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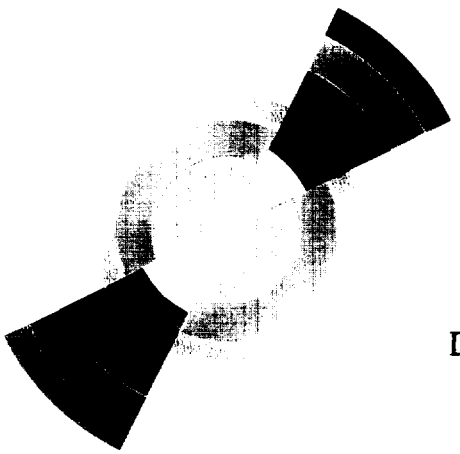

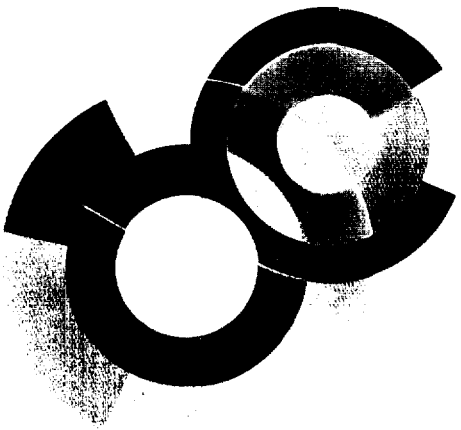

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MONTE CARLO SIMULATION OF VIRTUAL COMPTON SCATTERING AT MAMI

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The Monte Carlo simulation developed specifically for the VCS experiments taking place at MAMI is fully described. This simulation can generate events according to the Bethe-Heitler + Born cross section behaviour and takes into account resolution deteriorating effects. It is used to determine solid angles for the various experimental settings.

1 Introduction

Determining experimental differential cross sections for the $p(e,e'p)\gamma$ reaction is one of the necessary steps in reaching the final goal of the VCS experiments[1] at MAMI/Mainz: to measure for the first time the generalized polarizabilities[2] of the proton. To derive differential cross sections from the measured data the solid angle for a given data set has to be known. In general, if one performs an ideal experiment which is free of resolution effects one can write the ratio of the number of counts detected in a given phase space bin N_{bin} and the integrated luminosity L as:

$$
\begin{aligned}
\frac{N_{\text{bin}}}{L} &= \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega \\
&= \frac{\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega}{\int \mathrm{d}\Omega} \int \mathrm{d}\Omega = \left\langle \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \right\rangle . \Delta\Omega_1 \qquad (1) \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 \int \left(1 + \frac{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} - \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0}{\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0}\right) \mathrm{d}\Omega \\
&= \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 . (\Delta\Omega_1 + \epsilon) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_0 . \Delta\Omega_2 \qquad (2)
\end{aligned}
$$

Equation 1 shows that this ratio can be expressed as the mean differential cross section over the bin, multiplied by a solid angle $\Delta\Omega_1$ which is a pure "geometrical" quantity. On the other hand, equation 2 shows that one can also write this ratio as the product of the actual differential cross section value $(d\sigma/d\Omega)_0$ at a given point somewhere in the bin (eg. the centre point) multiplied by a solid angle $\Delta\Omega_2$, which deviates from $\Delta\Omega_1$ by a value ϵ depending on the relative cross section deviation from the $(d\sigma/d\Omega)_0$ value over the bin.

However, in reality resolution effects always play a role. The target thickness introduces effects such as multiple scattering and energy loss of incoming and outgoing particles and the particle detectors have an intrinsic resolution. As a result, the number of particles actually being detected in a phase space bin differs from the number of particles present in this bin at the point of interaction. To determine $\Delta\Omega_1$ or $\Delta\Omega_2$ taking into acount also resolution effects and the actual detection geometry, a Monte Carlo simulation can be used: events are generated in a given phase space according to a given cross section behaviour and all resolution effects and the actual detector geometry are implemented. The number of counts obtained in a bin combined with the integrated luminosity for the simulation and the used cross section yields a $\Delta\Omega$ corrected for the above effects. A simulation using a constant cross section behaviour yields $\Delta\Omega_1$ (ϵ becomes 0) while the use of the actual cross section behaviour yields $\Delta\Omega_2$. Determining $\Delta\Omega_1$ and especially $\Delta\Omega_2$ is the goal of the Monte Carlo simulation that has been written for the VCS experiments at MAMI. Below a desription of the present state of this simulation (which is still evolving) is given.

2 Cross section behaviour implementation

The $p(e,e'p)\gamma$ reaction is described by a five fold differential cross section $d\sigma^5/dk'd\Omega_{e'}d\Omega_{\gamma\gamma,cm}$ depending on the value of the incoming and outgoing lab electron momenta k and k', the lab electron scattering angle $\theta_{e'}$ and the cm outgoing real photon angles θ and φ. In the present version of this simulation, the actual behaviour of the $p(e,e'p)\gamma$ cross section is approximated by the Bethe-Heitler + Born (BH+B) contribution. The BH+B contribution to the cross section can be perfectly calculated once the elastic form factors of the nucleon are known. The polarizability contribution is estimated at most to be of the order of a 10% effect on the BH+B contribution and is neglected. Internal radiative corrections are not taken into account at present.

Events are generated in the phase space according to the BH+B cross section behaviour using the acceptance-rejection method [3] with a constant value as an envelope. However, since the BH+B cross section increases very rapidly when the outgoing real photon direction comes close to the incoming or outgoing electron directions, the efficiency of the acceptance-rejection method can be rather low (easily less than 1%), which means that one needs a fast method to calculate cross section values in order to generate events at an acceptable rate.

The BH+B cross section values are calculated using a theoretical code [4] yielding about 15 cross sections per second on a SUN SPARC10 workstation. To obtain cross sections at a usable rate for the acceptance-rejection method, a four dimensional cross section matrix has been constructed having k', $\theta_{e'}$, θ and φ as variables. The cross section value at each matrix point is calculated for k equal to the incoming beam momentum k_i using the above mentioned code. The matrix covers the total necessary phase space in the four variables for a full simulation: k' varies from the elastic peak value down to the minimum momentum acceptance of the electron spectrometer, $\theta_{e'}$ covers all possible electron scattering angles for a given spectrometer setting and the two real cm photon angles cover the half-sphere above the leptonic plane, which is sufficient because of the symmetry of the cross section around the leptonic plane. The cross section value at a random point in the phase space is obtained by logarithmic interpolation in the matrix, about 1000 times faster than the theoretical code can provide it. The accuracy of the interpolated cross section values is better than 0.5% in the region of interest. A cross section matrix contains about 410000 cross section values, and is contained in a binary file with a size of 1.6 MB.

To obtain cross section values for k-values lower than the incoming beam momentum k_i (due to energy loss of the incoming beam in the target before

the actual VCS-process takes place) without having to increase the matrix-size, at present the approximation $\sigma(k, k', \theta_{e'}, \theta, \varphi) \approx \sigma(k_i, k'_{dummy}, \theta_{e'}, \theta, \varphi)$ when $s(k, k', \theta_{e'}) = s(k_i, k'_{dummy}, \theta_{e'})$ is used: from the actual set (k, k') a set (k_i, k'_{dummy}) is determined using the cm-energy relationship, the latter set being covered by the cross section matrix. The error made by this approximation increases with increasing deviation of k from k_i, but for 97% of the events it is smaller than 3% in the region of interest.

3 The simulation procedure

The simulation procedure consists of three separate programs: VCSSIM, RESOLUTION and ANALYSIS.

The first program, VCSSIM, generates events according to a given cross section behaviour, and simulates the resolution deteriorating processes happening in the target and in the foils the particles have to pass through before arriving in a spectrometer. The target cell (liquid hydrogen) and spectrometer geometry (spectrometers A and B of hall A1 in Mainz are used to detect the proton and electron, respectively) are fully implemented. The code does not track the particles through the magnetic spectrometers, it only checks the acceptance of the particles by the collimators at the spectrometers' entrance and their momentum acceptance. A constant cross section for the $p(e, e'p)\gamma$ reaction, or as mentioned above, the BH+B contribution to the cross section can be used.

Roughly speaking, in order to obtain events the following steps are taken. First the beam position on the target is generated (to simulate the horizontal and vertical movement of the incoming electron beam needed to prevent local overheating) and uniformly distributed along the beam line an interaction point is chosen. Using the pathlength of the incoming electron through the target, the multiple scattering and energy loss of this particle is determined. This yields the incoming electron four vector k at the point of interaction. The electron energy loss contains a collision part and an (external) bremsstrahlung part. The latter contribution can be large, and therefore at this point a check is made in case this electron elastically scatters at the minimum scattering angle considered in the simulation phase space, whether its momentum is larger than the minimum momentum acceptance of spectro B or not. If this is not the case, everything up to this point starts all over again. If this momentum is indeed larger, the acceptance-rejection routine starts to generate an event. This yields a scattered electron four vector k', and also a cm real outgoing photon four vector. For the outgoing photon, the code actually generates the complete cm 4π phase space (except for the small regions around the electron directions

where the BH+B cross section is larger than the envelope value in case the BH+B cross section behaviour is chosen). The real photon is then transformed to the lab to obtain the four vector γ. The four vector of the recoil proton can now be calculated as p'=p+k-k'-γ. Using the path lengths of outgoing electron and proton through the target, their four vectors at the point of interaction k' and p' are subject to multiple scattering and energy loss, and afterwards the particles are tracked towards the spectrometers to determine whether they are accepted or not. Then the generation of a new event starts. For the simulation of the energy loss by collision, one can choose to use the mean energy loss, the most probable energy loss, or a more realistic energy loss distribution.

The output of the VCSSIM program is twofold. First, an Ntuple, containing the events accepted by both spectrometers (both with regard to angular acceptance and momentum acceptance). For each event the momentum components of the outgoing electron and proton in their respective spectrometer frames, and the coordinates of the point of origin of the event in the target are stored. Also a proton spectrometer index is stored. Indeed, since the code generates the complete cm 4π phase space for the outgoing photon, the complete recoil proton cone is reproduced. To take advantage of this, up to 10 different proton detector settings can be defined for which the result is obtained in one simulation run. The VCSSIM Ntuple contains what is physically presented at the spectrometers' entrance. Secondly, also a data file is produced containing the integrated luminosity, mean target thickness and some statistical information regarding the simulation run.

The second program, RESOLUTION, introduces the resolution effects of the spectrometers on the events in the Ntuple originating from VCSSIM. The momentum size and direction of the detected particles are changed using Gaussian distributed random numbers having the spectrometer resolution values at the target position as FWHM. Also the coordinate along the beam line of the point of origin is changed in the same way. The output is an Ntuple similar to the first one, but now the momentum components of the particles and the point of origin have been subject to the spectrometers' resolution effects. This Ntuple contains what one could call the equivalent of reconstructed experimentally obtained particle momenta at the reconstructed point of origin in the target. The effects of the spectrometer resolution on the data can as such be studied without the need to rerun the complete simulation over and over again.

The third and final part of the simulation, the ANALYSIS program, reconstructs using the particle momenta contained in the second Ntuple, for each event all physical observables that are also reconstructed from the experimental data. Before the reconstruction process, the particle momenta are corrected for the mean energy loss in the target, calculated using path lengths

derived from the coordinates of the origin. The simulation analysis procedure is as such equivalent to the analysis procedure of the experimental data. The physical observables are stored in an Ntuple, and the obtained distributions can then be compared qualitatively and quantitatively with the experimental ones.

4 Calculation of solid angles

Once the phase space bins for which differential cross section have to be obtained are well defined, the data in the output Ntuple from the ANALYSIS program, in combination with the integrated luminosity L from the VCSSIM program can be used to calculate solid angles for these bins using equation 3:

$$\Delta\Omega_{\text{bin}} = \frac{N_{\text{sim,bin}}}{L_{\text{sim}} \cdot \frac{\mathrm{d}^5\sigma}{\mathrm{d}k'.\mathrm{d}\Omega_{e'}.\mathrm{d}\Omega_{\gamma\gamma,cm}}} \tag{3}$$

In this formula $N_{\text{sim,bin}}$ is the number of simulated events present in the defined bin , while $d\sigma^5/dk'd\Omega_{e'}d\Omega_{\gamma\gamma,cm}$ stands for the differential cross section used in the simulation procedure. If a constant value has been used, equation 3 will yield $\Delta\Omega_{1,\text{bin}}$. Application of this $\Delta\Omega_{1,\text{bin}}$ to the measured data will give rise to the mean experimental cross section in the bin. On the other hand, if the simulation has been performed using the BH+B cross section, the value of $d\sigma^5/dk'd\Omega_{e'}d\Omega_{\gamma\gamma,cm}$ in equation 3 equals the BH+B cross section value at the centre of the bin. This procedure will give rise to $\Delta\Omega_{2,\text{bin}}$. Applying this $\Delta\Omega_{2,\text{bin}}$ to the measured data will give rise to a very good approximation of the actual cross section value at the centre of the bin, provided of course that the BH+B cross section has a "shape" very close to the actual cross section "shape".

The experimental mean differential cross section (obtained using $\Delta\Omega_{1,\text{bin}}$) should be compared with the theoretical mean differential cross section for a bin. To readily obtain the mean value of the BH+B differential cross section in each of the defined bins, the simulated BH+B data can be used as shown in equation 4:

$$\left\langle \frac{\mathrm{d}^5\sigma}{\mathrm{d}k'.\mathrm{d}\Omega_{e'}.\mathrm{d}\Omega_{\gamma\gamma,cm}} \right\rangle_{\text{bin}} = \frac{N_{\text{sim,bin,BH+B}}}{L.\Delta\Omega_{1,\text{bin}}} \tag{4}$$

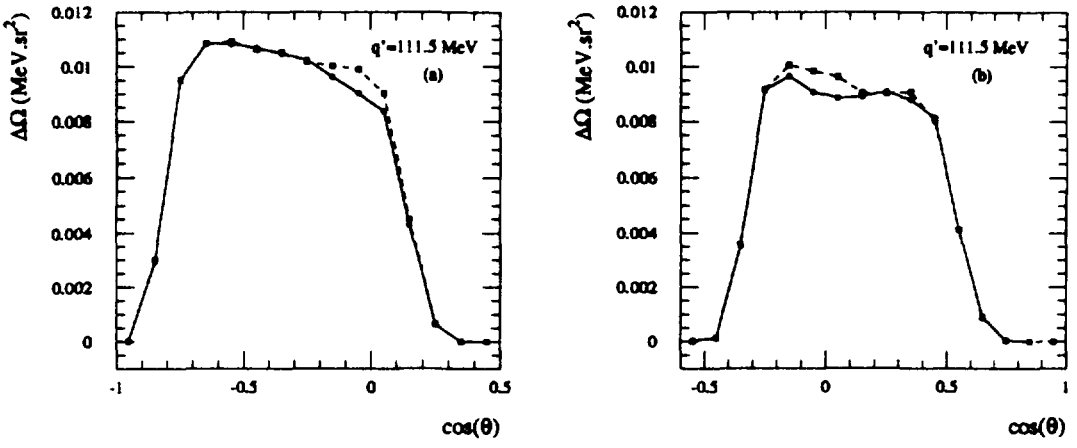

Figure 1: $\Delta\Omega_1$ (full line) and $\Delta\Omega_2$ (dashed line) for two proton spectrometer settings (labeled (a) and (b)) at q'=111.5 MeV

5 Results

As an example, figure 1 shows the obtained $\Delta\Omega_{1,\mathrm{bin}}$ (full line) and $\Delta\Omega_{2,\mathrm{bin}}$ (dashed line) for two different proton spectrometer settings at an outgoing cm real photon energy q' of 111.5 MeV. The bins are defined as $96.5MeV < q' < 126.5MeV$, $\Delta cos(\theta) = 0.1$, $158 < \varphi < 202$. The pure statistical error on the solid angles in the plateau region is about 1%. $\Delta\Omega_{2,\mathrm{bin}}$ is subject to an additional systematic error estimated to be about 2%, due to the approximations in the BH+B cross section behaviour. It turns out that for these examples the difference between the two $\Delta\Omega$s is at most of the order of 10%.

6 Summary and outlook

The simulation described above is flexible: all resolution deteriorating effects can independently be switched on or off and it is possible to use a constant cross section or the BH+B cross section to generate events. Due to the multiple proton spectrometer option several settings at the same real outgoing photon energy can be simulated in one run, while the modularity of the code gives the possibility to study spectrometer resolution effects in an efficient way. The implementation of the cross section matrix for the BH+B cross section option allows to obtain events at a very acceptable rate.

The simulation is still evolving. One of the features that certainly has to

be improved is the implementation of the k-dependence of the BH+B cross section. Also a very important improvement will be the implementation of internal radiative corrections[5]. The virtual part of these corrections will most probably be implemented as a corrected BH+B cross section, while for the real part two options are open: the use of an equivalent radiator approximation or the introduction of the (missing mass)^2 as an additional sampling variable.

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