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Suppression of Radiation Excitation in Focusing Environment*

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Abstract

Radiation damping and quantum excitation in an electron damping ring and a straight focusing channel are reviewed. They are found to be the two limiting cases in the study of a general bending and focusing combined system. In the intermediate regime where the radiation formation length is comparable to the betatron wavelength, quantum excitation can be exponentially suppressed by focusing field. This new regime may have interesting applications in the generation of ultra-low emittance beams.

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DISCLAIMER

INTRODUCTION

Many applications of particle accelerators require very low emittance beams. In an electron damping ring, synchrotron radiation created by bending magnets is utilized to damp the beam emittance in all three degrees of freedom. It is well known [1, 2] that the damping effect is counteracted by quantum excitation due to random photon emissions, which leads to an equilibrium beam emittance when the damping and excitation rates balance.

On the other hand, we have shown [3, 4] that in a straight, continuous focusing channel, the transverse damping rate is independent of the particle energy, and that no quantum excitation is induced. In fact, the final normalized emittance in a generic focusing system is limited only by the uncertain principle and is equal to one half of the Compton wavelength of the electron, which is much smaller than the equilibrium emittance achieved in a normal damping ring.

In this paper, we first review these distinct results. In order to illustrate the transition between bending systems and focusing ones, we study the radiation effects on particle beams in a system where both bending and focusing are present. We show that, in general, quantum excitation can be suppressed by the focusing environment because a photon emission does not take place instantaneously. Finally, we investigate the possibility of an ultra-low emittance damping ring based on this effect.

AN ELECTRON DAMPING RING

In an electron damping ring, the transverse focusing quadruples are present to confine the beam. Their contribution to the radiation effects is secondary relative to the bending dipoles. The typical length associated with a photon emission (the radiation formation length) is on the order of ρ/γ [1, 2], where ρ is the bending radius and γ is the electron energy in units of its rest energy. The standard treatment of quantum excitation can be quasi-classical because the radiation formation length is much shorter than the average beta function β. Thus, one can model the radiation to be instantaneous with a continuous spectrum of frequencies and treat the quantum nature of radiation as fluctuations about the average rate.

On the average, the radiation from the bending dipoles takes away the electron's momenta in all three degrees of freedom, while the rf acceleration only replenishes the longitudinal momentum of the electron. Thus, the transverse damping rate is comparable to the energy damping rate which is given by the characteristic damping constant

$$\Gamma_b = \frac{1}{E}\left|\frac{dE}{dt}\right| = \frac{2}{3}\frac{r_e c\gamma^3}{\rho^2}, \tag{1}$$

where $r_e = e^2/mc^2$ is the classical electron radius.

Although the position of the electron does not change instantaneously right after a photon emission, the horizontal betatron displacement is suddenly changed as a result of the equilibrium orbit shift

$$\delta x_\beta = -\eta\frac{u}{E}, \tag{2}$$

where u is the photon energy, η is the dispersion function, and we have assumed that $d\eta/ds = 0$ for simplicity. Because of the random nature of this sudden change, the transverse normalized emittance diffuses at a rate [1, 2]

$$\left(\frac{d\varepsilon_N}{dt}\right)_{\rm QE} = \gamma\left\langle \frac{(\delta x_\beta)^2}{2\beta}\frac{1}{u}\left|\frac{dE}{dt}\right|\right\rangle_{\text{averaged over the radiation spectrum}}, \qquad (3)$$

$$= \frac{55}{48\sqrt{3}}\frac{r_e\hbar\gamma^6}{m}\frac{\eta^2}{\beta\rho^3} = \frac{55\sqrt{3}}{96}\Gamma_b\frac{\lambda\!\!\!^{-}_c\gamma^3\eta^2}{\beta\rho}$$

where $\lambda\!\!\!^{-}_c = \hbar/mc$ is the Compton wavelength for electron. The equilibrium is reached when the damping rate is equal to the quantum excitation rate. Thus, we obtain

$$(\varepsilon_N)_{\min} \sim \frac{\lambda\!\!\!^{-}_c\gamma^3\eta^2}{\beta\rho} \sim \lambda\!\!\!^{-}_c\frac{\gamma^3}{\upsilon^3}. \qquad (4)$$

In the last step of Eq. (4), we have used the smooth approximation so that the dispersion function is $\eta \sim \beta^2/\rho$ and the betatron tune is $\upsilon \sim \rho/\beta$. Eq. (4) indicates that the equilibrium emittance increases with higher particle energy, and decreases with higher betatron tune. It already suggests that stronger focusing (i.e., increasing the tune) can allow for the lower emittance. Before we study the radiation effects due to focusing in a general system, we first look at the simpler situation of a straight focusing channel.

A STRAIGHT FOCUSING CHANNEL

Following Ref. 3 and 4, we consider a planar focusing system that provides a continuous parabolic potential $Kx^2/2$, where K is the focusing strength. An electron of energy E oscillates in the transverse x direction while moving freely in the longitudinal z direction with a constant longitudinal momentum p_z in the absence of radiation, i.e.

$$E = \sqrt{m^2c^4 + p_z^2c^2 + p_x^2c^2} + \frac{1}{2}Kx^2$$
$$\approx \underbrace{\sqrt{m^2c^4 + p_z^2c^2}}_{E_z} + \frac{p_x^2c^2}{2E_z} + \frac{1}{2}Kx^2, \qquad (5)$$

Defining the transverse frequency as $\omega_z = \sqrt{Kc^2 / E_z}$, we obtain from a simple quantum mechanical analysis that

$$E(n, p_z) = E_z + \hbar\omega_z\left(n + \frac{1}{2}\right), \tag{6}$$

where $n = 0, 1, 2,...$ is the transverse quantum level and is related to the normalized beam emittance by

$$\varepsilon_N = \lambda\!\!\!^{-}_c \left\langle n + \frac{1}{2} \right\rangle_{\text{beam}} \approx \lambda\!\!\!^{-}_c \langle n \rangle_{\text{beam}} \qquad \text{for large } n. \tag{7}$$

Eq. (6) indicates that n is another independent constant of motion besides p_z in the absence of radiation. Instead of building a semi-classical model for the photon emission process, we can calculate the change of the transverse quantum level (ultimately related to the evolution of the normalized beam emittance) directly by conservation laws before and after a photon emission (namely, the conservation of total energy and total longitudinal momentum). A simple kinematical argument shows [3, 4] that n must drop after an arbitrary photon emission. The existence of the focusing field suppresses the direct transverse recoil and absorbs the excess transverse momentum. Therefore, no quantum excitation is induced to the transverse emittance in this focusing system.

When the transverse oscillation amplitude is very small, the transverse motion looks like a one-dimensional harmonic oscillator in the co-moving frame of the electron. It is straightforward to obtain the damping rate in that frame and transform back to the lab frame, then we have

$$\frac{d\varepsilon_N}{dt} = -\frac{2}{3}\frac{r_e K}{mc}\varepsilon_N \equiv -\Gamma_c \varepsilon_N, \tag{8}$$

where $\Gamma_c = 2r_e K/(3mc)$ is the energy-independent damping constant given in Ref. 3 and 4.

In the case of arbitrary transverse oscillation, the transverse motion exhibits a figure of eight motion in the co-moving frame. The damping rate can be calculated using a semiclassical method or using the Lorentz-Dirac radiation damping force [4] and can be written in the form

$$\frac{d\varepsilon_N}{dt} = -\Gamma_c \varepsilon_N - \frac{3}{4}\frac{1}{E}\left|\frac{dE}{dt}\right|\varepsilon_N. \tag{9}$$

The second term of Eq. (9) comes directly from the energy loss, similar to the radiation damping in a normal damping ring. It is the dominant term when the oscillation amplitude is large. However, the direct momentum suppression due to the transverse focusing gives rise to an additional term for the damping (the first term in Eq. (9)), which becomes more significant as the oscillation amplitude becomes smaller.

In the absence of quantum excitation, the electron damps to the transverse ground state (n = 0) that corresponds to a theoretical minimum emittance for the beam:

$$(\varepsilon_N)_{\min} = \lambda\!\!\!^{-}_c/2 \sim 10^{-13}\,\mathrm{m}. \tag{10}$$

This ultimate emittance is limited only by the uncertainly principle, and is analogous to the diffraction limited photon beam emittance because the Compton wavelength here plays the role of natural wavelength for the electron.

We notice that the damping constant Γ_c is independent of energy and is proportional to the focusing strength K. The damping effect is usually negligible for any practical straight focusing device. Thus, we extend this effect to a bent focusing system where the electron beam can be recirculated for a long period of time.

A BENT FOCUSING SYSTEM

In a system where the radiation effects due to focusing are as important as those from bending, the quasi-classical picture of instantaneous photon emissions may not be valid because the oscillation wavelength can be the same order as the radiation formation length. In this case we can follow the treatment of the above section and calculate the evolution of constants of motion when the radiation is turned on. Let us consider a simple model with continuous focusing superimposed by a global bending field. Suppose a reference electron with momentum p_0 has a circular trajectory with radius ρ , then the vector potential for the uniform bending field in a curvilinear coordinates system (x, s, y) is [5]

$$A_s \equiv \left(\vec{A}\cdot\hat{s}\right)\left(1+\frac{x}{\rho}\right) = -\frac{cp_0}{e}\left(\frac{x}{\rho}+\frac{x^2}{2\rho^2}\right). \tag{11}$$

Let the continuous focusing force ($-Kx$) be in the transverse x direction and neglect the dynamics in the other transverse y direction, the total energy of the electron can be decomposed as

$$\begin{aligned} E &= \sqrt{m^2c^4 + p_x^2c^2 + \frac{\left(p_s - eA_s/c\right)^2 c^2}{\left(1+x/\rho\right)^2}} + \frac{1}{2}Kx^2 \\ &\approx \underbrace{\sqrt{m^2c^4 + p_s^2c^2}}_{E_s} + \frac{p_x^2c^2}{2E_s} + \frac{1}{2}\underbrace{\left(K + \frac{p_0^2c^2}{E_s\rho^2}\right)}_{K_e}x^2 - \left(p_s - p_0\right)c\frac{x}{\rho}, \\ &\approx E_s + \frac{p_x^2c^2}{2E_s} + \frac{1}{2}K_e\underbrace{\left(x - x_\varepsilon\right)}_{x_\beta}^2 - \frac{1}{2}K_e x_\varepsilon^2 \end{aligned} \tag{12}$$

where the equilibrium orbit displacement $x_\varepsilon = (p_s - p_0)c/(K_e\rho)$ and the betatron oscillation frequency $\omega_s = \sqrt{K_e c^2 / E_s} \equiv c/\beta$ are both functions of p_s. Similar to the straight channel analysis, the total energy of the electron

$$E(n,p_s) = E_s + \hbar\omega_s\left(n+\frac{1}{2}\right) - \frac{1}{2}K_e x_\varepsilon^2 \tag{13}$$

is a function of n and p_s, with n = 0, 1, 2,... being the transverse quantum level. Both n and p_s are constants of motion in the absence of radiation.

The change of the transverse quantum level n due to spontaneous radiation can be calculated with first-order, time-dependent perturbation theory. Since we are interested in the total radiation effects, we can integrate over the angular and frequency distribution of the radiated photons to obtain the total transition rate W_{fi}, it can be shown that [6]

$$\begin{aligned}\frac{dn}{dt} &= \sum_{f(n',p_s')}(n'-n)W_{fi} \\ &= -\frac{2}{3}\frac{e^2\gamma^3}{\rho^2 mc}\left(\chi^2-1\right)n + \frac{e^2\gamma^3}{\rho^2 mc}\frac{\exp(-2\sqrt{3}\chi)}{144\chi^3}F(\chi)\end{aligned}, \tag{14}$$

where

$$\begin{aligned}F(\chi) &= 55\sqrt{3} + 330\chi + 262\sqrt{3}\chi^2 + 300\chi^3 + 48\sqrt{3}\chi^4, \\ \text{and } \chi &\equiv \frac{\rho/\gamma}{\beta} = \frac{\text{radiation formation length}}{\text{reduced betatron wavelength}}.\end{aligned} \tag{15}$$

From Eq. (7), the evolution of the normalized emittance is then given by

$$\frac{d\varepsilon_N}{dt} = -\Gamma_b \left\{ \left(\chi^2 - 1\right)\varepsilon_N - \lambda\!\!\!^{-}_c \frac{\exp(-2\sqrt{3}\chi)}{96\chi^3} F(\chi) \right\}, \tag{16}$$

where Γ_b is the damping constant defined in Eq. (1). Equation (16) describes the general result of radiation (anti-)damping (the first term) and quantum excitation (the second term) in this combined function system. We can now take various limits for different situations. For example, when $\chi << 1$ or $\rho / \gamma << \beta$, Eq. (16) reduces to

$$\frac{d\varepsilon_N}{dt} = \Gamma_b \left\{ \varepsilon_N + \lambda\!\!\!^{-}_c \frac{55\sqrt{3}}{96\chi^3} \right\} = \Gamma_b \left\{ \varepsilon_N + \lambda\!\!\!^{-}_c \frac{55\sqrt{3}\gamma^3}{96\nu^3} \right\}, \tag{17}$$

where $\upsilon = \rho / \beta$ is the betatron tune in this smooth system. The first term of Eq. (17) is anti-damping instead of damping because the combined function system studied here has a negative horizontal damping partition number (J_x = -1) [1]. However, the second term of Eq. (17) gives the same quantum excitation rate as using the quasi-classical model in an electron damping ring (see Eq. (3) with $\eta \sim \beta^2 / \rho$).

When $\chi >> 1$ or $\rho / \gamma >> \beta$, Eq. (16) also predicts the correct result for a straight focusing channel ($\rho \rightarrow \infty$), i.e.,

$$\frac{d\varepsilon_N}{dt} = -\Gamma_b \chi^2 \varepsilon_N = -\Gamma_c \varepsilon_N . \tag{18}$$

As expected, no quantum excitation is induced in the straight focusing channel.

In the intermediate regime where the radiation formation length is on the order of reduced betatron wavelength ($\rho / \gamma \sim \beta$), the quantum excitation is exponentially suppressed according to Eq. (16) and starts to depart from Eq. (3) based on the quasi-classical model (see Figure 1). The transverse energy spectrum of the electron is highly discrete due to the strong transverse focusing force, and excitation (jumping up transverse levels) becomes impossible for almost all photon emissions. Therefore, the betatron oscillation is adiabatically suppressed to the new ideal orbit during the radiation process.

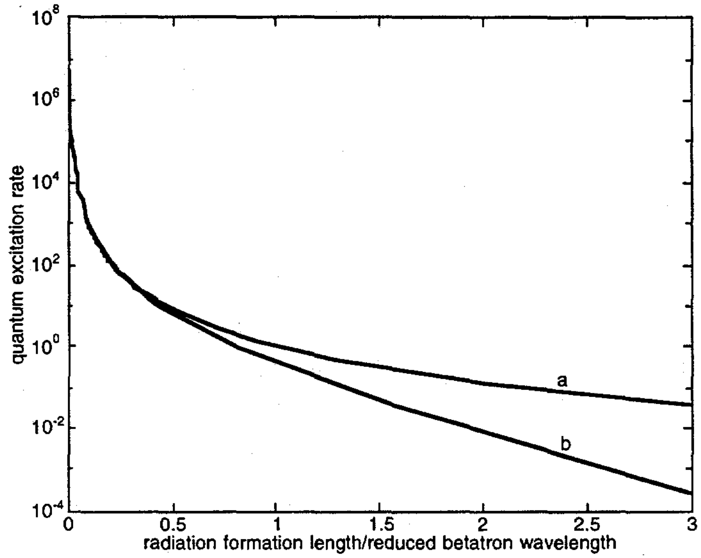

FIGURE 1. Quantum excitation rate in units of $\Gamma_b \lambda\!\!\!^{-}_c$, predicted by (a) quasi-classical model, i.e., Equation (3) and (b) quantum mechanical calculation, i.e., the second term of Equation (16).

Finally, we note that all of the above results can be extended to alternating-gradient and separated function systems when longitudinal variations of both bending and focusing fields are short compared with the radiation formation length [6]. Thus, the beam in such lattices will damp instead of anti-damp. We consider a realistic lattice design in the following section.

A FOCUSING-DOMINATED DAMPING RING

In this section, we study the parameters of a focusing-dominated damping ring where quantum excitation can be strongly suppressed. Suppose that the ring is composed of many repetitive cells. Each cell of length $4L$ consists of four basic elements of equal length L:

focusing quad, bend, defocusing quad, and another identical bend. Both quads have the same field gradient g. Furthermore, we assume that the phase advance per cell is 60 degrees. If we treat the bending as gradual and the cell as a basic FODO cell with drift space $2L$, we obtain

$$L[\text{cm}] \approx \left[\frac{E[\text{MeV}]}{6g[\text{Tesla/cm}]}\right]^{1/2}. \tag{19}$$

The averaged beta function (reduced betatron wavelength) for the 60 degrees cell is

$$\beta = \frac{24L}{2\pi} = \frac{12L}{\pi}. \tag{20}$$

By choosing $\chi \approx 1$ or the averaged ring radius $\rho \approx \gamma\beta = 12\gamma L/\beta$, quantum excitation is kept at the minimum level and the equilibrium emittance is on the order of the Compton wavelength.

These simple lattice scaling formulas suggest that in order to design a compact ring, it is favorable to use high-gradient focusing quads and low-energy electron beams. As an example, we assume that permanent magnet quads have a field gradient $g = 4$ Tesla/cm, and we take the electron energy to be 25 MeV, we then arrive at

$$L = 1.0 \text{ cm}, \;\; \beta = 3.9 \text{ cm}, \;\; \rho = 1.9 \text{ m}. \tag{21}$$

The transverse damping rate is about the same for both the focusing effect and the bending effect since $\rho/\gamma \approx \beta$. The two damping constants are

$$\Gamma_b = \Gamma_c = 0.11 \text{ sec}^{-1}. \tag{22}$$

The transverse size that corresponds to the Compton wavelength is

$$\sigma_x = \sqrt{\frac{\lambda_c}{\gamma}\beta} = 1.8\times 10^{-6}\ \text{cm}. \tag{23}$$

The energy loss per turn is mainly due to the bends, as long as the betatron amplitude is not too large. Thus, we have

$$(\Delta E)_{\text{per turn}} = \frac{2\pi\rho}{c}\Gamma_b E \approx 0.11\ \text{eV}. \tag{24}$$

It can be replenished by either radio-frequency or betatron-type acceleration. The equilibrium energy spread is determined by the effect of discrete photon emissions, and is given by

$$\frac{\sigma_E}{E} = \sqrt{\lambda_c \frac{\gamma^2}{2\rho}} = 1.6\times 10^{-5}. \tag{25}$$

However, at such low energy, space charge and intra-beam scattering effects are significant. It might be conceivable to operate the ring below the transition energy when $\rho/\beta = \nu \equiv \gamma_t > \gamma$ is satisfied, then the Coulomb interaction between electrons, together with the external focusing environment, tend to stabilize the beam by the crystallization effect [7]. Other collective effects such as wakefields and beam-gas scattering can also influence the stability of the system and may determine the final beam emittance. These effects have yet to be studied in this new regime of operation.

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