TRI-PP-95-58
August 1995

Chiral symmetry breaking and cooling in lattice QCD

R.M. Woloshyn and Frank X. Lee

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

Abstract

Chiral symmetry breaking is calculated as a function of cooling in quenched lattice QCD. A non-zero signal is found for the chiral condensate beyond one hundred cooling steps, suggesting that there is chiral symmetry breaking associated with instantons. Quantitatively, the chiral condensate in cooled gauge field configurations is small compared to the value without cooling.

(Submitted to Physical Review D)

PACS numbers: 12.38.Gc, 11.15.Ha, 11.15Kc

Recently Chu *et al* [1] presented evidence from a quenched SU(3) lattice gauge theory that instantons play a dominant role in determining hadronic correlation functions. Motivated by this work, a calculation was done in quenched SU(2) lattice gauge theory exploring the effect of cooling [2]. In Ref. [2] it was found that the spin-dependent potential and the chiral symmetry breaking order parameter disappear very quickly upon cooling. In a subsequent nonquenched SU(3) calculation by Edwards *et al* [3], it was found that cooling has a large effect on the hadron spectrum, tending, for example, to diminish spin-slittings. This supports the conclusions of Ref. [2]. However, Edwards *et al* [3] did not see the rapid disappearance of the chiral order parameter found in the quenched SU(2) calculation. For this reason we have decided to revisit the problem of chiral symmetry breaking and cooling. A calculation was done in quenched SU(3) lattice gauge theory to see if the different chiral symmetry breaking pattern observed in Ref. [2] and in Edwards *et al* [3] is due to the use of SU(2) versus SU(3) color or due to the quenched versus nonquenched nature of the simulation.

In this paper we present results for the chiral order parameter calculated in a set of quenched SU(3)-color gauge configurations to which local cooling [4,5] has been applied. A persistent non-vanishing signal for chiral symmetry breaking was found even for cooling of a hundred sweeps or more. This differs from what was observed in the earlier quenched SU(2) simulation [2] but is consistent with the finding of Edwards *et al* [3] in their nonquenched SU(3) calculation. Quantitatively, after cooling, the chiral order parameter is small (in lattice units) compared to the uncooled value.

The calculations were done using standard methods of Ref. [2] carried over to SU(3). The fermion action is

$$\begin{aligned} S_f &= \frac{1}{2}\sum_{x,\mu}\eta_\mu(x)\left[\overline{\chi}(x)\,U_\mu(x)\,\chi(x+\hat{\mu}) \;-\overline{\chi}(x+\hat{\mu})U^\dagger_\mu(x)\,\chi(x)\right] + \sum_x m\overline{\chi}(x)\chi(x), \\ &\equiv \overline{\chi}\,\mathcal{M}(\{U\})\,\chi, \end{aligned} \tag{1}$$

where $\overline{\chi}, \chi$ are single-component fermion fields, $\eta_\mu(x)$ is the staggered fermion phase [6], m is the mass (in units of the inverse of the lattice spacing a) and the $U's$ are gauge field links. Antiperiodic boundary conditions were used for the staggered fermion fields in all directions.

The chiral symmetry order parameter is calculated from the inverse of the fermion matrix $\mathcal{M}$ of Eq. (1)

$$\langle\overline{\chi}\chi\rangle = \frac{1}{V}\,\langle \mathrm{Tr}\,\mathcal{M}^{-1}(\{U\})\rangle, \tag{2}$$

where V is the lattice volume and the angle brackets indicate a gauge field configuration average. The random source method [7] was used to calculate $\mathrm{Tr}\mathcal{M}^{-1}(\{U\})$. Twelve Gaussian random sources were used for each gauge field configuration.

Calculations were done on a 10^4 lattice at $\beta = 5.7$ and on a 12^4 at $\beta = 5.9$. The SU(3) pseudo-heatbath method was used for updating the quenched gauge field configurations. After 4000 sweeps to thermalize, configurations were analyzed every 300 sweeps. A total of 30 configurations at $\beta = 5.7$ and 20 configurations at $\beta = 5.9$ were included in the analysis. Each configuration was cooled using a local cooling algorithm [4,5] and $\langle\overline{\chi}\chi\rangle$ was calculated along the way.

The results are tabulated in Table 1. The results for $\langle\overline{\chi}\chi\rangle$ as a function of quark mass at no cooling and after 20 cooling sweeps are shown in Fig. 1 and Fig. 2 for

$\beta = 5.7$ and $\beta = 5.9$ respectively. The value of $\langle\overline{\chi}\chi\rangle$ obtained in a free field theory is also plotted in the figures.

Following Bowler *et al* [8], the values at zero quark mass (extrapolated values) were obtained by fitting to the function

$$\langle\overline{\chi}\chi\rangle(m) = \langle\overline{\chi}\chi\rangle_0 + \langle\overline{\chi}\chi\rangle_1 m + \langle\overline{\chi}\chi\rangle_2 m^2. \quad (3)$$

The resulting chiral condensate values $\langle\overline{\chi}\chi\rangle_0$ are plotted in Fig. 3 versus the number of cooling sweeps. A clear nonzero signal for chiral symmetry breaking is obtained confirming the conclusion of Edwards *et al* [3]. Furthermore, this signal persists beyond 100 cooling sweeps where one expects to see isolated instantons in the gauge field configurations [9]. This is in clear contrast to the results of Ref. [2] (see Figs. 9-11) where the chiral order parameter was found to disappear very quickly with cooling. It is fair to say, however, that the value of the chiral order parameter, although nonzero after cooling, is small compared (in lattice units) to the uncooled value. Therefore we have an indication that, in an SU(3) gauge theory, instantons produce chiral symmetry breaking but we have no evidence that instantons dominate chiral symmetry breaking.

At present we have no explanation for the different behavior observed in the SU(2) and SU(3) calculations. One possibility is that the SU(2) calculation, which was done at $\beta = 2.4$, is just deeper in the weak coupling region than the SU(3) calculations done here at $\beta = 5.7$ and 5.9. Indeed, comparing the SU(3) calculations at the two β values may suggest a diminished persistence of chiral symmetry breaking at $\beta = 5.9$ but this conclusion can not be made definitively due to the smallness of the chiral condensate (and concomitant large uncertainty) at the larger β value. It may also be that there is still some more fundamental difference between the SU(2) and SU(3) theories which has yet to be found.

It is a pleasure to thank H.D. Trottier for helpful discussions. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

I. REFERENCES

[1] M.-C. Chu, J.M. Grandy, S. Huang and J.W. Negele, Phys. Rev. D **49**, 6039 (1994).

[2] H.D. Trottier and R.M. Woloshyn, Phys. Rev. D **50**, 6939 (1994).

[3] R.G. Edwards, K. Bitar, U.R. Heller and A.D. Kennedy, Nucl. Phys. B(Proc. Suppl.) **42**, 328 (1995).

[4] B. Berg, Phys. Lett. **104B**, 475 (1981); Y. Iwasaki and T. Yoshie, Phys. Lett. **125B**, 197 (1983).

[5] J. Hoek, Phys. Lett. **166B**, 199 (1986); M. Teper, Phys. Lett. **171B**, 86 (1986); E.-M. Ilgenfritz, M.L. Laursen, M. Müller-Preussker, G. Schierholz and H. Schiller, Nucl. Phys. **B268**, 693 (1986).

[6] N. Kawamoto and J. Smit, Nucl. Phys. B **192**, 100 (1981).

[7] R. Scalettar, D. Scalapino and R. Sugar, Phys. Rev. B **34**, 7911 (1986); S. Gottlieb, W. Liu, D. Toussaint, R.L. Renken and R.L. Sugar, Phys. Rev. D **35**, 3972 (1987); K. Bitar, A.D.Kennedy, R. Horsley, S. Meyer and P. Rossi, Nucl. Phys. **B313**, 348 (1989).

[8] K.C. Bowler, R.D. Kenway, D. Roweth and D.B. Stephenson, Nucl. Phys. **B301**, 304 (1988).

[9] M.I. Polikarpov and A.I. Veselov, Nucl. Phys. **B297**, 34 (1988).

Table 1: Chiral order parameter values calculated for different values of the quark mass along with the value extrapolated to zero mass.

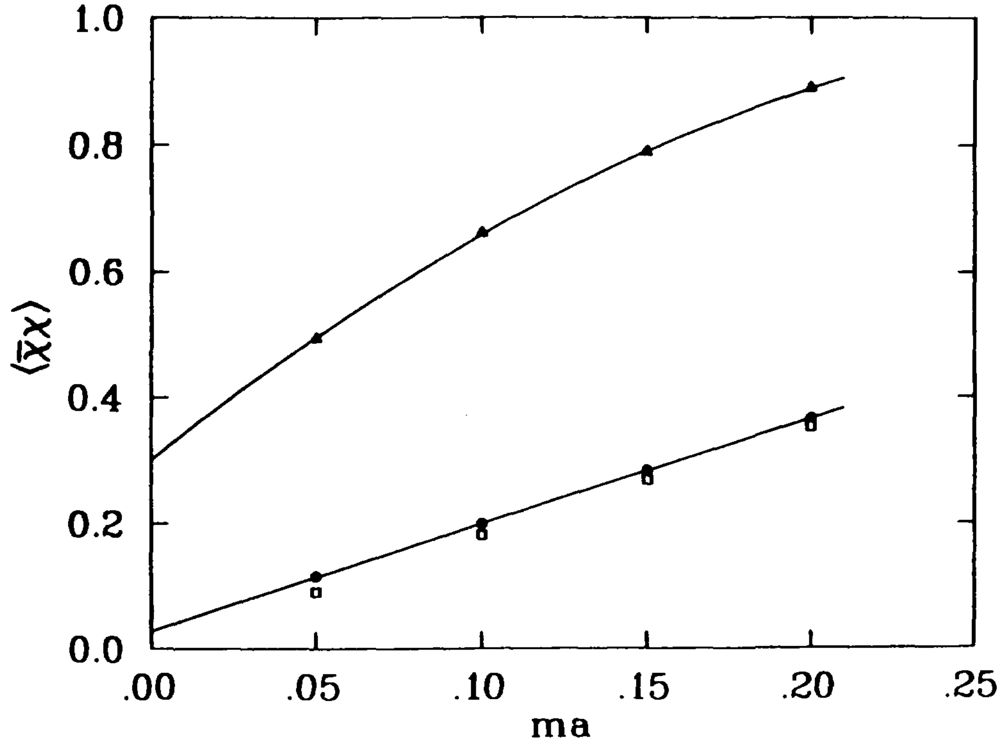
		ma				
β	Cooling step	0.2	0.15	0.1	0.05	Extrapolated value
5.7	0	0.889(2)	0.788(2)	0.660(2)	0.494(3)	0.030(1)
	20	0.3659(3)	0.2835(5)	0.1986(7)	0.115(2)	0.028(4)
	40	0.3613(5)	0.2783(5)	0.1930(8)	0.110(2)	0.023(5)
	60	0.3600(4)	0.2768(6)	0.1913(9)	0.109(2)	0.020(4)
	80	0.3593(5)	0.2761(6)	0.1907(9)	0.107(2)	0.020(4)
	100	0.3592(5)	0.2757(6)	0.1899(9)	0.107(2)	0.020(5)
5.9	0	0.7775(7)	0.6589(8)	0.5104(9)	0.316(1)	0.091(4)
	20	0.3595(2)	0.2760(2)	0.1892(3)	0.1009(5)	0.009(2)
	40	0.3583(2)	0.2745(2)	0.1874(3)	0.099(6)	0.007(2)
	60	0.3576(2)	0.2738(2)	0.1866(3)	0.098(6)	0.005(2)
	80	0.3576(2)	0.2736(2)	0.1862(3)	0.098(6)	0.005(2)
	100	0.3573(2)	0.2732(2)	0.1859(3)	0.097(6)	0.005(2)

FIGURE CAPTIONS

Fig. 1. Plot of the chiral order parameter (in lattice units) as a function of quark mass at $\beta = 5.7$ for no cooling ($\triangle$) and 20 cooling steps ($\bullet$). The lattice free-field values are also shown ($\square$). The lines are the extrapolations to zero quark mass using Eq. (3).

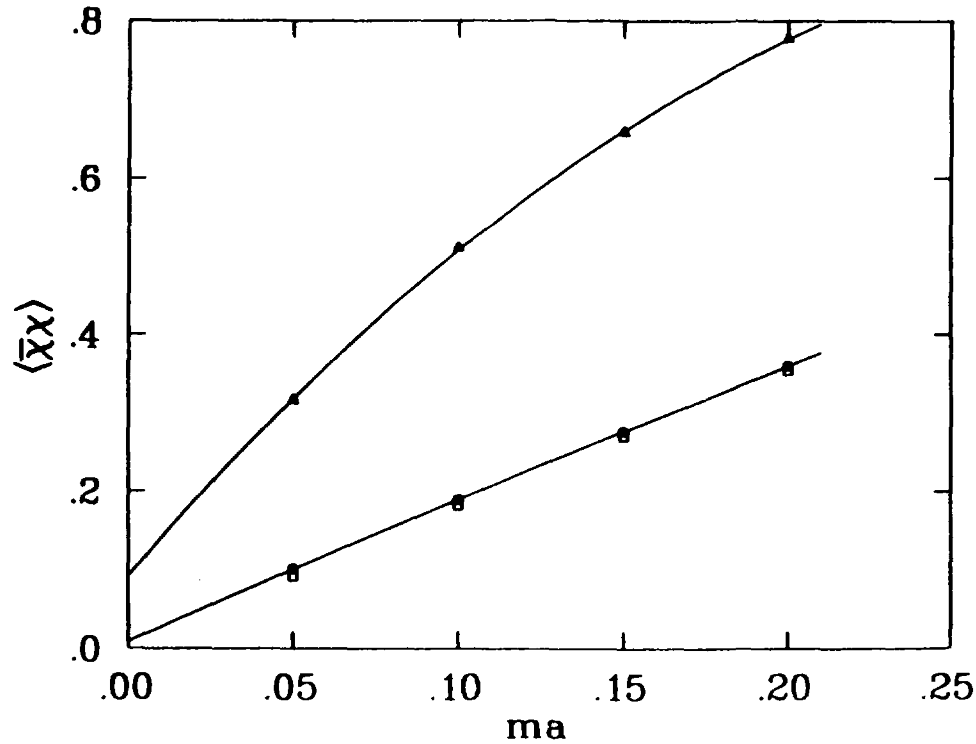
Fig. 2. Same as in Fig. 1, except for $\beta = 5.9$.

Fig. 3. The chiral order parameter at zero quark mass $\langle\bar{\chi}\chi\rangle_0$ versus number of cooling sweeps at (a) $\beta = 5.7$, and (b) $\beta = 5.9$.



1.0
0.8
0.6
0.4
0.2
0.0
$\langle\bar{\chi}\chi\rangle$
.00
.05
.10
.15
.20
.25
ma

Fig. 1



.8
.6
.4
.2
.0
$\langle\bar{\chi}\chi\rangle$
.00
.05
.10
.15
.20
.25
ma

Fig. 2

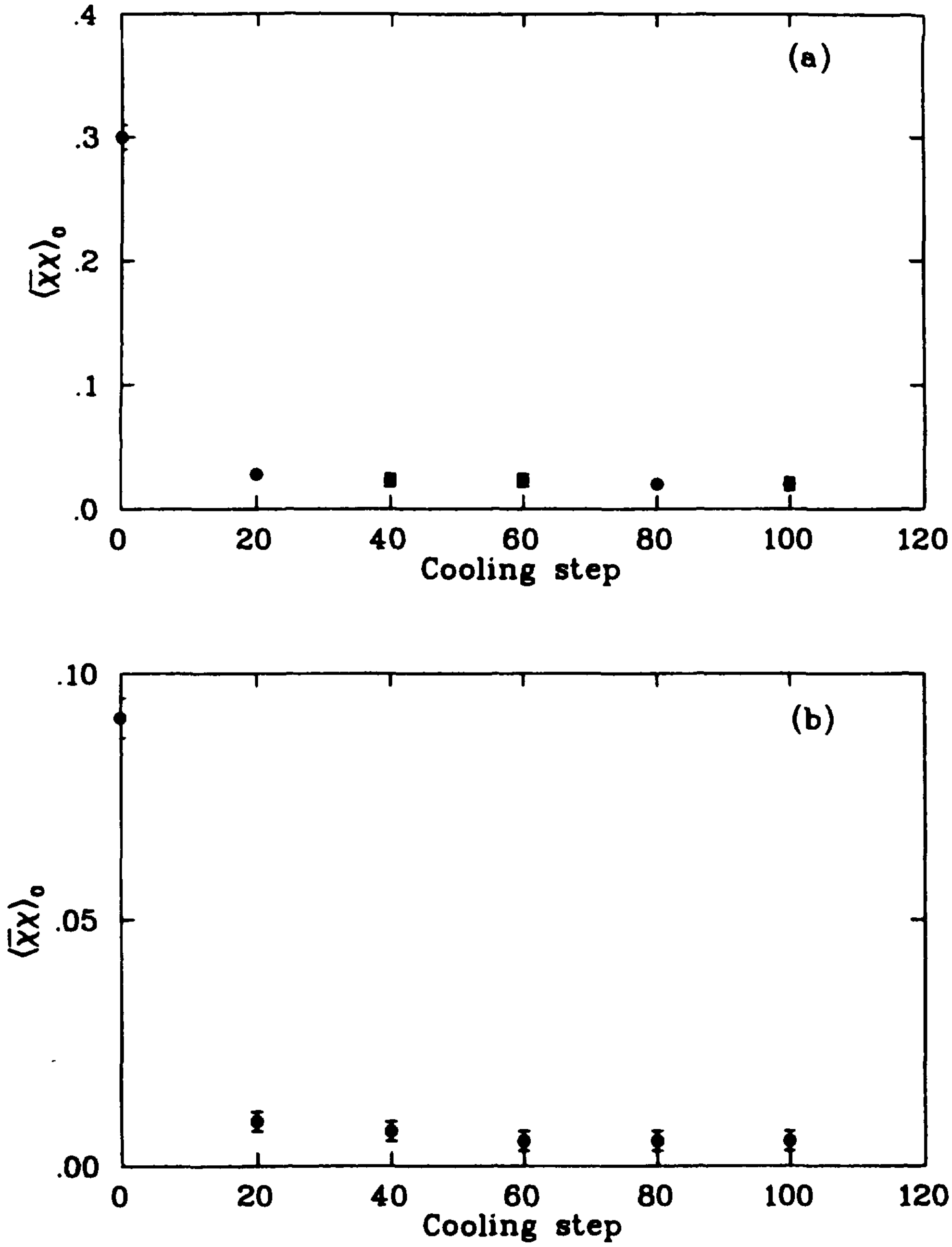

(a)
(b)
$\langle\bar{\chi}\chi\rangle_0$
Cooling step
.0
.1
.2
.3
.4
.00
.05
.10
0
20
40
60
80
100
120

Fig. 3