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Noise-Induced Drift in Systems with Broken Symmetry and Classical Routes to Superconductivity

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Abstract

We discuss concepts and mechanisms of particle motion in a variety of conditions of asymmetry towards spatial inversion that suggest an idea for the possibility of persistent currents within classical statistical considerations. We expose misapplications of Gibbs statistics and the Langevin approach and show that the idea does not contradict general principles. It gains support from the classical mechanism of capillary wave instability and keeps within the detailed balance and fluctuation-dissipation theorems.

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According to the Landau theorem [1] any internal macroscopic motion of a classical system at thermodynamic equilibrium is forbidden, so the phenomena of persistent currents like superfluidity and superconductivity are beyond the classical statistical physics. Such is the conventional paradigm. However, any postulate imposed on nature is an idealization and thus leaves loopholes. Search for conditions and mechanisms elucidating them may be of importance both in basic and applied aspects. First we describe a few mechanisms of directed motion forced by noise in conditions of broken parity symmetry. Then we consider arguments of the conventional paradigm banning classical routes to persistent currents and finally dwell on arguments supporting the phenomenon.

A. Gyro forces asymmetry

Let us consider a particle hopping upon a horizontal reflecting plate subjected to chaotic vibration. Gravity tends to bring the particle into contact with the plate but the vibration makes it experience Brownian dancing. Now let the particle be charged and a permanent magnetic field be applied in the horizontal direction z. This results in a net drift of the particle along the surface in direction $x \perp z$. When the parameters are such that the height of the jumps is large compared to the amplitude of surface vibration, the drift velocity amounts $V_x =< \Delta x/\Delta t >$ where Δx is the particle displacement for the time Δt of flight between two collisions estimated via the equation $\dot{\mathbf{v}} = \mathbf{g} + \mathbf{\Omega} \times \mathbf{v}$. In the weak magnetic field limit

$$V_x =< v_x > +\Omega < v_y^2 > /3g \ , \tag{1}$$

where g is the acceleration of gravity, Ω the Larmor frequency, and v_x and v_y are the components of particle velocity at the moments just after the collisions. The averaging is over the distribution of v_x, v_y. In case of isotropic scattering $< v_x >= 0$. Taking a number of charged particles, one arrives at macroscopic persistent currents not supplied by the work of regular fields.

Such a process seems to allow extraction of energy out of noise. However, a persistent extraction contradicts the second law of thermodynamics. The only consistent pattern is a coupled-with-noise state representing a limited energy resource. Accurate incorporation of the coupling is essential. We will return to this question later, after sections B and C, where similar questions arise.

B. Gradient forces asymmetry

Let the gyro forces be absent, instead we apply a static potential force field alternating in x asymmetrically, so that its potential $U(x)$ has the periodical ratchet form of Fig. 1. Say the particle is charged and the electric field is formed by an arrangement of identical electrodes charged oppositely in turn, as illustrated in Fig. 1 by circles. In the absence of noise a particle of energy below the level of the potential barrier will oscillate in a potential well, spending most of its time near the right edge of its oscillation. Therefore the ambient noise of weak intensity makes the probability of overcoming the right edge potential barrier larger than that for the left edge. As a result the particle should experience a net drift in the positive x direction. Let. e.g. the particle be in a buffer gas which is so dense that particle motion in the ratchet-form potential $U(x)$ is overdamped. Such an overdamped motion is of the kind discussed in [2]. The model [2] reads

$$\dot{x}(t) = -dU/dx + \xi(t) + \eta(t) , \tag{2}$$

with the $U = U(x)$ taken to be linear piecewise, $\xi(t)$ is a Gaussian white noise and $\eta(t)$ is a Gaussian finite-correlation time noise. The latter, according to numerical computations [2], gives rise to the net particle drift. Possibly, one is not satisfied by numerical results as far as it concerns the average asymptotic behavior at $t \to \infty$. We obtained exact analytical proof of the net drift effect for the system (2) with $\eta(t)$ modelled by a random telegraph function (it takes two values $\pm \mid \eta \mid$ in turn and the alternations occur randomly with a finite mean frequency). Thus, even strong frictional forces seem, from this reasoning, not to be an obstacle for the persistent current.

C. Noise field asymmetry and negative friction effect

The net drift effect considered in section B suggests an idea for the analogous effect caused by the ambient noise by itself, in the absence of static force fields, when the noise intensity is inhomogeneous and has a ratchet profile. As is well-known from long ago, going back to Chladni and Rayleigh [3], inhomogeneous noise gives birth to gradient mean forces (for extensions of the concept see [4]). So, arranging a ratchet form configuration of the noise environment, one arrives at a trend similar to that of the previous section.

In fact, the ambient noise system gives rise to both the gradient and frictional mean forces and they appear in two different ways: due to the back reaction and in neglect of the latter, i.e. when considering the noise influence as given. While the back reaction provides positive friction, this is not the case for the other contribution and it depends on the dispersion law of the noise system's elementary excitations. It appears, even in homogeneous medium, that the net frictional force exerted on a particle moving slowly, as compared to the characteristic velocities of ambient noise excitations, can be negative and will tend to accelerate, on the average, the particle passing a thin layer of the medium [5].

Again, as for the examples of previous sections, the statement about ultimate stationary states corresponding to persistent currents at thermal equilibrium requires for its verification thorough methods, more well-grounded than methods based on the equations like (2).

D. About Gibbs statistics and the Landau theorem

The persistent currents are in evident disparity with the Gibbs statistics. According to it, for a particle of dynamics governed by an arbitrary form of conservative force field as well as for a set of particles under conservative interactions the drift should totally vanish at thermodynamical equilibrium. Indeed, for the dynamics governed by the Hamiltonian function

$$H(\mathbf{r},\mathbf{p}) = \sum (p_i - A_i(\mathbf{r}))^2/2 + U(\mathbf{r}) , \tag{3}$$

(with $\mathbf{r} = \{r_i\}$, $\mathbf{p} = \{p_i\}$ and $A_i(\mathbf{r})$ and $U(\mathbf{r})$ of arbitrary form) the stationary probability distribution of the system's state at equilibrium is prescribed by the

Gibbs statistics to be of the form

$$P(\mathbf{r},\mathbf{p}) = \mathbf{N}\exp[-\beta H(\mathbf{r},\mathbf{p})] \, , \tag{4}$$

and, consequently, $< \dot{r}_i > \equiv 0$ for arbitrary component i, since $\dot{r}_i = p_i - A_i$ and P is even in $p_i - A_i$. This reasoning, which goes back to Niels Bohr, shows that any persistent current is forbidden. The incorporation of internal particle dynamics – and by extension the phase space so that the terms A_i and U in H in (3) become also functions of internal variables – does not change the conclusion.

However, the Gibbs statistics relies on the strong ergodicity hypothesis. Referring to our example of section A, the particle velocities in x and y direction are related by the relation (1) true for arbitrary statistics and thus imposing correlations between the fluctuations of v_x and v_y at equilibrium, while according to the Gibbs statistics these fluctuations should be absolutely independent statistically. Then, while H in (4) is taken exactly the same as the Hamiltonian function of the isolated system, the coupling may contribute to stochasticity and to regular conservative dynamics. Generally the accuracy of Gibbs statistics in the form presented is unknown and cannot be estimated. Being an empirical law, it requires examination for each concrete system.

The same concerns the Landau theorem [1] stating that a closed system of interacting parts in thermal equilibrium admits only uniform translation and rotation motion as a whole. This mathematical theorem does not proceed from any concrete form of the stationary distribution. Instead, it deals with the system's entropy S, taken in the form of a sum where each summand S_k is a function of internal energy equal to the difference between the total and kinetic energy only of part k. Obviously, the assumption of additivity does not apply for parts interacting by long-range forces like gravity and the electromagnetic forces of uncompensated charges. The questions about additivity are especially acute when dealing with surface phenomena.

E. Langevin approach

Let us consider the phenomenon of persistent currents from the standard classical Langevin approach to interacting particles. It proceeds from the equations

$$\dot{r}_i \equiv v_i = \partial H/\partial p_i, \quad \dot{p}_i = -\partial H/\partial r_i - \Gamma_i + \xi_i \, , \tag{5}$$

where H is of the form (3), $\xi = \{\xi_i\}$ is a Gaussian delta-correlated in t process, $< \xi_i(t)\xi_j(t') >= D_{ij}\delta(t-t')$, the time derivatives in (5) are treated in the Stratonovich sense, and $\Gamma = \{\Gamma_i\}$ represents the mean dissipative forces caused by the noise environment and compensating at equilibrium the stochastic pumping in accordance with the fluctuation-dissipation theorem. For this Γ is taken of the form $\Gamma_i = \beta D_{ik} v_k$ where β is the Boltzmann factor and summation over the double index is implied. The probability density distribution $P(\mathbf{r},\mathbf{p},t)$ for the stochastic system (5) obeys the Fokker-Planck type equation

$$\frac{\partial}{\partial t}P + \frac{\partial}{\partial r_i}v_i P - \frac{\partial}{\partial p_i}(\frac{\partial H}{\partial r_i} + \Gamma_i)P = \frac{\partial}{\partial p_i}D_{ik}\frac{\partial}{\partial p_k}P \, . \tag{6}$$

The stationary solution of (6) is exactly the Gibbs distribution given by (4). Therefore a persistent current has no chance to survive.

One might think that these conclusions are due to the delta-correlation of noise. However, when incorporating finite correlations by means of extension of the phase space by additional dynamic variables obeying stochastic equations of type (5), which is a conventional and rather natural way, one arrives at a kinetic equation of type (6) in the extended space and, since the number of variables does not matter for the reasoning presented above, one comes to the same conclusion of zero persistent currents. In particular, assuming in (2) the random force $\eta(t)$ of finite time decay correlations, the approach treats η as a dynamic (Hamiltonian) subsystem forced by white noise and interacting with the x system by the interaction energy term proportional to $x\eta$. In this way, the term $\eta(t)$ in (2) becomes a retarded function of x motion due to back reaction. It can be readily verified, in accordance with the general reasoning outlined above, that the modified force η in (2) acts so that the net drift effect will never occur. Thus, exploring the Langevin approach but in a more consistent way than in [2] – by taking into account the back reaction effects of the ambient noise subsystem, results in total vanishing of the noise energy extraction mechanism [2].

So, it is impossible to prove the existence and investigate the net drift effects analytically or by running computers, basing only on the Langevin equations in the presented form. Virtually, this approach cannot be a proof of Gibbs statistics, because the structure of the noise forces is postulated so as to describe the relaxation pertaining to the desired Gibbs distribution. Another basis should be used.

F. Routes to persistent currents

The crucial point resulting in the ban of persistent currents is that the stationary probability density $P(\mathbf{r},\mathbf{v})$ (4) peaks only at $\mathbf{v} = \mathbf{0}$. However, the universality of this feature follows from nothing.

In general, proceeding from reversible microscopic dynamics and the possibility of a contracted statistical description in terms of $P(\mathbf{r},\mathbf{v})$ for some basic variables at the same time, we have

$$P(\mathbf{r},\mathbf{v}) = P(\mathbf{r},-\mathbf{v})$$

for any set $w = (\mathbf{r},\mathbf{v})$ of basic many-dimensional variables, $\mathbf{r}$ even and $\mathbf{v}$ odd with respect to time reversal, provided the system evolves to a stationary distribution. Here external gyro forces are incorporated by extending the $\mathbf{v}$ space. If additionally we assume that the transition probability in space w obeys a Fokker-Planck equation, then its diffusion and drift terms satisfy a set of detailed balance conditions [6]. As a consequence, only the drift terms associated with the reversible motion enter the stationary Fokker-Planck equation determining $P(w)$. Denoting by $H_{ef}(w)$ the Hamiltonian function of the reversible motion, we arrive at the general solution

$$P(w) = P(H_{ef}(w), J(w))$$

where $J = \{J_k(w)\}$ are the invariants of the motion. That is, P and the associated effective potential Φ defined via $P(\mathbf{r},\mathbf{v}) = \mathrm{N}\exp[-\Phi(\mathbf{r},\mathbf{v})]$ with N a normalization constant have to be invariants of the symmetry group of the system. Here H_{ef} is an effective Hamiltonian, it accounts for the interactions with numerous variables, the truncation of whose space provides the statistical description within the basic collective variables w. It is worth noting that in general the dependence of P on J is ignored ad hoc or on the ground of the ergodicity hypothesis [6]. However, this is the relaxation of the transition probability $P(w,t)$ to the distribution $P(w)$, controlled by the diffusion and irreversible drift terms of the Fokker-Planck equation at hand, that should determine whether this or that integral J_i destroys for the time interval of interest.

If the fluctuations of w are small, the stationary probability will be sharply centered around minima of the potential Φ. Let e.g. the external gyro field be absent. Then Φ is even with respect to the system's velocities. The minima of Φ at $\mathbf{v}=0$ obviously correspond to zero persistent currents, while the minima at $\mathbf{v}\neq 0$ make a difference. Having lower symmetry, these minima are degenerate and the diffusion constitutes a symmetry restoring motion in the usual way. Changing external fields and boundary conditions we control critical points of the transition and the level of currents. In other words, all the well-known phenomenological techniques of the phase transitions can be applied for the case, as well as the proper Langevin equations, equivalent to the Fokker-Planck equations at hand. Still, this is only a phenomenology and one needs to determine under which physical conditions the minima in Φ of lower symmetry, which are obviously beyond the Gibbs statistics of interacting particles, do appear.

Strikingly, an example of such conditions lies on the "surface": it is in the classical physics of capillary waves, i.e. a ripple on a fluid surface, going back to Stokes (1847) and Rayleigh (1917). Let a capillary wave propagate in the x direction upon the fluid surface whose position at rest is $y = 0$. The reversible microdynamics of fluid particles in case of potential interactions is given by

$$m\dot{\mathbf{v}} = -\operatorname{grad} U(\mathbf{r},t)\ , \qquad (7)$$

where U is the self-consistent potential $U = \int K(|\,\mathbf{r}-\mathbf{r}'\,|)n(\mathbf{r}',t)d\mathbf{r}'$. Here $K(|\,\mathbf{r}-\mathbf{r}'\,|)$ is the interaction energy of two particles positioned at $\mathbf{r}$ and $\mathbf{r}'$, $mn(\mathbf{r},t)$ is the mass density of fluid, inside the fluid n may be taken constant and outside $n = 0$.

The capillary waves representing potential flows governed by this dynamics have wonderful kinematics independent of form $K(r)$: the profile $y = y(x,t)$ of a harmonic in the t wave has both oscillating and mean (over t and x) components. The mean $< y >$ is proportional to the wave intensity. It is always positive, if y is reckoned positive in the direction outside the fluid. Due to the elevation the wave transfers mass (charge, etc) along the surface in the direction of propagation. The most striking feature is that the elevation contributes to the potential energy of the wave motion by a negative term, provided

$$K_o = \int_o K(|\,\mathbf{r}_o - \mathbf{r}'\,|)d\mathbf{r}' < 0$$

where $\int_o$ means integration over the fluid volume at rest, $\mathbf{r}_o$ is a point on its surface $y = 0$. The inequality means nothing more than the attraction of surface particles to the bulk of fluid. The wave energy per unit length x of a small intensity harmonic capillary wave amounts to

$$W = < y > n(mc^2 + nK_o/2)\ . \qquad (8)$$

Here c is the velocity of the capillary wave depending on its wave number. The first term in the brackets of (8) is the sum of the kinetic energy $mc^2/2$ and the potential energy $mc^2/2$ due to the surface bending; it is positive. The second term in (8) is negative; it is due to the elevation caused by the wave. So the two terms have different structure and it may happen that the negative contribution prevails. The fluid at rest is energetically less preferable than the wave motion. The wave dynamics is nonlinear and there exists a maximum value of wave intensity and hence of $< y >$ over which the wave process is impossible.

Vlasov [7] considered the ripple as applied to superfluidity and superconductivity and presented estimates supporting the idea. The analysis [7] was based on Vlasov's kinetic equations and their exact solutions which, in fact, correspond to the purely dynamical (without introduction of random elements) results following directly from the reversible microdynamics of ideal inviscid fluid governed by (7). Further developments, in particular an analysis of statistical kinetics of the process and association with phase transitions, we have not found in the literature, so far. Below we briefly describe the trend in terms of the phenomenology outlined above in this section.

The capillary wave instability means that the effective potential Φ as a function of mass transfer changes from one-well to a double-well form, Fig. 2. For our example the mass transfer J_x per unit length (along z) amounts to $J_x =< y > nmc$. Associating Φ with the energy of the wave interactions results in the form of Φ characteristic of the formalism of Ginzburg-Landau equations. The minima of Φ at $J_x \neq 0$ having lower symmetry are degenerate and inevitable fluctuations restore the symmetry ultimately. However, since the talk is about self-arranged regimes of developed capillary wave instability, changing the regime may require high intensity fluctuations. In the conditions of small fluctuations an initial state at one of the minima will have a huge life time until the diffusion restores the symmetry. This time may be longer than the time of observation. The quasistationary regime, with dynamics of its small perturbations, is controlled by diffusion and irreversible drift terms (satisfying Casimir-Onsager relations and the fluctuation-dissipation theorem) so as to provide the quasistationary wave motion. This is the kind of regime below the critical area of transitions to superfluidity and superconductivity as we see them from the classical picture. In this light, reasoning in terms of the drift mechanisms outlined in sections A-C can serve as a heuristic tool for understanding transient behavior, in particular it indicates a variety of ways of symmetry breaking that may facilitate the transition.

Thus, we see that the investigation of classical routes to persistent currents represents an interesting and not groundless field of study which could possibly give new ideas in search for materials and devices.

References

[1] L.D. Landau and E.M. Lifshitz, *Statistical Physics* (Pergamon, New York, 1980).

[2] M.O. Magnasco, Phys Rev Lett **71**, 1477 (1993).

[3] J.W.S. Rayleigh, *The Theory Of Sound* (Dover Publications, New York, 1945).

[4] V.E. Shapiro, Phys Rev E **48**, 109 (1993).

[5] V.E. Shapiro, Phys Lett **102A**, 167 (1984).

[6] R. Graham and H. Haken, Z Physik **243**, 289 (1971); **245**, 141 (1971).

[7] A.A. Vlasov, *Statisticheskie Funktsii Raspredeleniya* (Nauka, Moscow, 1966).

Figure Caption

1. The combined effect of periodical force $-dU/dx$ and noise on the particle results in its drift in the positive x direction.

2. The Ginzburg-Landau model potential associated with the energy of form (8). The upper figure corresponds to the regime of stable surface state, and the lower figure to the ripple instability.

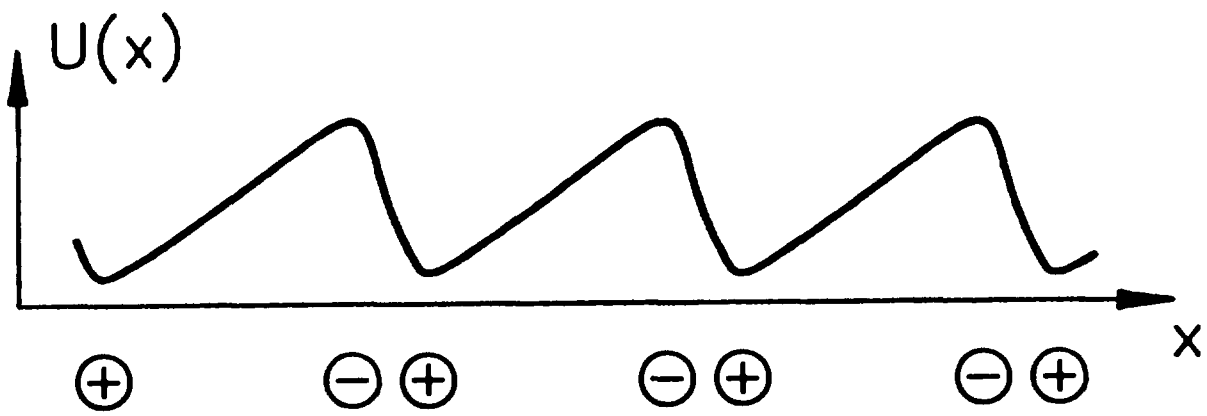

Fig. 1

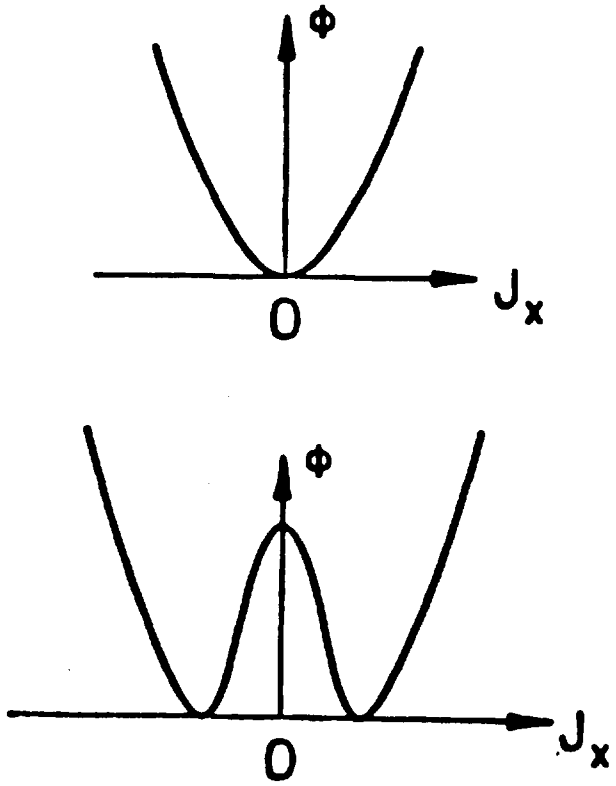

Fig. 2