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Title: Point Kinetics Modeling

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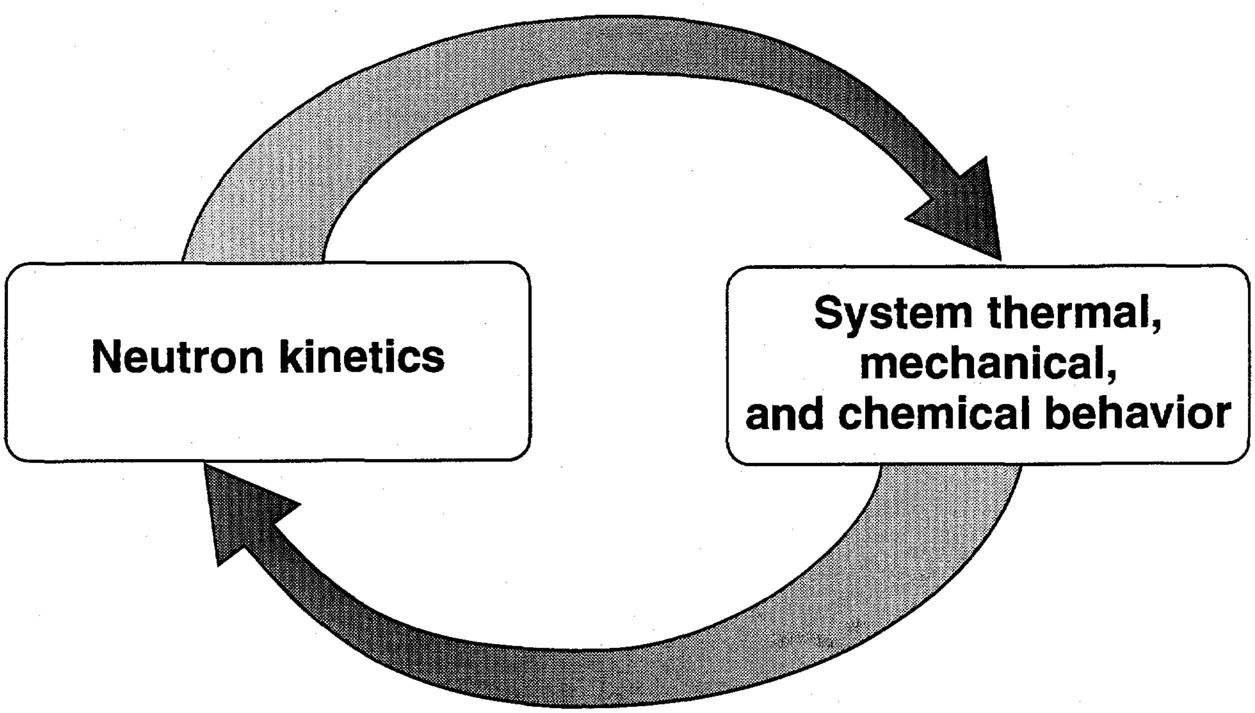
Point Kinetics Modeling

Dynamics of Nuclear Systems



POINT KINETICS MODEL

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \sum_i \lambda_i C_i + q$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n - \lambda_i C_i$$

Normalized Form

$$n = n_o N$$

$$\frac{dN}{dt} = \frac{\beta}{\Lambda}\left[(R-1)N + \sum_i \frac{\beta_i}{\beta} D_i - R_s\right]$$

$$\frac{dD_i}{dt} = \lambda_i (N - D_i)$$

where

$$D_i = \frac{C_i}{C_{io}} \qquad R = \frac{\rho}{\beta}$$

$$C_{io} = \frac{\beta_i n_o}{\Lambda \lambda_i}$$

$$R_s = \frac{\rho_s}{\beta} \qquad \rho_s = -\frac{q\Lambda}{n_o}$$

The main advantage of the normalized form of the point kinetics equations is convenience. Initial conditions are simple and easily changed. Subcritical steady-state conditions can be easily established. The units of n_O are arbitrary and are required only for reactivity feedback.

It may also be said that the normalized form of the point kinetics equations is more aesthetically pleasing.

OTHER USEFUL MODELS

Prompt Jump Approximation

$$N = \frac{\sum_i \frac{\beta_i}{\beta} D_i}{1 - R}$$

Nordheim-Fuchs Model

$$\frac{dN}{dt} = \frac{\beta}{\Lambda}(R-1)N$$

$$R = R_o - \alpha T$$

$$\frac{dT}{dt} = Kn$$

where K is the reciprocal heat capacity

For more details on these models see Ref. 1.

REACTIVITY FEEDBACK MECHANISMS

Volumetric Expansion

In all nuclear systems, a temperature increase will cause materials in the core (fuel, coolant, etc.) to expand. This expansion reduces atomic number densities, which in turn reduces macroscopic cross sections. This may increase neutron leakage, which results in a decrease of the core's neutron population. See Ref. 2.

Thermal Neutron Temperature Effect

A temperature rise in a thermal system, causes a hardening of the thermal neutron spectrum ($E_{neutron} \approx kT$). A shift in the thermal neutron spectrum to higher energies, causes the neutrons to "see" different thermal cross sections. This phenomenon can produce either positive or negative reactivity feedback. See Ref. 2.

Doppler Effect

Many nuclear fuels contain isotopes whose cross sections contain sharp resonances. The widths of these resonances are usually quite narrow, less than 1 eV. An increase in temperature (increased nuclear thermal motion) can significantly affect the energy dependance of the neutron cross section near the resonance. This phenomenon leads to an increase in resonance absorption and a decrease in self-shielding. See Refs. 1 and 3.

Void Formation

The formation of radiolytic gases or steam in aqueous fissile solution systems, causes the displacement of fuel. This displacement tends to move fuel from regions of high importance to regions of lower importance. Voids can radically alter the geometry of a solution system, and produce large negative reactivity feedback. See Ref. 4.

Sample Problem
Glovebox Deep Well Problem

A plutonium solution accidently forms in a glovebox deep well and an excursion occurs. See Fig. 1.

Assumptions

1. 4.2 kg (a double batch) of plutonium (5% Pu-240) is present and enters the deep well in an optimum manner.
2. A sufficient quantity of water is available to fill the entire deep well volume.

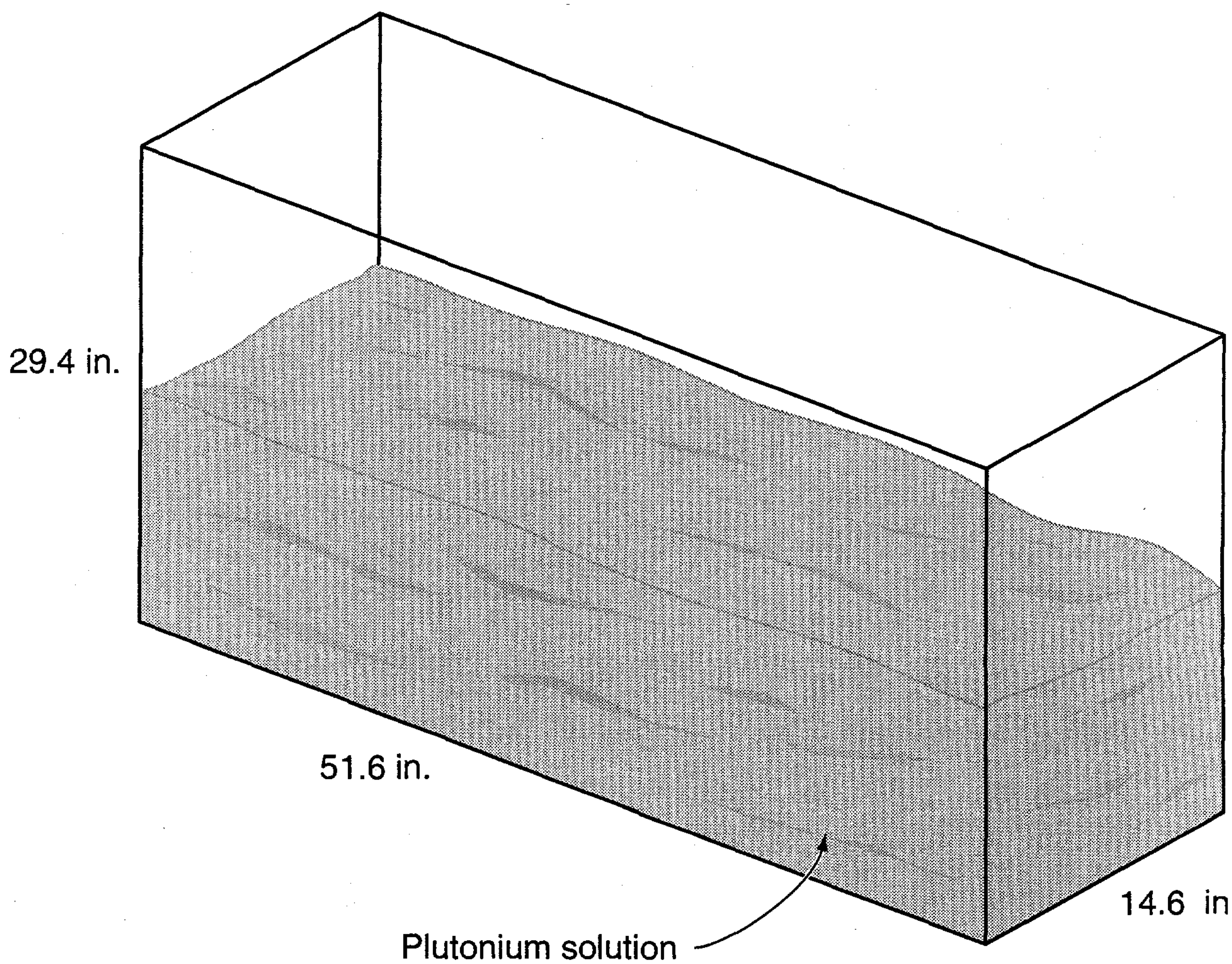


Figure 1. Glovebox deep well.

Step 1. Reactivity Analysis

With the use of a neutron transport code, such as TWODANT, determine the multiplication factor that 4.2 kg of plutonium and varying amounts of water could produce in the deep well. See Fig. 2.

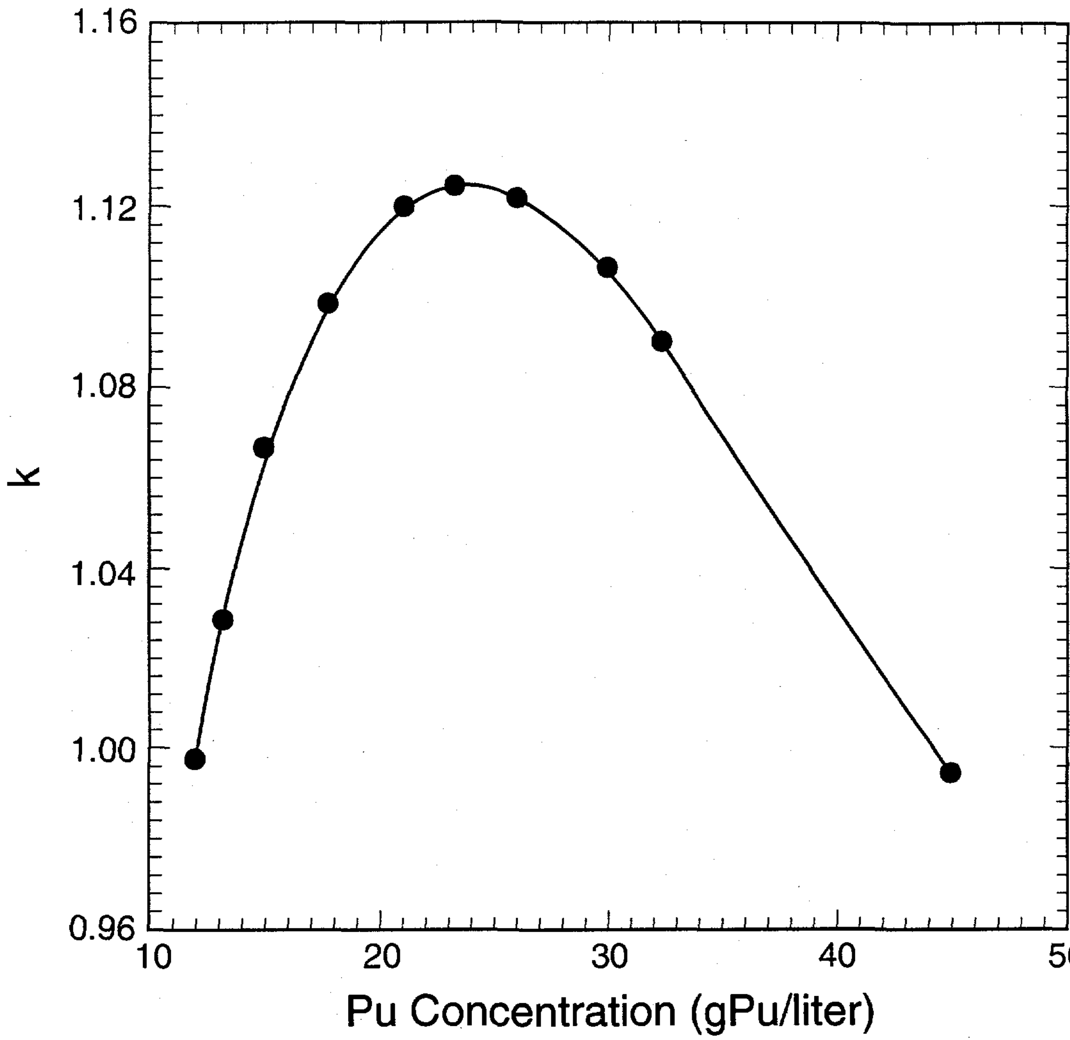


Figure 2. k vs plutonium concentration for solutions containing 4.2 kg of plutonium in the deep well.

Q: Which solution appears to be the worst?

A: The 180 liter 23 gPu/liter solution.

Q: What are the reactivity feedback mechanisms of such a solution?

A: Volumetric expansion, thermal neutron temperature effect, and void formation.

Determine the reactivity feedback coefficients for the solution. A thermal neutron temperature coefficient of -0.029 $/°C was calculated by Kornreich (Ref. 2). A combined volumetric expansion and void formation reactivity feedback coefficient was calculated using TWODANT. See Fig. 3. The combined expansion coefficient is given by

$$\phi = \frac{dR}{d\rho_s} = 0.543 - 0.429\rho_s \$ / \left(kg / m^3\right)$$

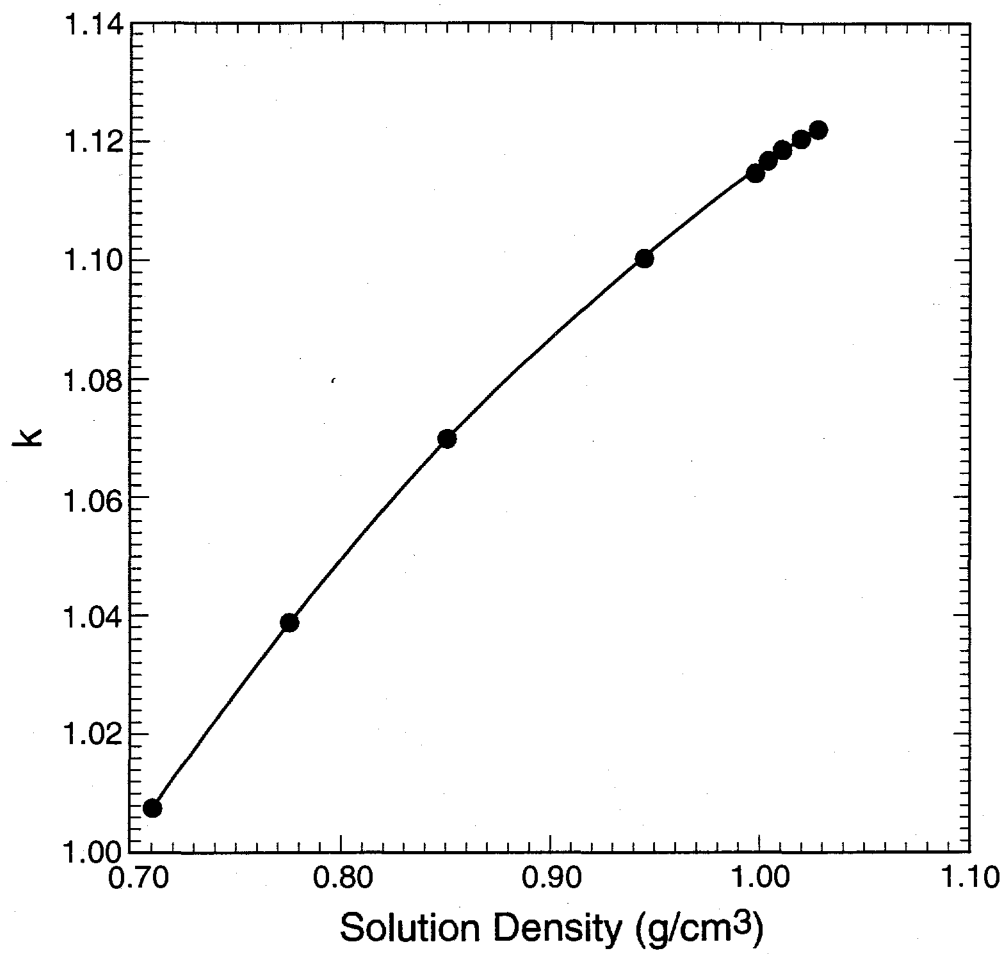


Figure 3. k vs solution density for the 180 liter solution in the deep well. In this set of calculations the height of the solution was continually raised and the density decreased to conserve the mass of the solution. The lower densities represent a homogeneous mixture of solution and steam void.

Step 2. Kinetics Model

Q: What is the neutron energy spectrum of the system?

A: Thermal.

Choose the appropriate values of the delayed neutron parameters.

Q: What is the neutron mean generation time of the system?

A: From TWODANT k_{eff} and α eiganvalue calculations,

$$\Lambda \approx \frac{k_{\text{eff}} - 1}{\alpha}$$

Determine the reactivity model. Put together the reactivity feedback mechanisms and the reactivity insertion model.

Q: How is reactivity inserted and at what rate?

A: Not known precisely.

In the case of unknown parameters, such as the reactivity insertion mechanism, perform a sensitivity study, analyze the effect of varying the reactivity insertion rate. The reactivity model for the solution is given by

$$R = \gamma t + \phi(\rho_s - \rho_{so}) + \alpha_T(T - T_o)$$

Q: What are the initial conditions?

A: Not known precisely.

It has been assumed that 2.1 kg of plutonium are already in the deep well initially. A TWODANT calculation has shown that this amount of plutonium would form a solution with a reactivity of approximately $30 below critical. The initial power of the solution would be given by

$$n_o = -\frac{S}{\beta R}$$

where S is the spontaneous fission rate of Pu-240 times the energy of fission.

Step 3. System Behavior

Q: What happens physically to the solution as a result of the nuclear heat generation?

A: Solution temperature increases, volume expands, and possible steam void generation.

Define an equation of state and an energy equation for the solution. Also, develop a steam production model. These equations are given as,

Energy

$$\frac{dT}{dt} = \frac{n}{MC_p}$$

Equation of State

$$\frac{dV}{dt} = \beta V \frac{dT}{dt}$$

where ß is the coefficient of volumetric expansion of the solution

Steam Generation

$$\frac{dV_g}{dt} = \frac{v_g n}{h_{fg}} - \frac{V_g}{l}$$

where V_g is the volume of steam in the solution, v_g is the specific volume of water vapor at STP, h_{fg} is the heat of vaporization at STP, and l is the mean lifetime of a steam bubble in the solution

Solution Density

$$\rho_s = -\frac{M}{V + V_g}$$

Step 4. Excursion Simulation

The model outlined above is a system of coupled differential equations. To solve this system of equations an interactive dynamic system simulator has been used. See Ref. 5.

A series of dynamic simulations were made with varying reactivity ramp rates. A summary of the peak powers and spike yields for each of the excursions is given in Table 1. Figure 4 shows the model's simulation of a 1.0 $/s ramp insertion rate.

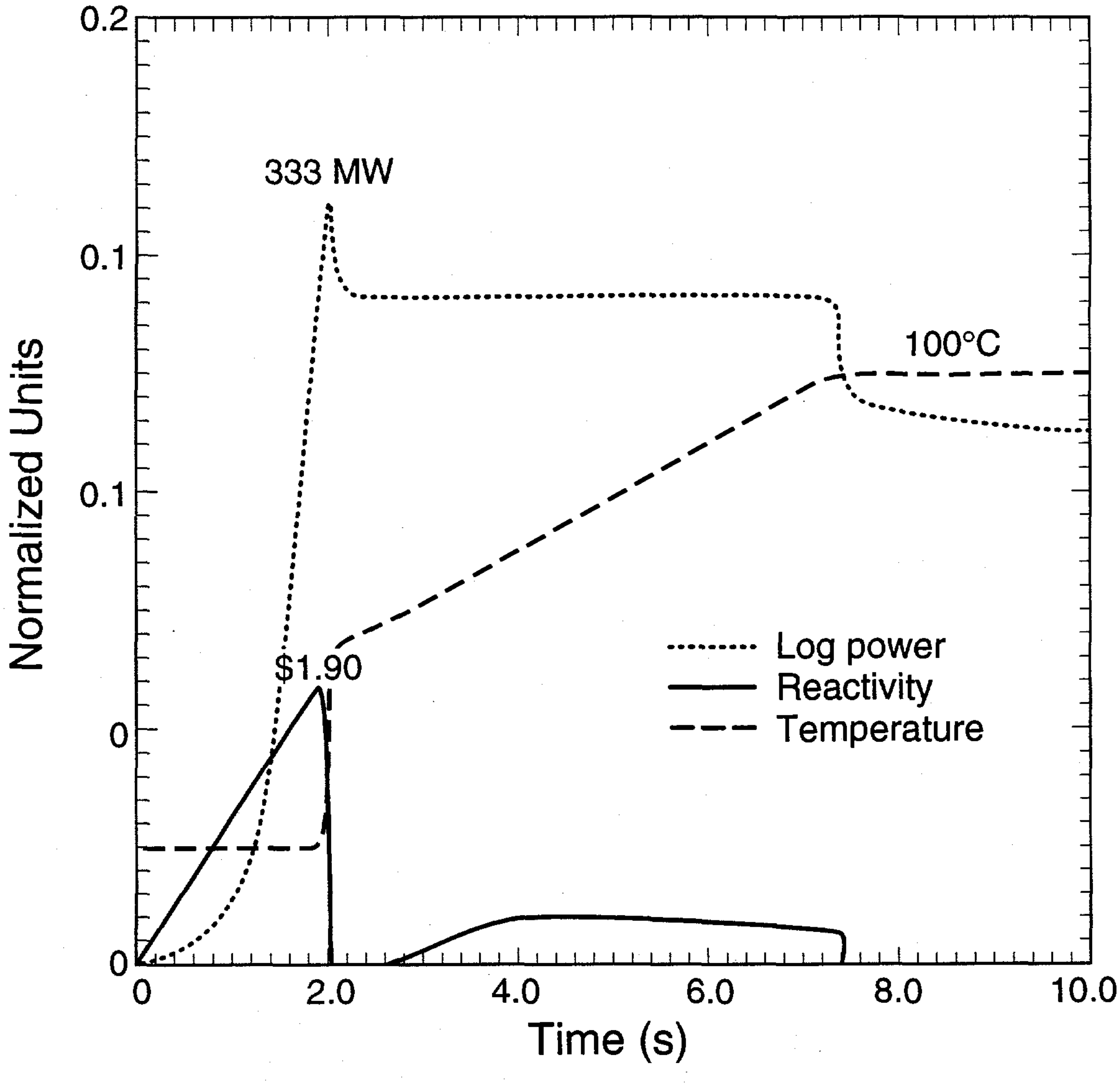


Figure 4. Model's simulation of 1.0 $/s ramp insertion in the 180 liter solution.

This simulation shows the power increasing rapidly, because of the ramp insertion. However, once the power starts heating the solution, the increase in the thermal neutron temperature and the expansion of the fuel quenches the reactivity momentarily. A maximum reactivity of $1.90 was reached. The power reaches a peak and drops, and the rate of fuel heating drops off as well. This allows the constant insertion of reactivity to "catch up" and overtake the reactivity feedback effects. The rate of reactivity insertion and reactivity feedback reach a temporary equilibrium. When the temperature reaches the boiling point however, the formation of steam voids in the solution introduces a large negative reactivity feedback, which quenches the excursion.

Q: Is this solution the worst case solution?

A: Consider the 320 liter 13 gPu/liter solution.

As in the case of the previous solution, a reactivity analysis was performed on the solution. A thermal neutron temperature feedback coefficient of +0.04 $/°C was calculated by Kornreich (Ref. 2) for this solution. Again, a combined volume expansion and void formation feedback coefficient was calculated using TWODANT. See Fig. 5. This feedback coefficient is given by

$$\phi = \frac{dR}{d\rho_s} = 0.436 - 0.313\rho_s \$ / \left(kg / m^3\right)$$

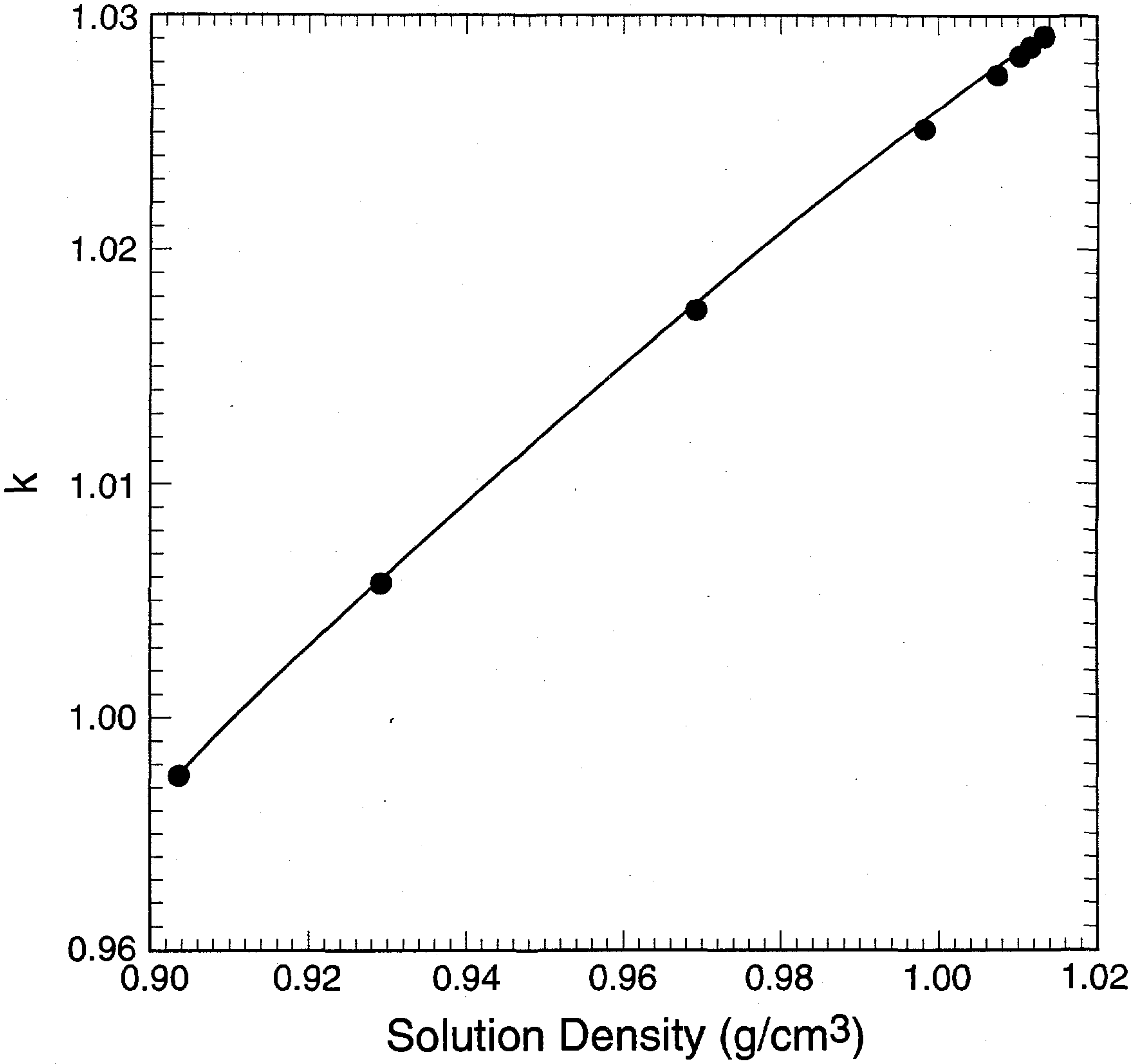


Figure 5. k vs solution density for the 320 liter solution in the deep well. In this set of calculations the height of the solution was continually raised and the density decreased to conserve the mass of the solution. The lower densities represent a homogeneous mixture of solution and steam voids.

The steps outlined above were again followed, and a new set of simulations were performed. Table 2 shows the results of these excursions for the 320 liter solution. Figure 6 shows the model's simulation for a 1.0 $/s rate of insertion.

Again, this simulation shows the power increasing rapidly, because of the ramp insertion. However, as the fuel temperature begins to rise, the reactivity insertion rate increases, because of the positive thermal neutron temperature feedback. For this solution the reactivity reaches a maximum of $2.44. The excursion is quenched only when the boiling point is reached, and steam voids are formed.

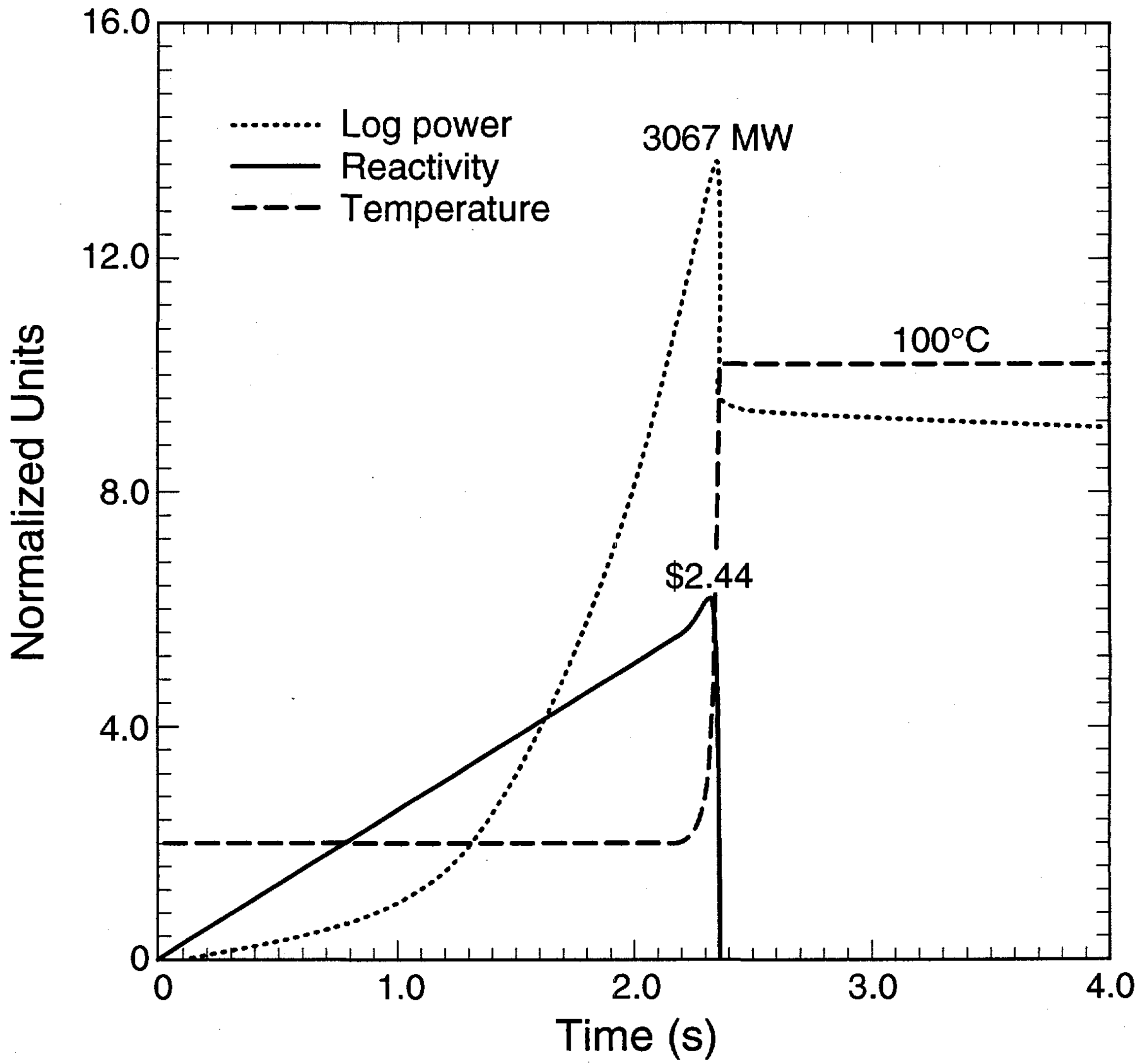


Figure 6. Model's simulation of 1.0 $/s ramp insertion in the 320 liter solution.

Table 1. Results for the 180 liter solution.

Insertion rate S/s	**Max. Reactivity $**	**Peak Power MW**	**Spike Energy Yield MJ**
0.1	1.20	25	17
1.0	1.90	333	25
5.0	3.16	1671	45

Table 2. Results for the 320 liter solution.

Insertion rate $/s	**Max. Reactivity $**	**Peak Power MW**	**Spike Energy Yield MJ**
0.1	1.44	664	110
1.0	2.44	3067	110
5.0	4.21	8770	110

Keys to Success

Account for all reactivity feedback mechanisms.

Model the reactivity feedback as realistically as possible. Use experimental data whenever possible.

Pay as much attention to the modeling of the thermal, mechanical, and chemical behavior of the system as to the modeling of the neutron kinetics.

In the case of unknown parameters, perform sensitivity studies.

Beware all positive reactivity feedback mechanisms.

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