ON RESPONSE OPERATOR IN SEMICONDUCTOR GAMMA RAY SPECTROMETRY

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Introduction

HPGe detectors having superior resolution are the best detectors available at present for low-level γ-ray spectrometry. However, their relatively lower efficiency in the MeV range resulting from technological difficulties in the production of detectors of large volumes, greatly limits the counting of ultra low activity samples (e.g. in studies of rare nuclear decays, environmental radioactivity investigations, etc. [1]).

Generally, in semiconductor γ-ray spectrometry [2], the most commonly used technique for evaluation of γ-ray spectra is the net area calculation of the full energy peak. The Peak Net Area (PNA) method provides greatly improved energy resolution, however, it takes into account only a fraction of the measured γ-ray spectrum.

The response operator method, mostly used in scintillation of γ-ray spectrometry [3], may be employed to obtain the real incident spectra. However, it is not possible to obtain an exact analytical formulation of the response function for various photon energies [3]. The application of the response operator to semiconductor γ-ray spectrometry is more complicated because of its higher resolution and, therefore, the greater number of spectrometric channels to be handled.

Response operator

The results of *c*-channel spectra measurements, using a matrix approximation of spectra [3], may be written by *c*-vectors put as columns in the matrix, **D**. Similarly, the incident γ-ray spectra associated with the physical spectra in **D** are represented by *c*-vectors put as columns into the matrix, **Q**.

A transformation operator converting the physical spectra (from the matrix **D**) into the unknown incident spectra (in the matrix **Q**) may be interpreted as the $c \times c$ response matrix, $\mathbf{K}_c$. Therefore, the transformation is described as follows

$$\mathbf{D}=\mathbf{K}_c\mathbf{Q}. \qquad (1)$$

The matrix $\mathbf{K}_c$ represents the response operator of the measuring device and it can be estimated by measuring of point standard sources which consist of *n* energy components. These calibration measurements can be described by the following matrix equation

$$\mathbf{S} = \mathbf{K}\,\mathbf{G}, \qquad (2)$$

where *c*-vectors as columns in the matrix, **S**, represent the standard physical spectra and *n*-vectors as columns in the matrix, **G**, represent the incident standard quantities (e.g. standard activities or photon fluence rates) corresponding to *n* energy components in the calibration spectra. The standard response matrix, **K**, is the $c \times n$ matrix of the single-energy component spectra as columns.

Complex analysis (SCFA model)

Because suitable single-energy standard sources, required for the calibration measurements (2), are not evidently available, two important questions in connection with a complex analysis application have to be drawn: how to obtain the single-energy components in operator **K** and how to interpolate the standard response operator, **K**, to the complex response operator, $\mathbf{K}_c$. A decomposition of the response operator into a set of latent determinants (factors) uniform for all energies of the γ-ray spectra in the investigated region could be a good answer. This procedure is described in detail elsewhere [4].

The latent structure of the response operator is based on a set of common factors resulting from fundamental photon interaction with matter as follows:

a) photoelectric factor,
b) Compton factor,
c) backscattering factor,
d) residual factor.

Furthermore, the escape peaks appearing in the spectra at higher energies, and the other factors (e.g. the annihilation factor) should be involved.

Scaling of the common factors distributions in the matrix **F** into corresponding fractional region in physical γ-ray spectra can be formulated as a two-level scaling product defined in Ref. [4] introducing the matrix of the scaling coefficients, **C**. Then the response operator **K** is reproduced by loading of scaling factors using weighting coefficients arranged in the matrix of factor loadings, **B**.

The latent structure of the reponse operator, given by **K(B,C,F)**, is very advatageous to formulate as a manifest covariance structure. This formalism enables us to employ many advanced computational procedures traditionally used in the confirmatory factor analysis [5]. Therefore, this method is called Scaling Confirmatory Factor Analysis (SCFA).

The latent loading, and scaling, coefficients in columns of matrix **C** and **B**, respectively, can be interpolated within measured energy range for energies which correspond to middles of *c* spectrometric channels. The complex *c x c* response matrix $\mathbf{K_c(B_c,C_c,F)}$ can be built up by the reproduction scheme (scaling and loading of the common factors) using the complete matrices, $\mathbf{B_c}$ and $\mathbf{C_c}$, instead of the incomplete standard matrices, **B** and **C** (see Figure 1).

In the second step, complex incident γ-ray spectra, **Q**, by an application of the complex response operator to unknown experimental spectra, **D**, are obtained. As an aspect of computational performance, it has been found that an application of the maximum likelihood followed by the least square method seems to be the best compromise for a complex γ-ray analysis [4].

Verification of the model

A comparison of the experimental γ-ray spectra and the SCFA spectra reproduced by the 4-factor hypothesis were used for a verification of the SCFA model. The verification was carried out for widely-ranged energy interval of standard point g-sources. An estimation of statistical and systematic relative errors of the model is listed in Table 1.

Comparison with traditional approach

A comparison of sensitivity of the traditional PNA method, the integral method and the latent SCFA method based on minimum significant activity (MSA) [6] calculations are shown in Table 2. The critical level set is related to the energy at 661.6 keV (^{137}Cs) for a point source at *10* cm from the detector forehead by using a *12.5%* HPGe detector placed in a shield of *15* cm of iron.

It has been found that the sensitivity of ^{137}Cs measurement for the PNA and SCFA method reaches approximately the same level in the absence of any interference. If some

interference being occured in the spectrum, the MSA for the SCFA model is 4 to 10 times lower (depending on interferences) than for the PNA model (see Table 2).

Tab. 1 The total, statistical and systematic errors of the SCFA model factorization.

Nuclide	t_s [s]	Σs_i	$\Sigma(s_i-\check{s}_i)$	χ^2_s	δ_{total}[%]	δ_{stat}[%]	δ_{syst}[%]
^{241}Am	503	763,2	23,93	15,77	0,65	0,16	0,62
^{57}Co	25 643	7,66	0,01	31,13	1,26	0,23	1,24
^{22}Na	3 008	59,82	0,04	1,86	0,32	0,24	0,22
^{137}Cs	1 041	2 203,4	12,83	6,06	0,16	0,07	0,15
^{88}Y	3 004	190,56	0,13	2,07	0,19	0,13	0,14
^{60}Co	1 190	5 851,8	37,61	7,64	0,11	0,04	0,1
Backg round	294 350	0,9	$3,09.10^{-6}$	1,01	0,2	0,19	0,02
Matrix S	1 000	1 296,7	10,65	8,21	0,25	0,09	0,24
^{152}Eu	1 087	2 920,7	307,6	114,5	0,6	0,06	0,6
<200 keV	1 087	1 388,3	306,8	240,2	1,26	0,08	1,26
>200 keV	1 087	1 532,4	1,57	1,11	0,08	0,08	0,03

Conclusion

Some results of the SCFA application in semiconductor γ-ray spectrometry presented in this contribution points out to a new ground for evaluation the γ-ray spectra. This whole-spectrum processing approach considerably increases detection sensitivity, especially, if significant interferences being present in the measured spectrum. Precision of the SCFA method is determined by choice of a sufficient number of suitable calibration γ-ray sources in the energy region of interest, by setting up an acceptable latent hypothesis and by chosen experimental quantification of spectra.

Tab. 2. *A comparison of MSA (^{137}Cs) for the Integral, PNA and SCFA methods (99% confidence level, counting time of 5000 sec).*

	MSA [Bq]				
Interference	background only	100 Bq of ^{60}Co	200 Bq of ^{152}Eu	13 kBq of ^{60}Co	219 kBq of ^{152}Eu
Integral method	6,4	535	1 286	69 735	$>10^6$
PNA method	2,6	5,3	5,4	53	136
SCFA method	0,7	0,85	0,89	5,4	18
PNA/SCFA ratio	3,71	6,24	6,07	9,9	7,6

The SCFA method is very advantageous to use, for instance, in ultra low-level g-spectrometry where counting rates in full energy peaks are extremly low as compared with background interferences. It enables to increase of the sensitivity by *5-10* times in comparison with the traditional full energy peak net area method.

References

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Figure 1. Factorization and determination in semiconductor spectrometry