a distribution of coarse grains spread throughout the structure. The rms (Figure 6) is similar to specimen 13 which had a similar distribution but at 3.7 MHz there was a peak amplitude of 110 recorded which was the largest signal from any specimen at that frequency (Table 6). This may be attributable on microstructural evidence to one of many coarse grains (up to 350 micron diameter) distributed through the structure. Whilst it is not demonstrated it is suggested that this very strong signal arises from reflection at a favourably oriented crystallographic twin.
Specimen 7 was also hot rolled but examination of the structure showed a band of coarse grains close to the surfaces and in the centre. There is an increase in the rms corresponding to the central area of the thickness of the specimen, at 15 to 25 mm (Figure 7), which corresponds with the presence of coarse grains.

Specimen 5 was the only forged material of the range. It was found to have a more or less uniform grain size but with a band of coarse grains 2 to 7 mm from each surface. There appears to be a correspondence with this structure in the form of the rms with higher levels indicated in that depth range (Figure 8).

Specimen 6 shows an rms level undulating with depth (Figure 9). The rms trace is similar to that of specimen 7 which has a similar structure.

Specimen 9 was hot rolled and produced a higher level of rms values than specimen 5 which had a similar grain size. There were also some regions of increased rms which suggest regions of coarser grains (Figure 10), however there was no evidence of such regions found during metallurgical examination. This behaviour is not explained.

Specimen 12 shows an increase in rms level (Figure 11) with increasing grain size when viewed with the results from other specimens. The peak value is not exceptional compared with other cases examined. In this case it is suggested that the “peakiness” in the rms signal is a function of the isolated large grain scatterers rather than with bands of coarse grains.

The increase in mean grain size for the next specimen in the range, specimen 2, is not substantial (from 186 to 228 microns) but it was reported that a small number of very large grains were distributed through an otherwise relatively fine grained structure. A check on mean intercept in this part of the work indicated that 30 grains had intercepts exceeding 400 microns and together they accounted for 23 mm or 40% of the specimen thickness. The associated rms and peak signals appear in figure 12 and table 6. Whilst rms and peak amplitudes follow the general trend with grain size there is no obvious additional effect arising as a result of the very mixed structure.

Specimen 4 (Figure 13) responded in a similar way to specimen 2 there is considerable variation in rms noise as a function of depth and the and peak noise level increases in line with increasing grain size (mean intercept of 298 compared with 228 for specimen 2). This
specimen also had a mixed grain size structure with bands of very coarse grains concentrated near each surface\textsuperscript{2}. Measurements made in this stage of the work indicated that in all 45 grains had diameters exceeding 400 microns and they accounted for 45\% of the specimen thickness. In this case the rms noise values are higher in the regions where coarse grains are segregated.

Specimen 1 continues the trend with increasing grain size producing increasing rms plots (Figure 14). The metallography showed bands of coarse grains from 6 mm. to 12 mm. from each surface\textsuperscript{2}.

Specimen 3 similarly continues the pattern of increasing rms and peak values with increasing grain size. There is good correlation between the bands of coarse grains at 6 to 12 mm from each surface\textsuperscript{2} and large rms and peak values (Figure 15).

Specimen 11 had a complex structure in which only six grains accounted for some 25 mm of the thickness in the mean intercept measurement\textsuperscript{2}. The remainder of the thickness consisted of very fine grains, thus reducing the indicated grain size to 836 microns mean linear intercept. The effect of this structure appears to result in a less "peaky" rms plot (Figure 16) although the mean rms is consistent with the mean grain size.

Specimen 10 consists of relatively few, very large grains, at some 1144 microns mean intercept, The largest grain measured was 5mm. diameter but there was no observed pattern to the grain size distribution. It appears that at these grain sizes the scattering phenomenon was reducing in effect (Figure 17) as demonstrated by reduced rms and peak values. However, interestingly the attenuation value was the highest recorded (Table 3). This suggests that the ultrasonic beam was being scattered but not back to the originating transducer.

A peakiness in rms plots has been referred to for most specimens. In principal we would expect the rms versus depth relationship to be smooth for a random structure because each point on the line is the average of the value at that depth from some 400 A scans (see appendices A.4-A.6). Each A scan takes signals from a circle of diameter 10-15 mm and therefore each individual point on the rms line corresponds to an average from an area of close to 20 mm. x 20 mm. The fact that it is not smooth suggests a periodicity in the reflectivity of the material with depth. This periodicity is in some cases identified with bands of coarse grains or possibly isolated coarse grains in the structure. The effect of this peakiness on the implementation of the ISMBB model could be to cause an expected variation in $\Gamma_{rms}$ with
depth of at least a factor of 2 for an individual specimen. This would lead to exactly the same factor in variation in the figure of merit (FOM) calculated using equation 3.1. It is clear therefore that if consistency is to be achieved across a group specimens the rms noise must be determined over a sample depth which takes in several cycles of the peaks and troughs.

From the information above and sequential scanning of figures 4 to 17 it is clear that there is a correlation between mean grain size and mean rms amplitudes which is not disturbed by large variations in the range of grain sizes in the specimen or by the segregation of coarser grains. Increasing mean grain size up to specimen 3 (356 microns mean grain diameter) are coupled with an increase in mean rms noise levels. The two specimens with mean grain sizes coarser than 356 microns showed progressively decreasing mean rms values with increasing grain size. There is also a suggestion that localised regions of coarser grains cause local increases in the rms level but that the effects of this are averaged out when mean rms values are compared with mean grain size. This is qualitatively in agreement with the predictions of the Rose model as shown in Figure 1.

Peak noise levels are plotted as a function of mean rms levels for all three frequencies used in figure 18. All but three points lie close to a straight line through the origin. Foister working with titanium reported a straight line correlation between peak and mean rms amplitudes and most ultrasonic inspections depend on this in order to recognise the levels above the noise base which correspond to defect signals. The trend line indicates that peak noise is likely to be close to five times higher than mean rms noise. The three points which obviously deviate from the general correlation are from specimen 8 at 3.7 MHz which is 12 times higher than the mean rms, specimen 6 at 2.2 MHz which is 9 times higher than the mean rms and specimen 3 at 5 MHz which is 6 times higher than the mean rms. These indications appear as defect indications where no defects were present. They have been correlated with regions of coarse grains in the specimens and it has been suggested that they may be caused by reflections from favourably oriented twins. This is an attractive explanation since twins might be expected to have ideal characteristics for reflectors and large twins are observed in the appropriate regions of the specimens.

It has been pointed out that increasing ultrasonic frequency (reducing wavelength) causes a general increase in background noise since smaller microstructural features start to reflect.
Thus changing to a higher frequency may cause a significant increase in background noise whilst having a lesser effect on an isolated reflector and the ratio of peak to rms may therefore decline with increasing frequency. Careful examination of the .vol files reveals that the peak noise value is, in many cases located in a region of the specimen where coarse grains have been observed, and examination of combined effects of varying grain size together with changing ultrasonic frequency in Table 6 supports the above as an explanation of why the same peaks were not observed at all three frequencies. For example specimen 6 has the highest ratio of peak to rms noise of all specimens at 2.1 MHz, it has a high, but not the highest ratio at 3.7 MHz and at 5 MHz it is no higher than the norm.

7.3 Modelling of Backscatter and Equipment Effects.

The objective of this project was to search for a relationship between the metallurgical structures (principally grain size) of a group of specimens and their ultrasonic backscatter noise and attenuation characteristics. Ultimately the establishment of the backscatter coefficient as a fundamental parameter could provide a means of predicting defect detectability. The literature study highlighted the importance of the characteristics of the test system and recognised that a satisfactory model for the interaction of ultrasound with structure requires a model to describe and account for the influence of the test system before it can be tested. The Rose and ISMBB models are devised to fulfil these functions and if they can be shown to work for the practical materials of the copper/steel canister then they provide not only a method for determination of defect detectability but also a method for optimisation of the test equipment and procedures for the practical case.

Rose has pointed out that his model was developed on the assumption that the grain size is essentially uniform within the structure of the material being modelled. Thus there would be a uniform effect of the scattering behaviour from any sample of material.

Thompson and his co workers (see section 6) have produced a good agreement between the Rose prediction and experimental measurements at 5 MHz (corrected using the ISMBB corrections) on their own specimens and on both the ideal specimens from this work (specimens 13 and 14). This indicates that, for ideal specimens tested at a frequency of 5 MHz at least, both models come close to describing the real situation. Unfortunately these
workers do not quote results at other frequencies for copper, (although they do for other materials) and they have only considered a narrow band of fine grain sizes. However the ISMBB model is independent of material structure, it is concerned only with the modification to noise signals arising from equipment parameters. It can therefore be concluded that the ISMBB corrections to experimental data provide input data to test the sensitivity and reliability of the “Rose” model. It is also concluded that it is valid to use the ISMBB model in selecting equipment parameters to optimise the sensitivity and reliability of testing.

In considering the limits on the range of applicability of the Rose model the agreement between the experimental data adjusted for equipment effects using the ISMBB model and the predictions of the Rose model must be examined together with errors arising in the implementation of the ISMBB model and the effects of structure in the test materials on the predictions of the Rose model. The predictions of the Rose model are compared with the experimental observations (corrected using the ISMBB model) in Figures 19, 20 and 21.

The fit is convincing for the 3.7 and 5 MHz cases but less so for the 2.1 MHz case.

There is uncertainty associated with the assumption in the implementation of the ISMBB model that the attenuation coefficient for backscattered noise is constant for all specimens and it is estimated that as a consequence of this certain of the figure of merit values may be underestimated by up to a factor of 2 (see Appendix B). This is the only source error which has been recognised in the implementation of the ISMBB model. It could account for some of the discrepancy between experimental and theoretical data at 3.7 and 5 MHz (Figures 20 and 21), but it does not account for the discrepancies at 2.1 MHz (Figure 19) where experimental values of FOM in the low grain size region exceed the predicted values.

The Rose model is a material model relying on materials properties, grain size and ultrasonic frequency only. Of these grain size is the only one measured for this work and it is assumed by the model to be uniform.

There is uncertainty in the measurement of grain size (see section 7.1) and it is far from uniform in most of the specimens studied (as detailed in section 7.2). There is also some uncertainty concerning whether twins should be treated as grain boundaries or be neglected.
It is quite possible that the quality of the fit is influenced by errors in grain size measurement but this would not cause a difference between the quality of the fit at 2.1 MHz and the quality of the fit at the other two frequencies.

In an earlier report\(^2\) mean grain size was measured by including twins as grain boundaries. Figures 22, 23 and 24 show FOM data calculated using these grain size values. In these cases as in the earlier cases \(\Gamma_{\text{ref}}\) has been treated as an adjustable parameter. The selected values for \(\Gamma_{\text{ref}}\) for the various cases are given below.

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The fit at 3.7 and 5.0 MHz is good for all specimens except specimen 10, and the fit at 2.1 MHz is better than under the without twins data.

Counting twins as grain boundaries has reduced the apparent grain size of all specimens except specimen 11 by an approximate factor of 2 and it has reduced the apparent grain size of specimen 11 by a factor of 5. Thus in figures 22, 23 and 24 the point for specimen 11 has joined the group of grain sizes below 110 microns radius which are now all more closely grouped in grain size and closer to the predicted line. The point for specimen 10 (which is absent on figures 19 and 22) is only reduced by a factor of 2 on the grain size axis and is therefore well off the line. For the 3.7 and 5 MHz cases the change in the adjustable parameter is small and therefore the effects on the FOM value is correspondingly small, for the 2.1 MHz case the change is large and therefore the change in FOM is correspondingly large. It is concluded that the apparent improvement in the fit of the experimental data arising from counting twins as grain boundaries is artificial and there is no evidence from this work to support the view that twins should be counted as grain boundaries.

The FOM from the experimental data has been plotted as function of mean grain radius in figures 19-24. In view of the serious variability in grain size it is surprising that the fit is so good at 3.7 and 5.0 MHz. It is clear from an comparison of the mean effective grain sizes and mean intercepts presented in Table 3 (and the corresponding table for with twins data in the ndtfinalr1).
earlier work\(^2\)) with Figures 20 and 21, and 23 and 24 that a correction of the type used to calculate mean effective grain size is inappropriate. It has the effect of moving the peak in the FOM grain size relationship to unrealistically high values (compared with the Rose prediction). It appears therefore that the average grain size has been an adequate measure of grain size for the Rose model in these cases. This is contrary to the assumption in the model but in agreement with the qualitative observations referred to in section 7.2.

It has to be concluded that figures of merit measured at 3.7 and 5.0 MHz can be fitted to the Rose model but measurements taken at 2.1 MHz can not. This is in spite of the qualitative observation on Figures 2-15 that the rms noise appears to vary according to the Rose model. The reason for the failure to fit the corrected data to the model may be that at this frequency structure noise is very low and the rms values measured are dominated by equipment effects.

### 7.4 Relationships between Microstructure and Attenuation

Figure 2 shows the measured relationships between attenuation and mean grain diameter at the three frequencies used. It is clear that attenuation increases with grain size for testing at 3.7 and 5 MHz but at 2.1 MHz increases in attenuation are only observed for the extreme grain sizes of specimens 10 and 11. When the attenuation data are plotted against grain diameter cubed the horizontal axis is simply extended with no change in the trend. Figure 3 shows the attenuation data plotted against the mean effective grain size \((<d^6>/<d^3>)^{1/3}\). This treatment also spreads the horizontal axis but in this case the odd size distribution in specimen 10 (a central region of very fine grains) causes it to have a lower mean effective grain diameter than specimen 11. The relationships between attenuation and grain size with this treatment shows an initial increase for effective diameters up to 1500 microns followed by a decrease. The wavelengths of ultrasound in copper at 2.1, 3.7 and 5 MHz are 2.25, 1.27, and 0.94 mm respectively. When these wavelengths are compared with the information plotted on Figure 2 it is possible to suggest that the turning points in Figure 2 are a result of transitions from the Rayleigh region to the geometric or diffusion region but there is far too little data to draw a firm conclusion.

What is clear is that across the grain sizes of interest attenuation at 2.1 MHz is almost constant whilst at 3.7 and at 5 MHz attenuation increases with increasing grain size. Attenuation at 5
MHz is higher than attenuation at 3.7 MHz whilst at 2.1 MHz attenuation is between the other two for most grain sizes. For a copper canister of thickness 50 mm. it is to be expected that attenuation is likely to be in the range 25 to 50 dB depending on frequency.

8. Conclusions

1. The Independent Scatterer Model for Broad Band inspections (ISMBB) developed by Thompson et al. provides a way to treat experimentally determined rms noise levels to determine a Figure Of Merit (FOM) for copper specimens. The FOM is a material parameter which quantitatively describes its ultrasonic back scattering characteristics (noisiness). It is dependent on material structure and independent of the test system used to derive it.

2. The ISMBB models the effect of the test system, it may be used together with an attenuation coefficient (measured using a back face echo) to select optimum ultrasonic test parameters from the point of view of signal to noise ratio.

3. Implementation of the ISMBB requires a value for an attenuation coefficient for noise which is less than the attenuation coefficient for the main signal. A method is available for determination of this coefficient and for fine equiaxed grained copper it may be taken as zero. Errors in the value of this attenuation coefficient lead to errors in the derived value for figure of merit but they do not impair the usefulness of the model for optimisation of test parameters.

4. When a measured value for the attenuation coefficient for noise (zero to 20 Nepers per metre) was used together with experimental rms noise values, details of the test parameters and a measurement of grain size, a figure of merit was derived which agrees within the bounds of experimental error with a theoretically determined value of FOM arising from a material model developed by Rose.

5. The Rose model predicts figure of merit as a function of grain size and ultrasonic frequency from material physical properties but it is limited by an assumption of uniform equiaxed grains. In this work it was apparent from visual inspection of noise records that the Rose model qualitatively describes the noisiness of specimens as a function of grain size and test frequency over a wide grain size range even when specimens have very uneven grain sizes.
6. It has not been possible to test agreement between the Rose model and practical measurements made on all specimens at all frequencies (adjusted to remove equipment effects) in this work with quantitative rigour owing to a failure to foresee all the information needs at the outset. It has been possible to test agreement at a qualitative level by assuming an attenuation coefficient for noise of zero for all specimens and by inserting a single adjustable parameter in the ISMBB model for all specimens at each frequency. When this is done a good fit between experimental observation and theoretical prediction is achieved for all specimens tested at 3.7 and 5.0 MHz. The fit at 2.1 MHz is not convincing, possibly because real noise levels are low compared with equipment effects.

7. The quality of the fit between experimental data indicates that the Rose model gives a good qualitative description of rms noise as a function of mean grain size even when the grain sizes within a specimen are very uneven.

8. Crystallographic twins do not backscatter ultrasound in the same way as grain boundaries and their role is not fully understood.

9. A quantitatively rigorous examination of the Rose model would require that the attenuation coefficients for noise and reference values of rms noise should be determined for each specimen at each frequency. This would allow the fit between experimental data and theoretical predictions at 2.1 MHz to be interpreted with more confidence and the role of crystallographic twins to be explored more fully.

10. There is a considerable variation in rms noise with depth in a specimen and this may be related to variations in grain size. For this reason it is necessary to average noise over a depth in the specimen which includes a representative sample of the structure. When this is done the mean rms varies with mean grain size according to the pattern predicted by the Rose model.

11. Peak noise levels are generally up to five times higher than the mean rms. However isolated peaks have been observed which are up to twelve times higher than the mean rms. These peaks are not associated with defects and it is considered that they may arise as reflections from large favourably oriented crystallographic twins. Such peaks appear as “false positive” indications and special care will be required if they are to be discriminated from real defects in routine inspections.
12. Attenuation coefficients for the main ultrasonic beam have been measured. The practically useful result is that attenuation increases with mean grain size for all three frequencies and grain sizes in the range of interest in spite of the within-

13. specimen variability of grain size. Expected attenuation levels for 50 mm. thick material with grain sizes in the range of interest will be 25-50 dB depending on frequency.

14. When attenuation coefficient is plotted against \((\frac{d^6}{d^3})^{1/3}\) the plots show a turning point between 1mm. and 2mm. for all frequencies. This suggests that the Rayleigh criterion applies for the lower grain sizes and the intermediate criterion applies to the higher grain size examined. This is in line with theoretical expectations.

9 References

2. Bowyer WH and Crocker RL-A Study of Attenuation and Scattering of Ultrasound in Polycrystalline Copper, SKI report 96:27
3. Day AB An investigation into Optimisation of NDT of Spent Nuclear fuel Canister - Phase II - Feasibility study ‘Effectiveness of Ultrasonic Weld Inspection Techniques’ TWI Report No29921/4/93
10 Tables
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Table 1 Details of Specimens Prepared for Ultrasonic Inspection
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<td>0.25</td>
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<td>724</td>
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<td>2623</td>
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<td>0.45</td>
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<td>572</td>
<td>1502</td>
<td>0.58</td>
<td>0.53</td>
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Table 3  Attenuation Coefficients in dB/mm and Mean Effective Grain Diameters Listed in order of Mean linear Intercept
### Table 4  Transducer Characteristics

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Type</th>
<th>Nominal Frequency MHz</th>
<th>Actual Frequency MHz</th>
<th>Band Width(^1) MHz</th>
<th>Diameter mm</th>
<th>Effective Diameter mm</th>
<th>Water path mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metrotec</td>
<td>Flat</td>
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<td>5</td>
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\(^1\) bandwidth to 6 dB drop

### Table 5  Standard Values

<table>
<thead>
<tr>
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<th>Copper</th>
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<tr>
<td>Velocity (longitudinal) ms(^{-1})</td>
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<tr>
<td>Density kg/m(^3)</td>
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Table 5  Standard Values
<table>
<thead>
<tr>
<th>Specimen No</th>
<th>Grain radius microns</th>
<th>Ultrasonic Data</th>
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<tr>
<td></td>
<td></td>
<td>2.1 MHz</td>
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<tr>
<td></td>
<td></td>
<td>rms amp</td>
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<td>5.55</td>
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<td>7.52</td>
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<tr>
<td>10</td>
<td>572 x x</td>
<td>9.35</td>
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Table 6  Ultrasonic data for all specimens (ordered by grain size)
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<tr>
<th>Specimen number</th>
<th>Grain Dia. mean intercept microns</th>
<th>BACKSCATTER COEFFICIENT</th>
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<td></td>
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<tr>
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Table 7  Backscatter coefficients for all specimens calculated using the ROSE and ISMBB models
11 FIGURES
Figure 1 Rose Model Predictions for Backscatter Coefficient versus Grain Diameter

Figure 2 Attenuation Coefficients versus Mean Grain Diameter
Figure 3 Attenuation Coefficients versus Mean Effective Grain Diameter

Figure 4 rms Amplitude versus depth-specimen 13
Figure 5  rms Amplitude versus depth-specimen 14

Figure 6  rms Amplitude versus depth-specimen 8
Figure 7  rms Amplitude versus depth-specimen 7

Figure 8  rms Amplitude versus depth-specimen 5
Figure 9 rms Amplitude versus depth—specimen 6

Figure 10 rms Amplitude versus depth—specimen 9
Figure 11 rms Amplitude versus depth-specimen 12

Figure 12 rms Amplitude versus depth-specimen 2
Figure 13  rms Amplitude versus depth-specimen 4

Figure 14  rms Amplitude versus depth-specimen 1
Figure 15  rms Amplitude versus depth-specimen 3

Figure 16  rms Amplitude versus depth-specimen 11
Figure 15  rms Amplitude versus depth-specimen 3

Figure 16  rms Amplitude versus depth-specimen 11
Figure 17  rms Amplitude versus depth-specimen 10

Figure 18  peak noise versus rms noise for all specimens at all frequencies
Figure 19  Theoretical backscatter coefficient versus experimental rms amplitude at 2.1 MHz

Figure 20  Theoretical backscatter coefficient versus experimental backscatter coefficient at 3.7 MHz
Figure 21  Theoretical backscatter coefficient versus experimental rms amplitude at 5.0 MHz

Figure 22  Theoretical backscatter coefficient versus experimental rms amplitude at 2.1 MHz when twins are counted as grain boundaries
Figure 23 Theoretical backscatter coefficient versus experimental rms amplitude at 3.7 MHz when twins are counted as grain boundaries

Figure 24 Theoretical backscatter coefficient versus experimental rms amplitude at 5.0 MHz when twins are counted as grain boundaries
Appendix A

A.1 Transducer Characteristics

The ISMBB model removes the effect of the measuring equipment from the backscatter coefficient calculation. As part of this process a correction is made for the divergence of the ultrasonic beam both in the water path to and from the transducer to the specimen, and in the specimen itself. As the calculation is based on experimental data the actual operating characteristics of the transducer must be found, rather than the more commonly used "nominal" values.

A.1.1 Frequency

The actual operating frequency was measured by taking the frequency spectrum of the signal reflected from a reference target in a Rolls Royce Reference Standard test block. The operating frequency is the frequency at which maximum amplitude occurs and the band width is taken as the width of the spectrum in Hz at 6 dB below peak amplitude.

A.1.2 Effective Diameter

The effective diameter of the transducer is the apparent diameter which appears to be in operation when assessed by examining the reflection of the beam from a target. This differs from the actual diameter of the piezoelectric crystal in the transducer because although the crystal is often considered as a "piston oscillator" in practice it never is. There is a non-uniform response over the oscillating surface. The effective diameter is calculated by working back from the measured response to a diameter that would be in operation to produce the beam that is actually measured.

The first step in the measurement is to find the Near Point. This is the last diffraction maximum along the beam axis moving away from the face of the transducer.

Next the actual wavelength, \( \lambda(w) \), in water is calculated using the frequency, \( f \), measured in Section A 1.1 and the velocity of sound in water, \( v(w) \), via the well known relationship:

\[
\lambda(w) = \frac{v(w)}{f}
\]
The relationship between crystal diameter, D, the wavelength and the Near Point, N, is given by:

\[ N = \frac{D^2}{l(w)} \]

So by measuring both N and l(w), (via the frequency measurement), the effective transducer diameter can be found.

### A.1.3 Water Path

The water path is the separation between the face of the transducer and the entry surface of the specimen.

This is measured by noting the time for ultrasound to travel to and from the specimen by monitoring the elapsed time between the excitation pulse, (colloquially the “main bang”), and the front surface reflection, and using the velocity of sound in water to give distance via:

\[ \text{Water Path} = \text{Transit Time} \times \frac{V(w)}{2} \]

The magnitude of the water path is normally set to be close to the Near Point.

### A.2 Attenuation

The attenuation is measured by setting the reflection from the entry surface of the specimen, known as the front surface signal, fss, to an arbitrary but standard amplitude.

In this work this standard amplitude was always set as half screen height which is given by a voltage of 300 mV. The signal amplitude is adjusted to be this magnitude using the receiver gain adjustment.

(It should be noted that there is no significance of this voltage other than it is an arbitrary absolute amplitude which enables the comparison of one signal with another by comparing the receiver gain required to produce that amplitude for each signal).

Having noted the gain required to set the fss to 300 mV the reflection from the back surface of the specimen, bss, is adjusted to be 300 mV using the receiver gain adjustment. The
ultrasound giving rise to this signal has made two transits of the specimen; from the front face to the back face and back again.

The Attenuation is then calculated as the difference in gain settings in decibels, divided by twice the specimen thickness, $t$, to give the attenuation in dB/mm:

$$\text{Attenuation (dB/mm)} = \frac{(bss-fss)}{2t}$$

A.3 Raw Data Acquisition

The ultrasonic equipment used for this work was selected because of its capability to provide high speed raw data capture. This technique is known as Volume Scanning.

The basic signal used in almost all ultrasonic investigations is the A-scan. This is the variation of echo height as a function of time. If the velocity of sound in the specimen is known this can easily be converted to echo height as a function of distance, or depth, in the specimen.

The A-scan represents reflections that have occurred as the sound pulse traverses the specimen. For each position of the transducer above the surface of the specimen there will be a unique A-scan. The A-scan contains all the information about reflections from surfaces, for instance the front and back, defects such as cracks and voids, normally the object of attention, and sound scattered by the material itself. In this case we are primarily concerned about the latter at this stage but only as an indicator towards the efficiency of defect detection.

It is highly desirable, therefore, to acquire and store the whole of every A-scan from a specimen as this represents the sum total of the ultrasonic information relating to the grain noise and defect population.

The equipment used in this work provides this capability using the Volume Scan. The operator can define a matrix of points at which an A-scan is generated, digitised and stored. The mechanical system will then move the transducer over the surface of the specimen and the recording process occurs at each matrix position. In this work A-scans were recorded at a 0.5 mm pitch.

The Flaw Detector provides facilities to recall each A-scan, along with an image of the specimen, the C-scan, so that any analytical tools can be used on the data.
A.4  A-scan/C-scan Image

The Volume Scan facility was used to create a bitmap image of the C-scan and a typical A-scan from the central area of each specimen. The purpose of these images is to provide a visual indication within the report of the “noisiness” of each block. It should be noted that every A-scan from the 0.5 mm pitch matrix is available in this form and as a numerical file.

A.5  Volume Scan File

The ISMBB model requires a single number from the set of A-scans for each block which represents the noisiness of that block. The developers of the model established that this number should be the root-mean-square amplitude of the grain noise signals, see Section 3.2. In order to obtain this number a subset of the A-scan set is used. This is extracted using facilities provided by the Flaw Detector.

The procedure consists of the definition of an area (typically 10 mm x 10 mm around the centre of the specimen surface) on the front surface of the specimen and the abstraction of the A-scans which lie within that area. The resulting data set is stored in a “.vol” file. A .vol file has been created for each block at each frequency.

A.6  Root Mean Square (rms) Amplitude

This is one of the principal values to be entered into the ISMBB model. It is the Root Mean Square of all the signals that occur between two positions in the A-scan, which represents a depth range within the specimen.

The procedure consists of recalling the .vol file into Microsoft Excel and then calculating the RMS of every data point contained in the file.

An interim procedure is carried out in which the value of the rms amplitude is calculated at each depth in to the specimen. Digitisation of the A-scan results in “planes” of data being created with each successive plane situated 0.1 mm deeper into the specimen. The results of this procedure are shown in Figures 1 to 14.

The rms value used as the input to the ISMBB model is then calculated by taking the arithmetic mean of all the rms values at the various depths into the specimen.
Intuitively the more "noisy" the specimen, the greater the rms value.

A.7 Peak Amplitude Signal Calculation

The complete .vol file is searched using spreadsheet functions to find and report the maximum amplitude.

A.8 Normalisation

The value of Backscatter Coefficient and Figure of Merit calculated by the ISMBB model from experimental data is independent of the equipment used to make the measurements. To eliminate the dependence of measured noise on equipment gain settings, dimensionless versions of the rms signal level are obtained by dividing by $I_{\text{ref}}$ which is the peak amplitude of the reference signal. For instance if the equipment gain is changed for one specimen compared with another, both the rms and reference amplitude will change but the ratios will be comparable between specimens.

The equipment gain setting used to measure the reference signal will in general be lower than that used for rms noise measurement. When the gain is increased to make a noise measurement the reference signal, normally the reflection from the entry surface, will be saturated. It is therefore necessary to calculate the equivalent magnitude of the reference signal at the gain setting used for noise measurement.

The procedure is described as follows:

a) The amplitude of the front surface (reference) signal ($f_{\text{ss}}$) is adjusted to a known level using the gain control of the equipment. The "gain" in dB is noted.

With the $f_{\text{ss}}$ at this level there is insufficient sensitivity to measure the grain noise from within the specimen. Consequently the gain must be increased.

b) The "gain" level used to measure grain noise is noted.

c) The difference between these two gain levels provides the multiplication factor to calculate the equivalent amplitude of the reference signal.
For instance the gain level to make the peak amplitude of the reference signal $f_{ss}$ to be, say, 300 mV might be 5 dB. The gain level to obtain sufficient sensitivity to grain noise might be 45 dB. Thus 40 dB of extra gain has been applied to the signals.

If a gain of 5 dB produced a signal of 300 mV, then an extra gain of 40 dB would produce an equivalent signal of 30 V, which would undoubtedly saturate the equipment amplifier but is a perfectly valid value. This value is then used as $\Gamma_{ref}$ to normalise the rms signal from that specimen.

### A.9 Standard Values

A number of parameters are required for the model which were taken as standard values. These included the speed of sound in water and copper and the density of water and copper. The values used are shown in Table 2.
APPENDIX B IMPLEMENTATION OF THE ISMBB MODEL

The main equation of the ISMBB model which has been quoted in earlier reports was simplified by Foister as follows:

\[ \frac{\Gamma_{\text{ref}(f)}}{\Gamma_{\text{ref}(f)}} = \eta^2 \frac{4\rho_0 v_0 \rho_1 v_1^2}{\left(\rho_0 v_0 - \rho_1 v_1\right)^2} \left( \int \int_{s/2} G(z) e^{-4\alpha z_i} dz_i \right)^{1/2} \]

where \( \Gamma_{\text{ref}(f)} \) and \( \Gamma_{\text{ref}(f)} \) are frequency components of the backscattered noise from the material and a reference reflector respectively.

This equation is a derivation from the original work of Professor Frank Margetan at the Centre for Non-destructive Evaluation at Iowa State University. The following analysis is based on Margetan’s original work, but incorporates the simplifications developed by Foister where appropriate. The principal equation of the original Margetan ISMBB model is:

\[ \frac{\Gamma_{\text{ref}(f)}}{\Gamma_{\text{ref}(f)}} = \eta^2 \left( \frac{2T_{01}^2 \rho_1 v_1}{R_{00} \rho_0 v_0 b^2 \pi \int |D(f)|} \right) \left( \int \int_{s/2} G(z) e^{-4\alpha z_i} dz_i \right)^{1/2} \]

where \( T_{01} \) is the Transmission Coefficient \( T_{01} = 2\rho_0 v_0 / (\rho_0 v_0 + \rho_1 v_1) \)

and \( R_{00} \) is the Reflection Coefficient \( R_{00} = (\rho_0 v_0 - \rho_1 v_1) / (\rho_0 v_0 + \rho_1 v_1) \)

Subscripts 0 and 1 refer to water and to metal. The exponential term in the integral accounts for attenuation in the material where \( \alpha \) is the attenuation coefficient and \( z \) is the axial distance.
distance from the probe. \( z_0 \) is the water path and \( z_1 \) is the distance in the specimen. \( \rho \) and \( v \) are the densities and velocities of sound respectively.

The model accounts for the beam profile by the term \( G(z_1) \) where \( b \) is the probe radius and \( F \) is the focal length:

\[
G(z_1) = \frac{4.356b^2}{\left(\frac{z}{F}\right)^2\left(1 + \left(\frac{0.56325v_oF}{b^2f}\right)^2\right) - 2\frac{z}{F} + 1}
\]

and a diffraction correction for the probe is given by the term:

\[
D(f) = \left|1 - e^{-\frac{\pi}{4} \left(J_0[s] + iJ_1[s]\right)}\right|, \quad \text{where} \quad s = \frac{\pi fb^2}{v_i z_0} \quad \text{and} \quad z = z_0 + z_1 \frac{v_i}{v_o}
\]

\[
B1.5
\]

\[
B1.6
\]
An alternative way to write equation B1.1 is in the form:

\[
\frac{\Gamma_{\text{meas}}(f)}{\Gamma_{\text{ref}}(f)} = \eta^2 \left[ \frac{2T_0^2 \rho V_1}{R_0 \rho_0 v_0} \right] \frac{1}{b^2 \pi f} \left[ \frac{1}{D(f)} \right] \left[ \left( \int_{n_0}^{n_1} G(z) e^{-4\alpha_n z} dz \right)^2 \right]^{1/2}
\]

or

\[
\frac{\Gamma_{\text{meas}}(f)}{\Gamma_{\text{ref}}(f)} = \eta^2 \left[ A \right] \left[ B \right] \left[ C \right] \left[ E \right]
\]

Where:

\[
\frac{\Gamma_{\text{meas}}(f)}{\Gamma_{\text{ref}}(f)}\quad\text{is the experimentally determined value}
\]

\[
A = \frac{2T_0^2 \rho V_1}{R_0 \rho_0 v_0} \quad\text{is a constant derived from established values for the properties of water and copper.}
\]

\[
B = \left[ \frac{1}{b^2 \pi f} \right]
\]

\[
C = \left[ \frac{1}{D(f)} \right]
\]

\[
E = \left[ \left( \int_{n_0}^{n_1} G(z) e^{-4\alpha_n z} dz \right)^2 \right]^{1/2}
\]

is the term which account for the effects of the Gaussian beam intensity profile in any plane.
We now define $F$ as a constant for each transducer and experimental arrangement:

$$F = A^*B^*C$$

Now the ISMBB equation becomes:

$$r \cdot m = r_j^2 \frac{F}{E}$$

Where $F$ is a constant for each probe.

The value for $E$ may be simplified as follows:

$$E = \left( \int \frac{r_j}{z} \int G(z) e^{-4a_{i_1}z} \, dz \right)^{1/2}$$

Where:

$$G(z) = \frac{4.356b^2}{\left( \frac{z}{F} \right)^2 \left( 1 + \frac{0.56325v_0F}{b^2f} \right)^2} - 2 \frac{z}{F} + 1$$

For our work unfocussed probes were used therefore the value of $F$ is infinity. Thus the denominator in the expression for $G$ reduces to:
The value of $G(z,\alpha)$ decreases with $z$ as expected because the beam diverges with increasing
distance from the transducer.

The exponential term in the integral includes $\alpha$, the attenuation coefficient, and $z_1$ a position
co-ordinate measured along the axis of the beam from the surface of the specimen. In our
work attenuation coefficients were measured and the measured values are available for each
specimen. Measurements made by Thompson et al on our specimens 13 and 14 indicated that
a value of zero was appropriate for $\alpha$. If this is accepted then the value of the exponential term
reduces to 1. This is discussed in more detail later in this section.

Limits of integration have been set at 3mm and 24mm to correspond with the thickness of
copper used in the derivation of rms noise

Thus if the attenuation is assumed to be zero or any constant for all specimens tested using any
particular transducer, $E$ is a constant for each transducer.

We can, therefore, define a constant $K$:

$$K_{fe} = F^*E$$

where the subscript $fe$ indicates a specific transducer and experimental arrangement

The ISMBB equation then becomes

$$\frac{\Gamma_{rmi}(r)}{\Gamma_{ref}(r)} = \eta^2 K_{fe}$$
and the value of the Backscatter Coefficient, \( \eta \), or the Figure of Merit (FOM), \( \eta^2 \), depends only on the ratio of the measured rms to reference signal amplitudes, and the value of \( K_{fe} \).

From the above analysis of the ISMBB model it is concluded that when the method of implementation assumes that the attenuation coefficient for backscattered noise is set to a constant and \( T_{ref} \) is set as an adjustable parameter, the ISMBB model may be reduced to a statement as follows,

\[
\Gamma_{ma} = \text{FOM} \times K
\]

where \( \Gamma_{ma} \) is rms noise, FOM is the figure of merit which is equal to the square root of the Rose backscatter coefficient and \( K \) is a constant. This being the case, if both models are good it should be possible to select a value for \( K \) such that the measured rms noise data may be fitted to the curve which represents the relationship between the FOM and grain size computed from the Rose model.

The use of the zero value for attenuation has been justified by Thomson on the basis that the process of attenuation of noise is not understood but it is different to the process for attenuation of the primary pulse. Further the figure of merit must, by definition, have a constant value throughout the thickness of the specimen. When figure of merit has been calculated from experimental data through the thickness of copper specimens using a zero value for attenuation coefficient, it has been shown that its value decreases slightly with increasing depth in the specimen, when a value of 20 Nepers per metre is used for attenuation coefficient the value of figure of merit increases very slightly with increasing depth in the specimen. On consideration of the results of this experiment it was judged by Thompson et al that the zero value was the most appropriate value to use. The difference in calculated value of figure of merit using the zero and 20 Nepers per metre value for attenuation coefficient is a factor of 0.5.

In the work by Thompson and co-workers, the fit between experimentally determined values and theoretically determined values for figure of merit was better than a factor of 0.5 and considering that some discrepancy between experiment and theory has to be expected owing to difficulty in satisfying the assumptions of uniform equiaxed grains and of measuring grain size reliably, this has to be seen as a good fit and the zero attenuation assumption is acceptable.
The fact that the method of calculating the constant in equation B12 above results in a close agreement between the "Rose" predicted value of FOM and the experimentally determined value lends credibility to both models and justifies this work which has explored the boundaries within which the models might be applied.

It is unfortunate that in this work front face (reference signals) were not properly recorded. It is quite certain however that they would be constant for each frequency (indeed the ISMBB model assumes that it is governed by classical reflection theory). As they were not recorded it has not been possible to determine figure of merit as a function of depth in each specimen for each case under different assumptions for attenuation coefficient. Consequently it has been necessary to use equation B.13 above which assumes that attenuation coefficient for backscattered noise (not the main signal) are invariant between specimens for any particular frequency when this may not be the case.

Examination of the data in table 3 reveals that the attenuation coefficient for the main signal was 17 Nepers per metre. This is within range of 0 and 20 Nepers per metre referred to earlier when choice of the zero value was explained. In Thompsons work the choice of zero in the analysis gave a very good fit for specimen 13 but there was a difference of a factor of almost two between predicted and experimental values for specimen 14. The attenuation coefficient measured on specimen 14 was 30 Nepers per metre. It is possible that this higher value for the primary pulse (compared with 17 Nepers per metre for specimen 13) indicates that a different value may be appropriate for the attenuation of backscattered noise for this specimen. All except one of the experimental measurements of backscattered noise lie in the range zero to 58 Nepers per metre (table 3). The sensitivity of the factor $K$ in equation B.13 above to variations from zero to 58 Nepers per metres has been investigated by making use of the full analysis. It shows that a change from zero to 58 Nepers per metre changes the value of $K$ by a factor of 0.29. Since Thompson et al have indicated that the actual backscatter coefficient for noise is less than the backscatter coefficient for the main pulse, it can be taken that 0.29 times is the maximum error in $K$ which might arise from this cause in any individual result. By the same argument the maximum error which might arise the value of FOM for a particular specimen at a particular frequency from this cause would be a factor of $0.29^{-1}$ (3.5). If the experimental points really should fit on the theoretical line, making $\Gamma_{\text{ref}}$ an adjustable parameter as we have
done will reduce the errors arising from the choice of attenuation coefficient by an averaging process so that the errors in any particular value of FOM from this cause are likely to be less than a factor of \( \pm 1.75 \) \((3.5/2)\). It would appear therefore that choice of value for attenuation coefficient could account for some of the scatter at low grain sizes at 3.7 and 5 MHz (figures 20 and 21) but it does not account for the scatter at 2.1 MHz (figure 19) where noise is generally at a higher value than that predicted by Rose.

Other possible sources of error are the measured values of grain size, the extreme variations of grain size within some specimens and the interpretation of the role of twins. These parameters are dealt with in more detail in other sections, they influence the “Rose” prediction of backscattered noise rather than the ISMBB correction on observed backscattered noise, thus if the ISMBB model fits for ideal specimens it should also fit for non ideal specimens. The importance of this observation is that the ISMBB has been shown to fit with the “Rose” predictions for the ideal case, since it is independent of material characteristics it may be concluded that it provides a method for accounting for the effects of the test system on ultrasonic noise for all cases. This also indicates that it may be used together with attenuation coefficients to optimise test systems from the point of view of noise.