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# POLE MASSES OF QUARKS IN DIMENSIONAL REDUCTION

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Regularization by dimensional reduction, a modification of the conventional dimensional regularization, was proposed [1] as a candidate for an invariant regularization in supersymmetric theories. The idea is very simple. Supersymmetry is only possible in certain integer dimensions (four, to be specific). Therefore, the vector and spinor algebra in the numerator of Feynman diagrams should be retained four-dimensional. On the other hand, regularization of momentum integrals is achieved in $N = 4 - 2\varepsilon$ dimensions. The possibility of canceling the squared momenta in the numerator and the denominator, crucial for maintaining the gauge invariance, requires then that the momenta form an $N$-dimensional subspace in four dimensions.

Such a dimensional reduction from a finite-dimensional space into the formal space of the regularization proves to be mathematically inconsistent [2]. The regularized space should be recognized as infinite-dimensional because antisymmetrization over an arbitrary number of indices does not give the identical zero. A consistent modification [3] disables any strictly four-dimensional objects, like the totally antisymmetric $\epsilon$ tensor, thus distorting the algebra above a certain order. On the other hand, the original version, in spite of being globally inconsistent, can provide unambiguous results until antisymmetrization over five indices actually comes into play. Besides, it is very convenient technically, to have the usual four-dimensional algebra of spinor matrices (including $\gamma_5$) and even their Fierz rearrangement [4]. This is why the regularization by dimensional reduction was very soon applied to nonsupersymmetric calculations [5] in the Standard Model. It was also used, to verify the consistency of the $\gamma_5$ prescription in a three-loop QCD-electroweak calculation [6].

In the present paper we demonstrate the two-loop calculation of the pole masses of quarks in quantum chromodynamics in the framework of the regularization by dimensional reduction. The corresponding calculation in the conventional dimensional regularization was done in refs. [7, 8]. The leading approximation of the present calculation in dimensional reduction (that ignored contributions of heavier quarks and masses of lighter quarks) was used in ref. [6], to verify the $\gamma_5$ prescription.

Here we evaluate the contributions of the heavier and lighter quarks by means of the asymptotic expansion, which is of a special methodical interest. While the structure of the large-mass expansion caused no particular doubts, being defined by the universal Euclidean rules [9], the expansion in a small mass is more complicated. Its definition needs an on-shell infrared extension which we construct explicitly for the present case.

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Let us discuss the dimensional reduction in some more detail. As the momenta become $N$-dimensional, 4-vectors naturally split into true $N$-vectors and so-called $\varepsilon$ scalars which fall into a complementary subspace of dimension $4 - N = 2\varepsilon$, orthogonal to the momenta. The $\varepsilon$ scalars are nothing else but matter fields. Their presence is the only difference from the conventional dimensional regularization. Order by order in perturbation theory, contributions of $\varepsilon$ scalars are equivalent to finite counterterms, that is, to a change of the renormalization scheme. When performing renormalizations, one should remember an important fact. Formal 4-covariance (valid at the stage of generating the 'bare' diagrams) may be broken by counterterms. Renormalizations of $\varepsilon$ scalars and of their interactions (including the pole contributions in $\varepsilon$ ) are not identical to those of the vectors.

Moreover, quantum corrections may also generate a mass for $\varepsilon$ scalars. There is an arbitrariness in choosing the renormalization scheme for this mass [10]. A consistent way is to choose the finite $\varepsilon$-mass counterterm so that the pole (and renormalized) mass of the $\varepsilon$ scalars be zero. The $\varepsilon$-scalar field renormalization is left minimal. Then no additional dimensional parameter is ever involved in renormalizations while the polynomial renormalization of other masses, independence of renormalizations of dimensionless coupling constants on masses, and gauge independence of all $\beta$ functions are retained. We follow that scheme in the present calculation. However, this may not be the only way. For example, in a softly broken supersymmetric Yang–Mills theory it would be natural trying to restore the maximum symmetry by relating the mass of $\varepsilon$ scalars to the gluino mass as a special solution to the renormalization-group equations in the minimal subtraction scheme.

One should also bear in mind technical complications that are inevitable when performing a massive calculation up to a finite part with an additional mass on previously massless lines.

Other renormalizations are done minimally, that is by subtracting only poles in $\varepsilon$. We use a modified $\overline{\text{MS}}$ definition [8], dividing each loop by $(4\pi)^{\varepsilon}\Gamma(1 + \varepsilon)$ rather than multiplying by $\Gamma(1 - \varepsilon)$. This definition is more convenient in calculations with masses because a simple massive loop is then a pure rational function of $N$ without any $\Gamma$ functions. It would be proper to call the scheme MMS (massive minimal subtractions).

The pole mass is a physical quantity [7]. Thus, it is renormalization- and gauge-invariant. We verify the gauge invariance by performing our calculations in the arbitrary covariant gauge.

The pole mass of a quark $m_P$ is defined as a formal solution for $\hat{p}$ (in the Minkowski metric) at which the reciprocal of the connected full propagator equals zero:

$$\hat{p} - m - \Sigma(\hat{p}, m) = 0, \tag{1}$$

where $\Sigma(\hat{p}, m) = \hat{p}\, A(p^2, m) + m\, B(p^2, m)$ is the one-particle-irreducible two-point function (including the $i$ factor for one of its legs); $m$ may stand for the bare or renormalized mass, $m_B$ or $m_R$, depending on the prescription used in evaluating $\Sigma$. The solution to eq. (1) is sought order by order in perturbation theory. To two loops

$$m_P = m + \Sigma_1(m, m) + \Sigma_2(m, m) + \Sigma_1(m, m)\, \Sigma_1'(m, m) + \mathcal{O}(\Sigma_3), \tag{2}$$

where $\Sigma_L$ is the $L$-loop contribution to $\Sigma$, and the prime denotes the derivative with respect to the first argument.

According to eq. (2), technically, we need to evaluate two-loop propagator-type diagrams on shell. For the one-loop diagrams, also the derivative with respect to the momentum is necessary. It can be expressed through the derivative with respect to the mass because the total dimension of the diagrams is known. Most of these calculations – those that involve just one mass and massless contributions – can be performed with the aid of the SHELL2 package [11]. We use a modified implementation of the package, advanced in the scope and efficiency. The original algorithm [11] for reducing scalar numerators is only applied to the first powers of the numerators. In other cases we use the strategy described in ref. [12] of employing a recurrence relation for a neighbor triangle.

In individual diagrams there may be on-shell infrared divergences. However, the complete pole mass is infrared-finite [7]. It only involves ultraviolet poles in $\varepsilon$, which are removed after introducing ultraviolet counterterms. They can be evaluated in some other infrared-secure off-shell configuration of momenta and masses. The renormalization of the Yukawa charge for $\varepsilon$ scalars and their (nonminimal) mass counterterm are calculated separately. The point is that in a nonsupersymmetric theory, like QCD, the (ultraviolet divergent) renormalization constants for the Green functions of $\varepsilon$ scalars are quite different from those of the vectors (the later being restricted by gauge invariance). Renormalizing a physical quantity, as the pole mass, we can accumulate the difference in the bare charge

counterterms alone, ignoring the field renormalizations, if we evaluate the contributions of counterterms for the whole sum of the diagrams.
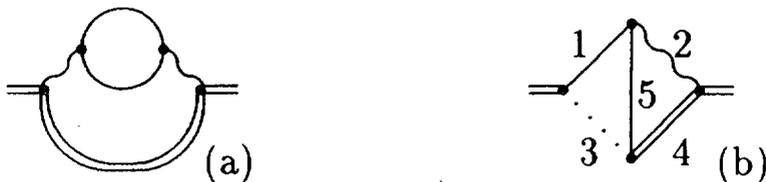


Figure 1: The diagram with a quark loop (a) and its scalar prototype (b).

Let us concentrate on the diagram with a quark loop (fig. 1(a)). When the mass of the quark in the loop $m_1$ is different from the mass of the external quark $m$, this diagram cannot be calculated by the SHELL2 algorithms. In the conventional dimensional regularization, the result involves complicated transcendental functions [8]. We are going to demonstrate how to evaluate the diagram by means of expanding it in the ratio of the masses. If the mass in the loop is large as compared to the mass of the propagating quark, $m_1 \gg m$, then the structure of the asymptotic expansion [9] established in case of purely Euclidean expansions does not require any modifications. The rules are as follows. The expansion is a sum over 'ultraviolet' subgraphs of the diagram. An ultraviolet subgraph must contain all lines with large masses, the points were large external momenta (if any) flow in/or out. The large momenta ought to go only through the ultraviolet subgraph and obey the momentum conservation law. And last, an ultraviolet subgraph should be one-particle irreducible with respect to lines with small and zero masses also may consist of several disconnected parts. An ultraviolet subgraph is Taylor-expanded in its small parameters (external momenta and internal masses) and then shrunk to a point and inserted in the numerator of the remaining Feynman integral.

The large-mass expansion of the diagram fig. 1(a) involves only two ultraviolet subgraphs: the whole graph and the loop of two lines with the heavy mass. As result of the expansion two-loop bubble integrals and propagator-type integrals on shell arise.

The case $m_1 \ll m$ is more difficult. Naively applying the standard expansion, we would encounter actual infinity for the subgraph that includes only the internal 'on-shell' line with the mass $m$. Thus, the expansion of this ultraviolet subgraph needs some extended definition.

It is convenient to perform further considerations in terms of the scalar prototype of the diagram (fig. 1(b)) in Euclidean space (after the Wick

rotation). The prototype involves arbitrary integer powers of the scalar denominators $c_L = k_L^2 + m_L^2$ on the lines. Their powers $j_L$ are called indices of the lines. The mass-shell condition for the external momentum now is $p^2 = -m^2$. Any scalar products of the momenta in the numerator are reduced to changed powers of the scalar invariants in the denominator. Thus, the indices may sometimes become negative. In particular, an auxiliary line. of the prototype, line # 3, is generated with an always non-positive index $j_3 \leq 0$. From the viewpoint of the $R$ operation or asymptotic expansion, line 3 represents just a vertex (local in co-ordinate space). It is convenient to choose $m_3^2 = m^2 + m_1^2$, so that $pk_1 = \frac{1}{2}(c_1 - c_3)$. Other masses are $m_2 = 0$, $m_4 = m$, $m_5 = m_1$.

The index of the auxiliary line $j_3$ can always be reduced to zero by means of recurrence relations. The recurrence relations are derived by the Chetyrkin–Tkachov method of integration .by parts [13]. We use a shorthand notation $\{XYZ\}$ of ref. [12] to denote the relation for the triangle formed of lines # $X$, $Y$, and $Z$ (the latter is optional, a degenerate case being allowed):

$$\int \frac{d^N k_X}{c_X^{j_X} c_Y^{j_Y} c_Z^{j_Z}} \left( N - 2j_X - j_Y - j_Z + j_X \frac{2m_X^2}{c_X} + j_Y \frac{m_X^2 + m_Y^2 - m_{XY}^2 + c_{XY} - c_X}{c_Y} \right.$$

$$\left. + j_Z \frac{m_X^2 + m_Z^2 - m_{XZ}^2 + c_{XZ} - c_X}{c_Z} \right) = 0, \qquad \{XYZ\}$$

where a double index $XY$ refers to the line that starts at the point where lines $X$ and $Y$ meet. For an external line on the mass shell, the value of $c_L$ is equal to zero. Expressing some term of eq. $\{XYZ\}$ through other terms, we obtain a relation between Feynman integrals of the same prototype but with varied indices of the lines.

In order to eliminate a numerator $j_3 < 0$, we apply the following relations. If $j_5 \neq 1$, we solve equation eq. $\{425\}$ with respect to the $c_3/c_5$ term. Other wise, if $j_4 \neq 1$, the $c_3/c_4$ term of eq. $\{524\}$ is used. If $j_1 \neq 1$, the $c_3$ numerator is reduced using eq. $\{315\}$ + $\{425\}$. In case $j_1 = j_4 = j_5 = 1$ we apply eq. $\{135\}$ + $\{245\}$, solved with respect to the free term, to create a denominator for which the above relations are applicable.

At $j_3 = -1$, one can derive a simpler formula which is more efficient than the above algorithm. First of all, $c_3 = c_1 - 2pk_1$. On integration over $k_1$ with just lines 1 and 5 involved, by Lorentz covariance a single component of $k_{1\mu}$ turns into $k_{2\mu}$ with the coefficient $k_1 k_2 / k_2^2$. Hence, $pk_1$ is equivalent

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to $(pk_2)(k_1k_2)/k_2^2 = \frac{1}{4}(c_2 - c_4)(c_1 + c_2 - c_5)/c_2$. Thus, the first power of the numerator on line 3 can be replace as $c_3 = \frac{1}{2}[c_1 + c_4 + c_5 + (c_1 - c_5)c_4/c_2 - c_2]$.

We assume that the general $R$-operation-like structure of asymptotic expansion is retained in the case of the on-shell expansion in a small mass. The set of ultraviolet subgraphs of our diagram (after eliminating line 3) is: $\{1, 2, 4, 5\}$, $\{1, 2, 4\}$, $\{2, 4, 5\}$, $\{4\}$. The first three subgraphs do not cause any technical complications in their evaluation. The whole graph becomes a two-loop on-shell propagator-type integral which is a particular case of a SHELL2 prototype with two massive lines, one of the latter being contracted. The two succeeding subgraphs both yield a product of a one-loop bubble integral by an on-shell propagator type integral. The last subgraph $\{4\}$ is infrared-singular, its formal Taylor-expansion being infinite on mass shell. Let us try to derive an extended definition for the asymptotic expansion of this subgraph. We need to expand the propagator $1/c_4 = 1/(-2pk_2 + k_2^2)$ in the small momentum $k_2$. It is reasonable to make use of the expansion

$$1/(-2pk_2 + k_2^2) = \sum_{n=0}^{\infty}(k_2^2)^n/(-2pk_2 + 0)^{n+1}, \tag{3}$$

where $+0$ keeps the correct causal (Euclidean) rule of going around the singularity at zero which appears as we perform the on-shell expansion.

The even powers of the scalar product in eq. (3), for which $+0$ is inessential in the context of dimensional regularization, can be worked up by the formula

$$\int d^N k_2 \, (pk_2)^{2n} f(k_2^2) = \frac{\Gamma(N/2)\Gamma(1/2 + n)}{\Gamma(N/2 + n)\Gamma(1/2)} \int d^N k_2 \, (p^2 k_2^2)^n f(k_2^2), \tag{4}$$

where $f$ is an arbitrary function which depends on $k_2$ through $k_2^2$ only. The formula can be proved by induction based on the Lorenz structure of the result, for positive integer values of $n$ and then analytically continued to arbitrary complex values. It is regular for negative integer $n$. We assume that $j_3$ has been brought to zero by recurrence relations before the expansion. Then, subgraph $\{1, 5\}$ after integration over $k_1$ depends only on $k_2^2$. This fact permits us to apply eq. (4) to even negative powers of $pk_2$ in eq. (3). The integral can then be explicitly evaluated and transformed to the form

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$$\int \frac{d^N k_1\, d^N k_2}{\pi^N \Gamma^2(3 - N/2) c_1^{j_1} c_2^{j_2} c_5^{j_5} (pk_2)^{2j}} =$$
$$\frac{(m_1^2)^{N-j_1-j_2-j_5-j}\Gamma(1/2)\Gamma(N/2 - j_2 - j)}{(-p^2)^j \Gamma(j_1)\Gamma(j_5)\Gamma(1/2 + j)\Gamma(N/2 - j)} \times$$
$$\frac{\Gamma(j_1 + j_2 + j - N/2)\Gamma(j_5 + j_2 + j - N/2)\Gamma(j_1 + j_2 + j_5 + j - N)}{\Gamma(j_1 + j_5 + 2j_2 + 2j - N)} . \tag{5}$$

Odd negative powers reveal on-shell infrared singularities of the propagator, which ought to generate some counterterms as a part of the asymptotic expansion. They become manifest as we remember that

$$1/(pk_2 + 0)^{2n+1} = 1/(pk_2)^{2n+1} + \frac{\pi}{(2n)!}\, \delta^{(2n)}(pk_2) \tag{6}$$

for purely imaginary $p$ as we have it. The first term on the right-hand side of eq. (6), which generates the integral in the sense of the principal value, yields just zero with a function of $k_2^2$. The second term picks out exactly the $2n$'th coefficient in the Taylor expansion of the rest of the diagram in $pk_2$, that is the component of $k_2$ parallel to $p$. This is the kind of infrared counterterms, only partially local in momentum space, which are characteristic of the non-Euclidean asymptotic expansion. The delta function itself takes off one integration over the corresponding component of $k_2$, leaving the integral in dimension $N - 1$.

The seemingly independent treatment of the odd powers of the scalar product in the denominator by eq. (6) is in fact firmly bound to formula (4) for the even powers in the general framework of dimensional regularization. The point is that the latter always implies an auxiliary intermediate analytic regularization in the line indices [3]. One seeks a correlated region of all the parameters, the powers and the space-time dimension, in which the integral under consideration is well defined and analytic in the parameters. Then the resulting function is analytically continued to the whole complex plane, and eventually, the limit is taken in the indices to the values that we started with. As a rule, the regular limit does exist while the space-time dimension is kept noninteger. According to this schedule, we have evaluated the integral on the right-hand side of eq. (4) for the relevant diagram, using

the power of the scalar product as an additional regulator rather than a natural number and taking the limit $n \to -j$, eq. (5). Having identically transformed the result, we can try to take the limit for half-integer values of $j$ as well. The limit does always exist and exactly coincides with the value that we get from $\int d^{N-1}k_2$ according to the eq. (6). It is interesting to note that the odd coefficients of the expansion turn out to be ultraviolet-finite, that is allow a smooth $N \to 4$ limit. Moreover, the limiting odd series happens to terminate because of a $\Gamma$ function in the denominator.
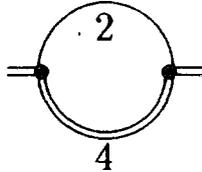


Figure 2: The one-loop diagram where the asymptotic on-shell expansion in $m_2 \ll m_4$, $p^2 = -m_4^2$, is performed.

The use of eq. (6) gives us an alternative way of achieving the same result, without specifying the weight function and avoiding the necessity to evaluate the indeterminacy that arises at half-integer negative $n$ in eq.(4) because of a singular coefficient. We have verified both procedures in yet another case when the integration in eq. (4) can be performed analytically in terms of $\Gamma$ functions. This is the one-loop diagram with one light mass and a line on mass shell (fig. 2) [4]. For odd expansion coefficients we obtain the same results either by analytically continuing eq.(4) or by reducing the dimension of integration to $N - 1$ via eq. (6). The complete expansion, which includes the contributions of subgraphs {4} and {2, 4}, perfectly reproduces the exact integral in the Feynman parameter

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[4]On preparing the present paper, we became aware that a general method of constructing typically Minkowskian and on-shell asymptotic expansions was proposed by V. A. Smirnov [14] and demonstrated by one-loop examples (including (fig. 2 ) ) and by a non-trivial two-loop master-integral [15]

$$(\mu^2)^\epsilon \int \frac{d^N k}{\pi^{N/2} \Gamma(3 - N/2)(k^2 + m_2^2)(-2pk + k^2)}$$

$$= \frac{1}{\varepsilon} + 2 - \ln\left(\frac{-p^2}{\mu^2}\right) - r^2 \ln r - r^2 \sqrt{4/r^2 - 1} \arctan\left(\sqrt{4/r^2 - 1}\right) + O(\varepsilon)$$

$$= \frac{1}{\varepsilon} + 2 - \ln\left(\frac{-p^2}{\mu^2}\right) - \pi\left(r + \frac{r^3}{8} + \frac{r^5}{128}\right) - r^2(\ln r - 1) - \frac{r^4}{12} + \dots, \quad (7)$$

where $r = m_2^2/(-p^2)$. It is worth pointing out the fact that analytic continuation recovers just the causal rule of getting around the singularity, as in eq. (6), which is automatically implied in the Euclidean space where $k_2^2 \geq 0$ after the Wick rotation.

Relations (3)–(6) are sufficient with a trivial generalization of eq. (3) by binomial coefficients as index $j_4$ is arbitrary for constructing the asymptotic expansion terms for the ultraviolet subgraph $\{4\}$ of the prototype fig. 1(b). The procedure does not require any modifications for a higher-loop analog of fig. 1(a) with an arbitrary insertion in line-2.

The expansion of ultraviolet subgraph $\{4\}$ suffers from on-shell infrared singularities. Therefore, the expansion is not perfectly local in coordinate space (or polynomial in momentum-space). Along with regular Taylor-like terms, $(k_2^2)^n$, partially local on-shell infrared counterterms $\delta^{(2n)}(pk_2)$ are produced in odd coefficients. At $j_4 > 1$, also nonlocal negative powers of $k_2^2$ appear in even coefficients. However, the general structure of the expansion as a sum over ultraviolet subgraphs remains valid.

In case of the conventional dimensional regularization the sum over all the ultraviolet subgraphs of the diagram (fig. 1) exactly reproduces the expansion presented in ref. [8].

The calculations in dimensional reduction do not generate any new prototypes of Feynman diagrams. Just the coefficients before the same scalar integrals are slightly changed for the bare diagrams. If we use $N$-dimensional indices, the full contribution of the vectors and $\varepsilon$ scalars to a diagram without external gluon lines is obtained by replacing $N$ in the numerator of a scalarized expression with 4. The contribution that involves $\varepsilon$ scalars can be extracted by replacing $N^n$ with $4^n - (4 - 2\varepsilon)^n$.

If we need to separate the proper-vector and $\varepsilon$-scalar propagators we should proceed as follows. The gluon propagator diagram is scalarized by contracting it with the metric tensor either in the formal four-dimensional space that contains the $(4 - 2\varepsilon)$-dimensional vector and $\varepsilon$-scalar subspaces,

or just in $4 - 2\varepsilon$ dimensions. According to the Ward–Slavnov–Taylor identities, the vector part (for the sum of all diagrams in the relevant order of perturbation theory) ought to be transverse with respect to the external momentum, while the $\varepsilon$-scalar part is always diagonal as there are no momenta in this subspace. Again, we perform the algebra using the formal N-dimensional indices. As the external indices are yet free, we should set $N = 4$, in order to sum up the vector and $\varepsilon$-scalar contributions in the internal lines of the diagram. Then the external indices are contracted, which produces $N^n$ with $n = 0, 1$. The $\varepsilon$-scalar propagator is then obtained by replacing $N^n$ with $[4^n - (4 - 2\varepsilon)^n]/(2\varepsilon)$. The scalar coefficient before the transverse projection operator in the vector propagator can be evaluated by replacing $N^n$ with $(4 - 2\varepsilon)^n/(3 - 2\varepsilon)$. The momenta are always assumed to belong to $4 - 2\varepsilon$ dimensions.

Here are our results in the framework of the dimensional reduction MMS scheme with the zero pole mass for $\varepsilon$ scalars. We have evaluated the one-loop vertex and field renormalization constants:

$$Z_{ffv} = 1 - \frac{h}{\varepsilon}\left[\left(\frac{\xi - 1}{4} + 1\right)C_a + \xi C_f\right] + \mathcal{O}(h^2), \tag{8}$$

$$Z_{ffs} = 1 - \frac{h}{\varepsilon}\left[\frac{\xi - 1}{2}C_a + (\xi + 1)C_f\right] + \mathcal{O}(h^2), \tag{9}$$

$$Z_v = 1 - \frac{h}{\varepsilon}\left[\left(\frac{5}{3} - \frac{\xi - 1}{2}\right)C_a - \frac{2}{3}N_f\right] + \mathcal{O}(h^2), \tag{10}$$

$$Z_s = 1 - \frac{h}{\varepsilon}\left[(3 - \xi)C_a - N_f\right] + \mathcal{O}(h^2). \tag{11}$$

Indices $f$, $v$, $s$ denote the type of the particles, fermions, vectors, and $\varepsilon$ scalars. The expansion parameter is $h = g^2/(16\pi^2) = \alpha_s/(4\pi)$; $\xi$ is the parameter of the covariant gauge, $\xi = 1$ corresponds to the Feynman gauge, $\xi = 0$ to the Landau gauge. We assume an arbitrary simple Lie gauge group of colors and an arbitrary irreducible representation for the quarks. In case of $SU(n)$ and the fundamental representation, the group coefficients would be: $C_a = n$, $C_f = (n^2 - 1)/(2n)$. The trace normalization was chosen as $T_f = 1/2$. The number of quark flavors is $N_f$. The field renormalization constants are defined for the one-particle-irreducible two-point diagrams, including one leg. They enter the charge renormalization in the positive powers:

$$Z_h = Z_{ffv}^2 Z_f^2 Z_v = 1 + \frac{h}{\varepsilon}\left(-\frac{11}{3}C_a + \frac{2}{3}N_f\right) + \mathcal{O}(h^2), \tag{12}$$

$$Z_Y = Z_{ffs}^2 Z_f^2 Z_s = 1 + \frac{h}{\varepsilon}\left(-2C_a - 2C_f + N_f\right) + \mathcal{O}(h^2). \tag{13}$$

As expected, the gauge and Yukawa charges are renormalized differently, eqs. (12) and (13), because there is now supersymmetry to protect the tree-level coincidence of the $\varepsilon$-scalar and vector coupling constants. We everywhere replace the Yukawa charge with the corresponding function of the gauge charge, that is, exploit a special solution of the renormalization-group equations.

The nonminimal mass counterterm for the $\varepsilon$ scalars

$$\Delta m_s^2 = -h \sum_{f=1}^{N_f} m_f^2 \left(\frac{2}{\varepsilon} + 2 - 2\ln\frac{m_f^2}{\mu^2}\right) + \mathcal{O}(h^2) \tag{14}$$

is necessary to insure that their pole mass is zero.

The fermion field and mass renormalization constants were calculated up to two loops:

$$Z_f = 1 - \frac{h}{\varepsilon}\left(-\xi C_f\right)$$
$$- h^2\left\{\left[\frac{1}{\varepsilon^2} - \frac{13}{4\varepsilon} + (\xi-1)\left(\frac{5}{4\varepsilon^2} - \frac{5}{4\varepsilon}\right) + (\xi-1)^2\left(\frac{1}{4\varepsilon^2} - \frac{1}{8\varepsilon}\right)\right]C_a C_f \right.$$
$$\left. + \left[\frac{1}{2\varepsilon^2} + \frac{7}{4\varepsilon} + \frac{\xi-1}{\varepsilon^2} + \frac{(\xi-1)^2}{2\varepsilon^2}\right]C_f^2\right\} + \mathcal{O}(h^3), \tag{15}$$

$$Z_m = 1 - \frac{h}{\varepsilon}3C_f - h^2\left[\left(-\frac{11}{2\varepsilon^2} + \frac{79}{12\varepsilon}\right)C_a C_f\right.$$
$$\left. + \left(-\frac{9}{2\varepsilon^2} - \frac{1}{4\varepsilon}\right)C_f^2 + \left(\frac{1}{\varepsilon^2} - \frac{1}{3\varepsilon}\right)C_f N_f\right] + \mathcal{O}(h^3). \tag{16}$$

In spite of the fact that there are several masses in the theory, there exist a common mass renormalization constant for all flavors. Let us remind that in the conventional dimensional regularization the mixing of the masses through the diagram of fiq. 1(a) was prevented by the gauge invariance which forbids a mass term in the vector propagator. In case of the $\varepsilon$ scalars,

there is no such ban in general. But subtracting the $\varepsilon$-scalar propagator at zero, we, at the same time, get rid of the $m^2/\varepsilon$ term in the incomplete $R$ operation. This is related to the fact that the subtraction at zero insures the absence of an infrared singularity in the renormalized subgraph $\{1, 2, 5\}$.

The final result for the pole mass of a quark with a given flavor is

$$
\begin{aligned}
\left(\frac{m_P}{m}\right)_f = {} & 1 + h\Big[5 - 3\ln\frac{m_f^2}{\mu^2}\Big]C_f \\
& + h^2\Big\{\Big[\frac{1093}{24} + \frac{11}{2}\ln^2\frac{m_f^2}{\mu^2} - \frac{179}{6}\ln\frac{m_f^2}{\mu^2} \\
& \quad - 6\zeta(3) - 8\zeta(2) + 24\zeta(2)\ln 2\Big]C_a C_f \\
& \quad + \Big[-\frac{59}{8} + \frac{9}{2}\ln^2\frac{m_f^2}{\mu^2} + \frac{3}{2}\ln\frac{m_f^2}{\mu^2} \\
& \quad + 12\zeta(3) + 30\zeta(2) - 48\zeta(2)\ln 2\Big]C_f^2 \\
& \quad + \sum_{j=1}^{N_f}\Big[-\ln^2\frac{m_f^2}{\mu^2} + \frac{13}{3}\ln\frac{m_f^2}{\mu^2} + E(m_j/m_f)\Big]C_f\Big\} + \mathcal{O}(h^3)
\end{aligned}
\tag{17}
$$

The sum over flavors represents the renormalized contribution of the diagram with the quark loop, fig. 1(a). We know this contribution in three kinds of expansions. The small-mass expansion, the construction and meaning of which was discussed above, gives

$$
\begin{aligned}
E(r) = {} & -\frac{37}{6} - 4\zeta(2) + 2\pi^2 r - 12r^2 + 2\pi^2 r^3 \\
& + \Big[-\frac{151}{18} - 4\zeta(2) + \frac{13}{3}\ln r^2 - \ln^2 r^2\Big]r^4 + \Big[\frac{76}{75} - \frac{8}{15}\ln r^2\Big]r^6 \\
& + \Big[\frac{1389}{9800} - \frac{9}{70}\ln r^2\Big]r^8 + \Big[\frac{3988}{99225} - \frac{16}{315}\ln r^2\Big]r^{10} \\
& + \mathcal{O}(r^{12}\ln r^2).
\end{aligned}
\tag{18}
$$

Numerically the most essential correction due to the mass of a light quark corresponds to the linear term in the mass ratio.

The large-mass expansion, performed according to the Euclidean rules, yields

$$E(r) = \frac{20}{9} + \frac{13}{3}\ln r^2 + \ln^2 r^2 - \left(\frac{76}{75} + \frac{8}{15}\ln r^2\right)/r^2$$

$$- \left(\frac{1389}{9800} + \frac{9}{70}\ln r^2\right)/r^4 - \left(\frac{3988}{99225} + \frac{16}{315}\ln r^2\right)/r^6$$

$$- \left(\frac{1229}{78408} + \frac{5}{198}\ln r^2\right)/r^8 - \left(\frac{184452}{25050025} + \frac{72}{5005}\ln r^2\right)/r^{10}$$

$$+ \mathcal{O}(r^{-12}\ln r^2). \tag{19}$$

In most calculations, people simply ignore the quarks that are heavier than the characteristic energy scale of the problem. They use thus an effective low-energy theory with the reduced number of particles. The running charges and masses of this effective theory are related to those of the full theory by the so-called matching conditions that the running parameters are supposed to change their running rate at some value of the renormalization scale $\mu$ of the order of the heavy-particle mass. Of course, the low-energy effective theory is no longer valid at that scale. The matching condition just ensures the equivalence of the two theories at low energies where the heavy particles ought to decouple, up to power corrections, irreproducible in the low-energy theory. The coincidence of the pole mass in the two theories provides the two-loop matching condition for the running mass. The charge matching is done by means of another physical quantity, like the invariant charge. In the MMS scheme (as well as $\overline{\text{MS}}$), on the one-loop level, relevant here, it is done at $\mu = m_j$ [16]. One should bear in mind, however, implicit numeric ambiguities of any matching condition, similar to the scheme dependence, owing to the fact that higher orders of perturbation theory are unavailable. The result, thus, depends on the choice of a physical quantity and on the detailed prescription of equating it at low energy. When defining the two-loop mass matching, the term of order zero in the ratio of the masses is absorbed into the matching point. The actual correction starts from the leading power $m_f^2/m_j^2$.

The third available expansion is performed in the difference of the squared masses. Such an expansion covers the intermediate region between eqs. (18) and (19). In the present case the intermediate expansion is just a regular Taylor expansion involving only the whole diagram as the ultraviolet subgraph. A surprising remark is, however, that generating this
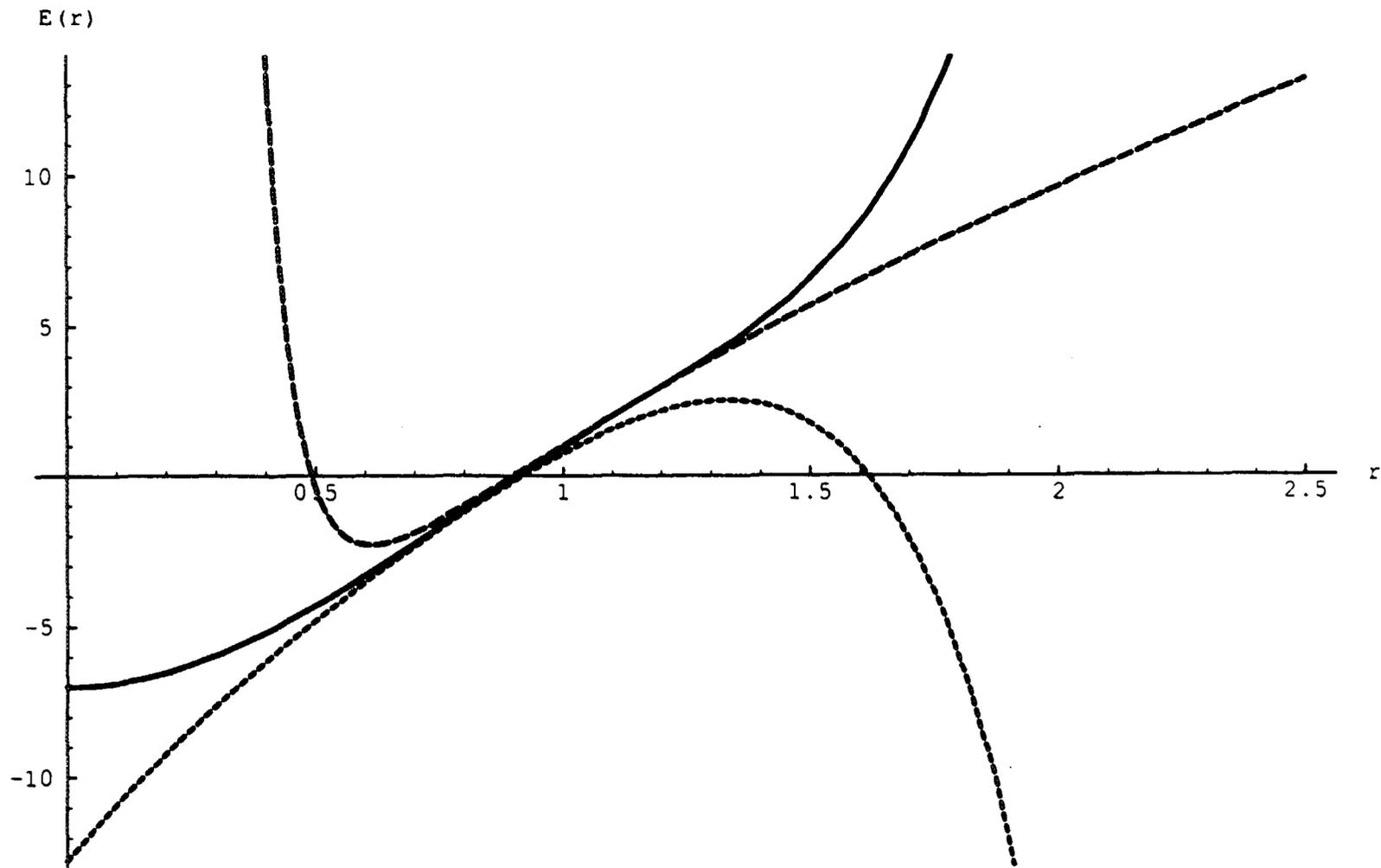
Figure 3: Three expansions for the renormalized diagram of fig. 1(a) in the dimensional reduction MMS scheme. The dotted line corresponds to the small-mass expansion up to $r^6$, the dashed line to the large-mass expansion up to $1/r^6$. the solid line to the intermediate expansion up to $(r^2 - 1)^3$.

trivial expansion is the most time-consuming of the three, while the most complicated small-mass expansion is the fastest.

$$
\begin{aligned}
E(r) &= -\frac{73}{6} + 8\zeta(2) + \left[ -8 + 8\zeta(2) \right](r^2 - 1) - \left[ 1 + \frac{1}{2}\zeta(2) \right](r^2 - 1)^2 \\
&+ [r^2 - 1]^3 - \left[ \frac{1}{2} + \frac{3}{32}\zeta(2) \right](r^2 - 1)^4 + \left[ \frac{19}{60} + \frac{3}{32}\zeta(2) \right](r^2 - 1)^5 \\
&+ \mathcal{O}[(r^2 - 1)^6].
\end{aligned}
\tag{20}
$$

As the masses of different quarks are quite different, higher terms of eq. (20) are of little practical use. The value that is important is $E(1)$, which defines the self-contribution of a quark to its pole mass. The intermediate expansion just demonstrates the means of recovering the function $E(r)$ in the whole region of the mass ratio. The matching of the three expansions is demonstrated in fig. 3. We terminated the expansions at the same relatively low order for the sake of clearness. Keeping more terms, we get a wider region around $r = 1$ where the curves practically coincide.

All the three expansions reproduce the same dependence of the diagram on $\mu$. This confirms the fact that our renormalization scheme, though being nonminimal, remains massless. The latter implies that the renormalization-group functions do not depend on masses and other dimensional coupling constants but polynomially. The nonminimality nevertheless reveals itself by the fact that the $\beta$ function of the fermion mass cannot be extracted from the first-order pole of the renormalization constant (16). The correct way of defining the mass beta function is to extract it from the condition of the renormalization invariance of a physical quantity, the pole mass, by differentiating eq. (17) with respect to $\ln \mu^2$. The result

$$
\beta_m = m\left[ h\left( -3C_f \right) + h^2\left( -\frac{23}{2}C_a C_f - \frac{3}{2}C_f^2 + C_f N_f \right) + \mathcal{O}(h^3) \right]
\tag{21}
$$

does not coincide with what was obtained in ref. [17], although the renormalization constant (16) and the one-loop counterterms, (8)–(13), and the one-loop contribution to eq. (15), agree with refs. [17, 18]. The explanation is quite evident indeed. The bare mass of the fermion ceases to be a renormalization-invariant quantity. The point is that the physical mass involves contributions from two mutually correlated bare masses – of the quark and of the $\varepsilon$ scalar. Both *do* depend on $\mu$ to ensure that the physical

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quantity is invariant. The true reason is thus an extra bare parameter as compared to the number of the physical renormalized parameters. In fact, the renormalized mass of the $\varepsilon$ scalar becomes one more independent quantity. However, we keep it equal to zero, while in the minimal subtraction scheme it would be running as well.

The results of the three expansions (18)–(20) differ from the corresponding expansions in the conventional dimensional regularization MMS scheme only by a constant $-\frac{1}{4}$ in the zeroth order, which confirms that these are just two different renormalization schemes.

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