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STATUS OF CHIRAL PERTURBATION THEORY

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ABSTRACT

A survey is made of semileptonic and nonleptonic kaon decays in the framework of chiral perturbation theory. The emphasis is on what has been done rather than how it was done. The theoretical predictions are compared with available experimental results.

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1 Physics at Low Energies

By decision of the Organizing Committee of this meeting, the strange quark was declared to be a heavy quark. Although we have benefitted from this decision, the strange quark is really a light quark on most other accounts. Of course, all quarks except for the top quark are light compared to the natural scale M_W of the Standard Model. However, there is an important practical distinction between bottom and charm quarks on one side and up, down and strange quarks on the other side.

As we come down from the "fundamental" scale M_W to lower energies, we can rely on perturbative QCD for the operator product expansion to describe physics at energies down to about m_c [1]. At this scale, the Lagrangian of the Standard Model has broken up into different pieces, such as the strangeness changing Lagrangian with $|\Delta S| = 1$ relevant for kaon physics. The degrees of freedom in those Lagrangians are the gluons and the quarks with masses below 1 GeV. From here on, the picture changes drastically as far as the theoretical framework is concerned. Because of confinement, it does not make sense to use perturbative QCD to describe the interactions of "light" quarks at energies below 1 GeV.

Among the many models and methods that have been employed to describe physics at low energies, two of them have the best theoretical credits by far: lattice gauge theories [2] and chiral perturbation theory [3, 4, 5, 6] (CHPT). The strategy of CHPT is that of an effective field theory for the actually observed degrees of freedom, i.e. for the hadrons (and leptons). CHPT uses only the symmetries of the Standard Model to construct the effective field theory in the nonperturbative domain. The advantage of such an approach is its generality: the predictions of CHPT are rigorous predictions of the Standard Model. The drawback is that this effective field theory has a score of a priori undetermined coupling constants (often called low-energy constants) that are not constrained by the symmetries. As is true for any effective field theory, these low-energy constants are remnants of the "short-distance" structure. The notion "short distances" encompasses all degrees of freedom that are not included as explicit fields in the effective Lagrangians. For kaon physics, only the pseudoscalar meson octet is contained in the CHPT Lagrangians. All other effects at higher scales such as meson resonances or short-distance effects in the usual terminology like CP violation are incorporated in the low-energy constants. Let me note at this point that CP violation will not be discussed in this survey.

An essential ingredient of CHPT is the spontaneously broken chiral symmetry, an approximate symmetry of the Standard Model for light quarks. The structure of the CHPT Lagrangians is constrained by this symmetry that gives rise to a systematic low-energy expansion. The relevant expansion parameter is

$$\frac{p^2}{16\pi^2 F_\pi^2} = \frac{p^2}{M_K^2} \cdot \frac{M_K^2}{16\pi^2 F_\pi^2} = 0.18 \, \frac{p^2}{M_K^2} \tag{1}$$

where p is a typical momentum and $F_\pi = 92.4$ MeV is the pion decay constant. Therefore, higher-order corrections in CHPT amplitudes for kaon decays are naturally of the order of 20%. This is not a very small parameter, but it is small enough to talk of a perturbative expansion unlike for the strong coupling constant in this energy range.

The status of CHPT is discussed in several recent reviews [7, 8, 9, 10]. For kaon decays

Table 1: *Phenomenological values and source for the renormalized coupling constants $L_i^r(M_\rho)$. The errors include estimates of the effect of higher-order corrections.*

i	$L_i^r(M_\rho) \times 10^3$	source
1	0.4 ± 0.3	$K_{e4}, \pi\pi \to \pi\pi$
2	1.35 ± 0.3	$K_{e4}, \pi\pi \to \pi\pi$
3	-3.5 ± 1.1	$K_{e4}, \pi\pi \to \pi\pi$
4	-0.3 ± 0.5	Zweig rule
5	1.4 ± 0.5	$F_K : F_\pi$
6	-0.2 ± 0.3	Zweig rule
7	-0.4 ± 0.2	Gell-Mann-Okubo,L_5, L_8
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}, L_5,$ $(2m_s - m_u - m_d) : (m_d - m_u)$
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$

in general and for the chiral description in particular, the standard reference is the Second DAΦNE Handbook of Physics [11]. In fact, most of what I am going to cover here can already be found there. More recent accounts can be found in the Proceedings of the Workshop on K Physics held at Orsay this spring [12].

2 Semileptonic K Decays

All semileptonic K decays that can be measured in the foreseeable future have been calculated at one–loop level. In the standard CHPT terminology, this corresponds to $O(p^4)$ for amplitudes without an ε tensor and $O(p^6)$ for those with an ε tensor. In some cases, higher-order corrections have been at least partly included with the help of dispersion relations. For semileptonic decays, the Second DAΦNE Handbook [11] is still up-to-date. I will therefore restrict myself to a few illustrative examples.

At lowest order in the low–energy expansion, the chiral Lagrangian for the strong, electromagnetic and semileptonic weak interactions contains a single parameter F, equal to $F_\pi = F_K$ at this level, and the meson masses. At the next order, $O(p^4)$, there are 10 new low–energy constants [5] L_1,. . .,L_{10}. Since they have all been determined phenomenologically, we have a completely predictive scheme to this order. In Table 1, the current status of these low–energy constants is summarized.

As a first example, we consider the decays K_{l2ll}, i.e. K_{l2} decays with a virtual photon producing a lepton pair. There are four form factors characterizing the decay amplitudes. The three axial ones dominate the rates. In Table 2, the chiral predictions [13] are compared with available experimental results. The decay mode with an electron neutrino is especially

Table 2: *Branching ratios for K_{l2ll} from theory [13] and experiment [14, 15]. For the modes with an e^+e^- pair, a cut $m_{e^+e^-} \geq 140$ MeV has been applied.*

	$K^+ \to \mu^+\nu_\mu e^+e^-$	$K^+ \to e^+\nu_e e^+e^-$	$K^+ \to \mu^+\nu_\mu \mu^+\mu^-$
tree	$5.0 \cdot 10^{-8}$	$2.1 \cdot 10^{-12}$	$3.8 \cdot 10^{-9}$
1-loop	$8.5 \cdot 10^{-8}$	$3.4 \cdot 10^{-8}$	$1.35 \cdot 10^{-8}$
experiment	$(1.23 \pm 0.32) \cdot 10^{-7}$	$(2.8^{+2.8}_{-1.4}) \cdot 10^{-8}$	$\leq 4.1 \cdot 10^{-7}$

Table 3: *Branching ratio for $K_L \to \pi^\pm e^\mp \nu_e \gamma$ from theory [13] and experiment [17]. Cuts on the photon energy $E_\gamma \geq 30$ MeV and on the electron-photon opening angle $\Theta_{e\gamma} \geq 20°$ have been applied.*

	BR($K_L \to \pi^\pm e^\mp \nu_e \gamma$)
Bremsstrahlung	$3.6 \cdot 10^{-3}$
$O(p^4)$ (L_i only)	$4.0 \cdot 10^{-3}$
$O(p^4)$ total (L_i and loops)	$3.8 \cdot 10^{-3}$
experiment (NA31)	$(3.61 \pm 0.14 \pm^{0.21}_{0.15}) \cdot 10^{-3}$

interesting because practically the whole amplitude is generated at $O(p^4)$ due to the helicity suppression of the lowest-order amplitude (Bremsstrahlung). In both channels where events have been found the effects of $O(p^4)$ are definitely seen.

The second class of decay modes I want to mention are radiative K_{l3} decays. The theoretical analysis [13] involves altogether ten form factors two of which appear also in the non-radiative decays. The final conclusion after quite some work is that the effects of $O(p^4)$ are relatively small: the amplitudes are still dominated by Bremsstrahlung. Let me emphasize that this is a definite prediction rather than an unfortunate mishap. As we saw in the previous case and as we shall see again for K_{l4} decays, the corrections of $O(p^4)$ are by no means negligibly small in general. Instead of a comprehensive comparison with experiment (for the status as of 1995 see Ref. [16]), I concentrate in Table 3 on the decays $K_L \to \pi^\pm e^\mp \nu_e \gamma$ where a new experimental result [17] has become available. The chiral prediction that the decay is dominated by Bremsstrahlung is supported by the data.

As a final example of semileptonic decays let me turn to K_{l4} decays. As for K_{l2ll}, there are three axial and one vector form factor. Two of the axial form factors dominate the amplitudes. In addition to the $O(p^4)$ calculation [18], the dominant higher-order effects were estimated using dispersion relations [19]. In this case, the corrections of the leading current algebra amplitudes of $O(p^2)$ are large. In Table 4 taken from the talk of Bijnens at the Orsay Workshop [12], the chiral predictions [19] for the various decay widths are confronted with experiment. The channel $K^+ \to \pi^+\pi^- e^+ \nu_e$ with the highest statistics [20] was used to

Table 4: *Predictions for the various K_{l4} decay widths [19]. The last two columns are normalized to K_L decays. Full includes estimates of higher-order corrections beyond $O(p^4)$. Errors are in brackets and all values are in s^{-1}.*

$\pi\pi$ charge	+−	00	+−	00	0−	0−
leptons	$e^+\nu$	$e^+\nu$	$\mu^+\nu$	$\mu^+\nu$	$e^+\nu$	$\mu^+\nu$
tree	1297	683	155	102	561	55
p^4	2447	1301	288	189	953	94
full	input	1625(90)	333(15)	225(11)	917(170)	88(22)
exp.	3160(140)	1700(320)	1130(730)	–	998(80)	–

extract the three low-energy constants L_1, L_2 and L_3 together with information from $\pi\pi$ scattering (cf. Table 1). The agreement with experiment in the remaining channels is quite impressive although the statistics is limited. Note the big corrections in going from tree level to $O(p^4)$ and the still sizable higher-order corrections (third line of theoretical predictions in Table 4 denoted "full").

The rates are mainly determined by the real part of the form factors. Through the imaginary parts, K_{l4} decays also allow for accurate measurements of some of the $\pi\pi$ scattering phase shifts. The present experimental status [20] is shown in Fig. 1 together with the theoretical predictions up to $O(p^6)$ (two-loop level) for the difference between the $I = l = 0$ and the $I = l = 1$ phase shifts [21]. We are looking forward to precision data on K_{e4} from DAΦNE and other kaon facilities to test CHPT to $O(p^6)$. This is a calculation based on chiral $SU(2)$ (pions only) where the natural expansion parameter is much smaller than for kaon decays, at least near threshold.

3 Nonleptonic K Decays

Already at the level of the operator product expansion, the semileptonic and nonleptonic weak decays are described by different Lagrangians. The same holds for the effective description of CHPT. The nonleptonic weak interactions of kaons require an additional chiral Lagrangian with low-energy constants that have a priori nothing to do with the strong constants L_i in Table 1.

At lowest order, again $O(p^2)$, the chiral Lagrangian for $|\Delta S| = 1$ nonleptonic weak interactions is characterized by two coupling constants G_8 and G_{27}, responsible for the octet and the 27-plet part of the effective Hamiltonian. The dominant decay modes $K \to 2\pi$, 3π are determined by these constants at lowest order (current algebra level). From $K \to 2\pi$ decays one extracts

$$|G_8| = 9 \cdot 10^{-6}\mathrm{GeV}^{-2} \, , \qquad G_{27}/G_8 = 1/18 \, . \tag{2}$$

The small ratio between G_{27} and G_8 is a manifestation, but of course not an explanation of

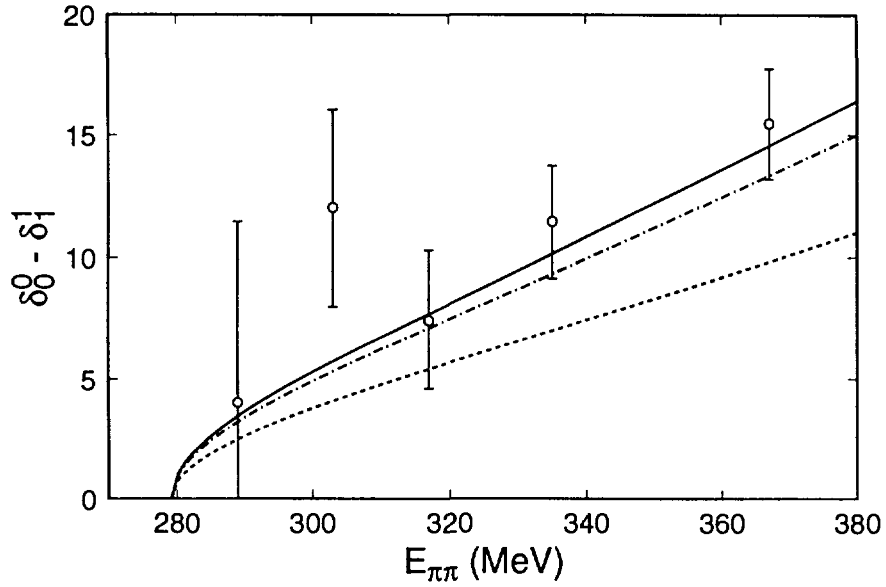

Figure 1: *The phase shift difference $\delta_0^0 - \delta_1^1$ (in degrees) as a function of the center-of-mass energy of the two incoming pions. The dotted (dash-dotted) line displays the tree (one-loop) approximation, whereas the solid line denotes the two-loop result [21]. The data are from Rosselet et al. [20].*

the $\Delta I = 1/2$ rule. For CHPT, this small ratio is input that allows to neglect the 27-plet contribution in most cases where the octet also contributes. With the values (2) one can predict 7 measurable quantities in $K \to 3\pi$ decays (amplitude and slope parameters). The conclusion has been known for many years: the agreement is qualitative only and there are sizable deviations on the order of 20 - 30 % in amplitude, precisely of the order expected in CHPT.

Before we move on to the next order in the chiral expansion, we can ask ourselves whether there are other channels for which predictions can be made at lowest order. I am not aware of another framework where one could prove the following statement as easily as in CHPT: there is no additional information in nonleptonic kaon amplitudes at lowest order in the momentum expansion beyond Bremsstrahlung which is of course determined by the non-radiative $K \to 2\pi$, 3π decays. In other words, most of the interesting physics in nonleptonic kaon decays starts at $O(p^4)$ only.

At next-to-leading order, $O(p^4)$, many new couplings enter. In the octet sector alone, there are 22 new low-energy constants [22] in addition to the ones we have already encountered. The suspicion seems well-founded that such an approach cannot have much predictive power. I will try to convince you that this suspicion is in general not justified.

Let us first turn to the dominant decay modes. It turns out [23] that there are only seven combinations of coupling constants for the altogether[1] 12 observables in $K \to 2\pi$, 3π decays. The resulting five relations [24] can be expressed as predictions for some of the quadratic slope parameters in the $K \to 3\pi$ amplitudes. As shown in Table 5, the agreement is very good

[1] At lowest order, five of the slope parameters vanish which explains the number seven mentioned before.

Table 5: *Predicted and measured values of the quadratic slope parameters in* $K \to 3\pi$ *amplitudes, all given in units of* 10^{-8}. *The table is taken from Kambor et al. [24] and is based on the CHPT calculation [23] to* $O(p^4)$.

parameter	prediction	exp. value
ζ_1	-0.47 ± 0.18	-0.47 ± 0.15
ξ_1	-1.58 ± 0.19	-1.51 ± 0.30
ζ_3	-0.011 ± 0.006	-0.21 ± 0.08
ξ_3	0.092 ± 0.030	-0.12 ± 0.17
ξ_3'	-0.033 ± 0.077	-0.21 ± 0.51

for the two $I = 1/2$ parameters where the data are most precise. The remaining predictions for the $I = 3/2$ slope parameters clearly need higher experimental precision for a meaningful comparison. Thus, even for the dominant nonleptonic K decays there are predictions of the Standard Model that remain to be tested.

All other nonleptonic K decays are put into the category of rare decays. The following classification takes into account the different structure of chiral amplitudes for the various transitions.

3.1 Short–distance dominated transitions

Here, CHPT cannot do much more than list the possible low–energy constants (where all the short–distance structure resides) and estimate to which extent the long–distance parts are suppressed. In addition to the well-known decays [1] $K \to \pi\nu\overline{\nu}$, the process $K_L \to \pi^+\pi^-\nu\overline{\nu}$ has recently been investigated [25].

3.2 Transitions with completely calculable $O(p^4)$ amplitudes

In this group, none of the 22 low–energy constants occurring in general at $O(p^4)$ actually appear in the amplitudes. There are still two different cases to distinguish: either the amplitude vanishes altogether at $O(p^4)$ or it does not.

Among the first transitions is $K_L \to \pi^0\pi^0\gamma$ where only an upper limit is available for the branching ratio [26]. However, there is also $K_L \to \gamma\gamma$ which is well–measured and yet cannot be counted as a success for CHPT. The amplitude for this decay vanishes at $O(p^4)$ due to the Gell-Mann–Okubo mass formula for the meson masses, but the corrections are large and not reliably calculable at present. Although formally of $O(p^6)$, the actual size of the decay amplitude as extracted from experiment is more like a typical p^4 amplitude. Unfortunately, this theoretical uncertainty influences also the decay $K_L \to \mu^+\mu^-$ where the dispersive part of the two–photon intermediate state cannot be reliably estimated [1] for the same reason.

Fortunately, $K_L \to \gamma\gamma$ is an exception rather than the rule in nonleptonic K decays.

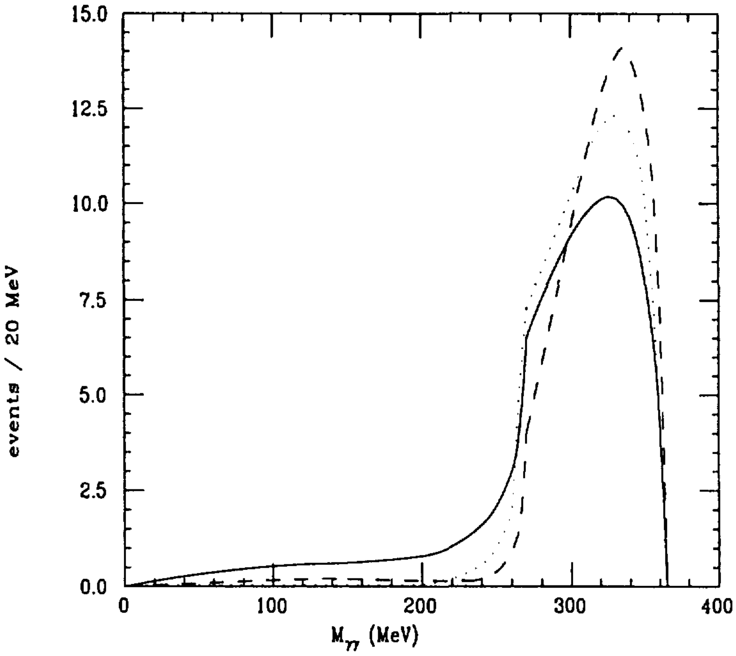

Figure 2: *Theoretical predictions for the* 2γ *invariant-mass distribution in* $K_L \to \pi^0\gamma\gamma$. *The dotted curve is the* $O(p^4)$ *contribution, the dashed and full curves correspond to the* $O(p^6)$ *calculation [31] without and with the appropriate vector meson exchange contribution to reproduce the measured rate, respectively. The spectra are normalized to the* 50 *unambiguous events of NA31 (cf. Fig. 3).*

There are also transitions with non-zero $O(p^4)$ amplitudes which are completely calculable in terms of the leading-order couplings G_8, G_{27} appearing in loop amplitudes, among them $K_S \to \gamma\gamma$, $K^0 \to \pi^0\gamma\gamma$ and $K^0 \to \pi^0\pi^0\gamma\gamma$.

Let me briefly review the status of the two decays that have already been measured. For $K_L \to \pi^0\gamma\gamma$, the experimental branching ratios [27, 28]

$$BR(K_L \to \pi^0\gamma\gamma) = \begin{cases} (1.7 \pm 0.2 \pm 0.2) \cdot 10^{-6} & \text{NA31} \\ (1.86 \pm 0.60 \pm 0.60) \cdot 10^{-6} & \text{E731} \end{cases} \tag{3}$$

are substantially bigger than the chiral prediction [29] $BR \simeq 0.7 \cdot 10^{-6}$. Higher–order corrections have been estimated by several groups [30, 31, 32]. The overall conclusion is that the enhancement of the rate can be understood but not really predicted by CHPT because of the uncertainties appearing at $O(p^6)$ and higher. However, and this is really the main message, the following two statements are then parameter–free predictions of CHPT:

- The two–photon mass spectrum can be predicted unambiguously once the rate is fixed [31] and it is in perfect agreement with the NA31 spectrum [27] as shown in Figs. 2 and 3.
- When the same kind of analysis is applied to $K_S \to \gamma\gamma$, there are essentially no corrections [31, 32] of the type occurring in $K_L \to \pi^0\gamma\gamma$. Therefore, the CHPT prediction

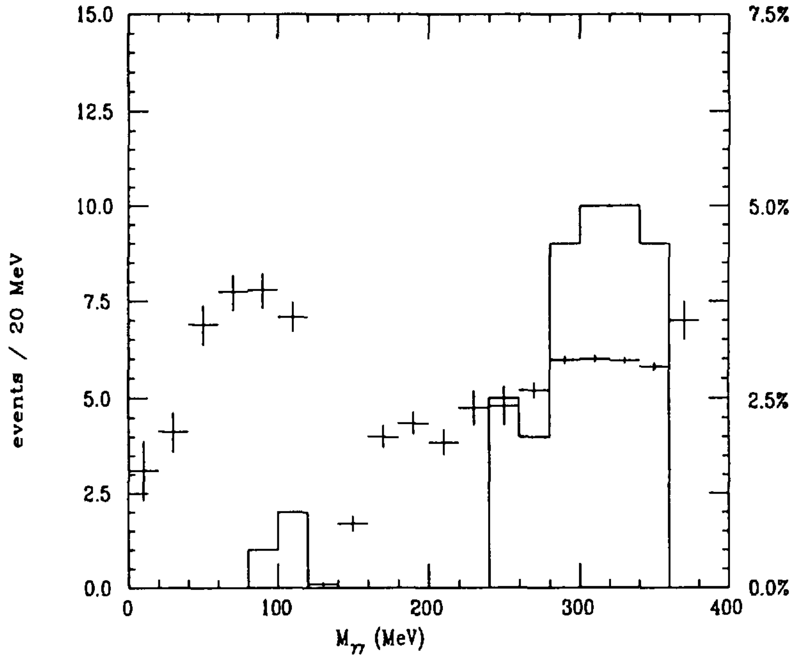

Figure 3: 2γ *invariant–mass distribution for unambiguous* $K_L \to \pi^0\gamma\gamma$ *candidates from NA31 [27] (solid histogram).*

[33] of $O(p^4)$ for the branching ratio

$$BR(K_S \to \gamma\gamma) = 2.0 \cdot 10^{-6} \tag{4}$$

remains practically unchanged. The present agreement with the experimental value [34]

$$BR(K_S \to \gamma\gamma) = (2.4 \pm 0.9) \cdot 10^{-6} \tag{5}$$

should therefore pass a more stringent test with better statistics. The KLOE experiment at DAΦNE will certainly have enough statistics for this purpose and is expected to improve the accuracy substantially [35].

3.3 Transition amplitudes with new couplings at $O(p^4)$

By far the biggest group of nonleptonic kaon decays is characterized by amplitudes where in addition to the already known constants L_i of Table 1 some of the 22 (octet) couplings N_i of the nonleptonic weak Lagrangian of $O(p^4)$ appear. In Table 6, a fairly complete list of such transitions and their dependence on the nonleptonic low–energy constants is given. Without explaining the seemingly obscure numbering of the N_i, it is easy to see that only 9 of those constants enter the various amplitudes: $N_{14},\ldots,N_{18}$ in so–called electric amplitudes (without an ε tensor) and $N_{28},\ldots,N_{31}$ in magnetic amplitudes (with an ε tensor). The four magnetic constants are sensitive to the chiral anomaly [36, 37].

This is not the place for a comprehensive discussion [38] of the transitions listed in Table 6. Before discussing a few examples, let me state the main conclusions:

Table 6: *Decay modes to which the coupling constants N_i contribute. For the 3π final states, only the single photon channels are listed. For the neutral modes, the letters L or S in brackets distinguish between K_L and K_S initial states in the limit of CP conservation. γ^* denotes a lepton pair in the final state. If a decay mode appears more than once there are different Lorentz structures in the amplitude.*

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*$ (S)	$\pi^0\pi^0\gamma^*$ (L)		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma$ (S)		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma$ (S)	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma$ (L)	"
		$\pi^+\pi^-\pi^0\gamma$ (S)	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$
	$\pi^+\pi^-\gamma^*$ (L)		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*$ (S)		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma$ (L)	$\pi^+\pi^-\pi^0\gamma$ (S)	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma$ (S)	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma$ (L)	$6N_{28} + 3N_{29} - 5N_{30}$

- All electric couplings $N_{14},\ldots,N_{18}$ can in principle be determined phenomenologically.
- In contrast, only three combinations of the magnetic constants $N_{28}, \ldots, N_{31}$ appear in measurable decay amplitudes. Fortunately, the theoretical expectations for these constants [36] are better founded than for the electric counterparts.
- A great number of interesting relations contained in Table 6 remain to be tested. To the considered order in the low-energy expansion, these relations are unambiguous predictions of the Standard Model (low-energy theorems).

3.3.1 $K^+ \to \pi^+ l^+ l^-$

The decay amplitude for this process ($l = e$ or μ) depends, in addition to explicitly known contributions, on the difference $N_{14} - N_{15}$ (essentially the constant w_+ used previously [39]). Extracting this constant from the rate $\Gamma(K^+ \to \pi^+ e^+ e^-)$, a two-fold ambiguity remains that was resolved by the measurement [40] of the spectrum in the invariant mass of the lepton pair. Once this constant is determined, both rate and spectrum for the decay in the muon channel

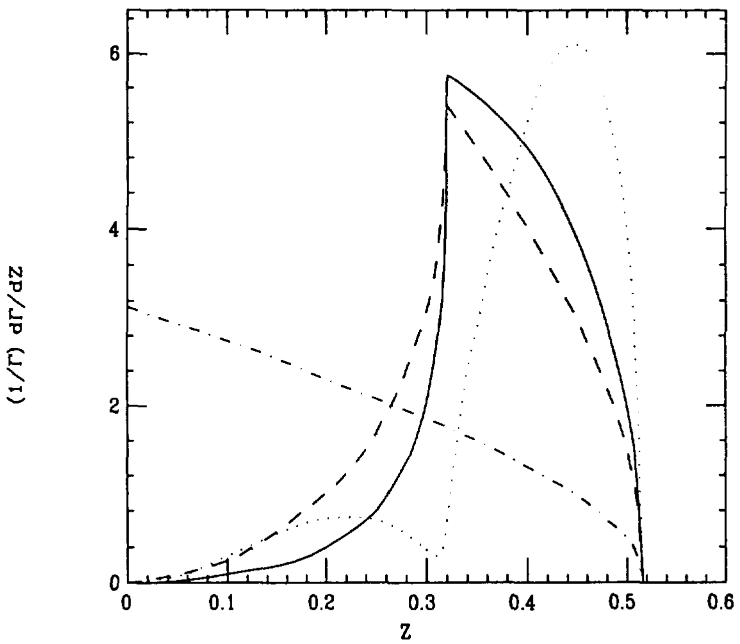

Figure 4: *Normalized distribution for the two-photon invariant mass squared ($z = M^2_{\gamma\gamma}/M^2_K$) in $K^+ \to \pi^+\gamma\gamma$ from CHPT [42] for several values of $\hat{c}$: $\hat{c} = 0$ (full curve), $\hat{c} = 4$ (dashed curve) and $\hat{c} = -4$ (dotted curve). The dash–dotted curve is the phase space distribution.*

are completely specified. The preliminary value $BR(K^+ \to \pi^+\mu^+\mu^-)$=$(5.0 \pm 0.4 \pm 0.6) \cdot 10^{-8}$ reported by the BNL-787 Collaboration at the Orsay Workshop [41] is in excellent agreement with the theoretical prediction $BR(K^+ \to \pi^+\mu^+\mu^-)$=$(6.2^{+0.8}_{-0.6}) \cdot 10^{-8}$. The second prediction [39] remains to be tested: unlike for the electron channel, the invariant-mass distribution of the muon pairs should be indistinguishable from phase space.

3.3.2 $K^+ \to \pi^+\gamma\gamma$

Although this decay shares many features with $K_L \to \pi^0\gamma\gamma$, Table 6 shows that it depends on an unknown combination of low-energy constants that is moreover different from the previous case of $K^+ \to \pi^+l^+l^-$. The combination $N_{14} - N_{15} - 2N_{18}$ is related to the constant $\hat{c}$ introduced originally [42]. For a reasonable range of values of this constant, the spectrum in the two-photon invariant mass squared shown in Fig. 4 has a very characteristic shape [42] similar to $K_L \to \pi^0\gamma\gamma$. Preliminary results from the BNL-E787 Collaboration [41, 43] are consistent with this prediction. Of course, the rate is correlated with the spectrum depending on the same constant $\hat{c}$. A recent estimate of higher-order corrections [44] along similar lines as for $K_L \to \pi^0\gamma\gamma$ suggests an increase of the rate by some 30 to 40 % over the value of $O(p^4)$.

3.3.3 $K \to 3\pi\gamma$

There are four different modes in this channel only two of which (for the charged kaon) have so far been observed experimentally. The full calculation to $O(p^4)$ has just been completed

[45]. To take full advantage of the available information on the nonradiative transitions, it is useful to generalize [46] the concept of Bremsstrahlung. This is certainly the case for $K \to 3\pi\gamma$, but it will also be very useful in other reactions with four particles plus a photon. In the present case, it turns out [45] that generalized Bremsstrahlung is an extremely good approximation[2] to the amplitude of $O(p^4)$. The effect of other contributions such as the coupling constants given in Table 6 are completely hidden in the present experimental errors for the nonradiative amplitudes. Much more precision would be needed to be sensitive to those other contributions. As always, this can also be phrased in a different, more positive way: the rates and spectra of such processes are precisely predicted by the Standard Model in terms of the $K \to 3\pi$ parameters.

4 Outlook

The main success of CHPT in the field of kaon physics has been the unified treatment of all decay channels within the same framework and the direct connection to the underlying Standard Model. For semileptonic decays, the theory is in excellent shape. As the data improve, some of the low-energy constants L_i will become even better known, but already now we have a very predictive scheme where to $O(p^4)$ all parameters are known with reasonable accuracy.

We are still far from this state of affairs in the nonleptonic sector. From the theoretical point of view, we need a better understanding not only of the values of the low-energy constants N_i, but also of their origin (as is the case [47] for the L_i). There are several attempts in this direction: $1/N_c$ expansion, lattice gauge theories, sum rules, chirally inspired models, ... Even with the limited knowledge we have of those constants, CHPT has been quite successful in the comparison with experiment also in the nonleptonic sector.

However, most importantly of all, we need more data to test the existing predictions of CHPT for K decays. The experimental program is well under way as we heard again during this meeting. Allow me to close this talk with another plea to our experimental colleagues: when you are out there searching for the few "gold-plated" events (exotic channels, CP violation, ...), please do not neglect the many "standard" events. Many interesting tests of the Standard Model in K decays are still ahead of us.

Acknowledgements

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[2] Less so for $K_S \to \pi^+\pi^-\pi^0\gamma$ where the nonradiative amplitude is suppressed at lowest order.

References

[1] G. Buchalla, these Proceedings.

[2] S. Güsken, these Proceedings.

[3] S. Weinberg, Physica **96A**, 327 (1979).

[4] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).

[5] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[6] H. Leutwyler, Ann. Phys. (N.Y.) **235**, 165 (1994).

[7] G. Ecker, Progr. Part. Nucl. Phys. **35**, 1 (1995).

[8] A. Pich, Rep. Prog. Phys. **58**, 563 (1995).

[9] E. de Rafael, in *CP Violation and the Limits of the Standard Model*, J.F. Donoghue, Ed., (World Scientific, Singapore, 1995).

[10] V. Bernard, N. Kaiser and U.-G. Meißner, Int. J. Mod. Phys. **E4**, 193 (1995).

[11] *The Second DAΦNE Physics Handbook*, Eds. L. Maiani, G. Pancheri and N. Paver, (INFN, Frascati, 1995).

[12] J. Bijnens, Chiral perturbation theory for DAΦNE physics, NORDITA preprint 96/45 N,P, hep-ph/9607304;
A. Buras, Theoretical review of K–physics, Munich preprint TUM-HEP-255/96, hep-ph/9609324;
A. Pich, Rare kaon decays, Valencia preprint FTUV/96-65, hep-ph/9610243.

[13] J. Bijnens, G. Ecker and J. Gasser, Nucl. Phys. **B396**, 81 (1993).

[14] A.M. Diamant-Berger et al., Phys. Lett. **62B**, 485 (1976).

[15] M.S. Atiya et al., Phys. Rev. Lett. **63**, 2177 (1989).

[16] J. Bijnens, G. Colangelo, G. Ecker and J. Gasser, Semileptonic kaon decays, in Ref. [11].

[17] F. Leber et al. (NA31), Phys. Lett. **B369**, 69 (1996).

[18] J. Bijnens, Nucl. Phys. **B337**, 635 (1990);
C. Riggenbach, J. Gasser, J.F. Donoghue and B.R. Holstein, Phys. Rev. **D43**, 127 (1991).

[19] J. Bijnens, G. Colangelo and J. Gasser, Nucl. Phys. **B427**, 427 (1994).

[20] L. Rosselet et al., Phys. Rev. **D15**, 574 (1977).

[21] J. Bijnens, G. Colangelo, G. Ecker, J. Gasser and M.E. Sainio, Phys. Lett. **B374**, 210 (1996).

[22] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. **B394**,101 (1993).

[23] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. **B261**, 496 (1991).

[24] J. Kambor, J.F. Donoghue, B.R. Holstein, J. Missimer and D. Wyler, Phys. Rev. Lett. **68**, 1818 (1992).

[25] C.Q. Geng, I.J. Hsu and Y.C. Lin, Phys. Rev. **D50**, 5744 (1994);
L.S. Littenberg and G. Valencia, $K_L \to \pi^+\pi^-\nu\bar{\nu}$ within the Standard Model, Brookhaven preprint BNL-62549, hep-ph/9512413.

[26] G.D. Barr et al. (NA31), Phys. Lett. **B328**, 528 (1994).

[27] G.D. Barr et al. (NA31), Phys. Lett. **B284**, 440 (1992).

[28] V. Papadimitriou et al. (E731), Phys. Rev. **D44**, 573 (1991).

[29] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. **B189**, 363 (1987);
L. Cappiello and G. D'Ambrosio, Nuovo Cimento **99A**, 155 (1988).

[30] L. Cappiello, G. D'Ambrosio and M. Miragliuolo, Phys. Lett. **B298**, 423 (1993).

[31] A.G. Cohen, G. Ecker and A. Pich, Phys. Lett. **B304**, 347 (1993).

[32] J. Kambor and B.R. Holstein, Phys. Rev. **D49**, 2346 (1994).

[33] G. D'Ambrosio and D. Espriu, Phys. Lett. **B175**, 237 (1986);
J.L. Goity, Z. Phys. **C34**, 341 (1987).

[34] G.D. Barr et al. (NA31), Phys. Lett. **B351**, 579 (1995).

[35] P. Franzini, private communication.

[36] J. Bijnens, G. Ecker and A. Pich, Phys. Lett. **B286**, 341 (1992).

[37] G. Ecker, H. Neufeld and A. Pich, Nucl. Phys. **B413**, 321 (1994).

[38] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, Radiative nonleptonic kaon decays, in Ref. [11].

[39] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B291**, 692 (1987).

[40] C. Alliegro et al. (BNL-E777), Phys. Rev. Lett. **68**, 278 (1992).

[41] T. Shinkawa, Contribution to the Workshop on K Physics, Orsay, France, May 30 – June 4, 1996.

[42] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303**, 665 (1988).

[43] T. Nakano, First observation of the decay $K^+ \to \pi^+\gamma\gamma$, Proc. Moriond 1996.

[44] G. D'Ambrosio and J. Portolés, Unitarity and vector meson contributions to $K^+ \to \pi^+\gamma\gamma$, Naples preprint INFNNA-IV-96/12, hep-ph/9606213.

[45] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, in preparation.

[46] G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, Phys. Lett. **B380**, 165 (1996).

[47] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321**, 311 (1989).