CONF-970503--66 BNL-64403

Multiphoton processes in the field of two-frequency circularly polarized plane * electromagnetic waves

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(April 4, 1997)

We solve Dirac's equation for an electron in the field of a two-frequency plane electromagnetic wave, deriving general formulae for the probabilities of radiation of a photon by the electron, and for the probabilities for pair production by a photon when the two-frequency wave is circularly polarized. In contrast to the case of a monochromatic-plane electromagnetic wave, when an electron is in the field of a two-frequency circularly polarized wave, besides the absorption of multiphotons and emission of simple harmonics of the individual waves, stimulated multiphoton emission processes and various composite harmonic-photon emission processes are occurred; when a high-energy photon is in a such a field, multiphoton processes also follow the pair production processes.

PACS numbers: 41.60.-m, 32.80.Wr, 12.20.-m, 23.20.Ra

I. INTRODUCTION

There have been many investigation of multiphoton processes in strong electromagnetic fields since the invention of the laser in the early 1960s. Reiss formulated the multiphoton absorption processes(no emission of multiphotons) that generate electron pairs when a photon collides with an intense wave field [1]. Nikishov and Ritus derived formulae both for multiphoton absorption of an electron in a plane electromagnetic field, and the multiphoton absorption processes of pair production [2]. Narozhnyî *et al.* extended these formulae to the case of a circularly polarized photon beam [3]: such effects were observed recently [4]. Other theoretical studies made by

* This work was done under the auspices of the Power Reactor and Nuclear Fuel Development Corp.

the classical approach [5] [6] and by the semiclassical scattering theory as reviewed by Ehlotzky [7], or by quantum electrodynamics (QED) [8], which describe the multiphoton processes of the harmonic-photon generation also have made enormous advaces in our understanding. The predicted second harmonic-photon emission phenomenon was observed by Englert and Rinehart [9]. Besides these simple harmonics of a monochromatic wave, it is possible to generate varied harmonics by using a two-frequency intense laser beam. We noted that Puntajer and Leubner considered the specific case of this problem by Classical Electrodynamics [10](constraining the frequency of the two waves $\omega_2 = 2\omega_1$; the wave and electron beam are counterpropagating). In the present paper, we consider a more general case within the QED theory, with an arbitrary frequency for the two waves and an arbitrary angle between the wave and electron beam. We expect that through these investigations of electrons and of photons in intense electromagnetic fields we may be reveal some of their unknown properties.

The theoretical research with QED on multiphoton absorption processes in an intense laser beam is based on Dirac's equation. The success of the QED theory in describing multiphoton processes demonstrates that the semi-classical treatment of the laser beam as an external unquantized electromagnetic field is a good approximation. In fact, a way to describe an intense electromagnetic field in quantized terms has not yet been realised nor is there a reliable perturbation theory for treating electron

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and intense electromagnetic-wave scattering. Using the semi-classical treatment of electromagnetic fields for an electron in a plane electromagnetic wave, an exact solution of Dirac's equation has been found [11]. However, the exact solution of Dirac's equation for more than one plane waves has not been obtained and it seems difficult resolve, because there are collisions between photons when we consider two-plane waves. To remove this complication but, at the same time not to lose generality in physics, we consider a two-frequency plane wave; that is, two different plane waves propagating in the same direction. With this approach, we can find an exact solution of Dirac's equation for an electron in these waves.

Having obtained an assessment of an electron's wave function in intense electromagnetic waves, multiphoton absorption and emission processes are not difficult to understand; for example, an electron's wave function in an intense monochromatic electromagnetic wave can be expanded as (see Sec. 101 of [12]) $\psi_p = (\sum_{s=-\infty}^{\infty} A_s e^{-iskx}) \frac{u(p)}{\sqrt{2q_0}} e^{-iqx}$, a state that is sometimes called a "dressing" state. In fact, this is some kind of composed state between a free electron with "quasi-momentum" q and those multiphoton states, so that the electron can absorb or emit multiphotons.

In this paper, we describe our study of a two-frequency plane electromagnetic wave with four-vector momentums, k_1 and k_2, which collide with a single free electron. This has led us to a new prediction of multiphoton absorption and emission processes, and nonlinear harmonic-photon generation; that is, there are n_1 number of photons k_1, and n_2 number of photons k_2, absorbed or emitted by the electron when it is emitting approximately harmonic-photons of $n_1 k_1 + n_2 k_2$(in reference to an electron that is at rest), where n_1 and n_2 are integer; as we can see from formulae given later in this paper, there are interference terms in the scattering rate. This may be an interesting new phenomena. The substitution rule gives the other effects of multiphoton absorption or emission processes for pair production when a photon collides with a two-frequency plane electromagnetic wave. The scattering probabilities depend on the relative rotary directions of the two circularly polarized waves. Recently, a high intensity laser beam was produced, and we expect that these predicted nonlinear effects soon will be observed.

This paper is organized as follows: In Sec. 2, we solve Dirac's equation for an electron in a field of a two-frequency laser beam which is circularly polarized; in Sec.3, we derive harmonic-photon emission probabilities for the electron in such a field. Sec. 4 describes our use of the substitution rule to derive the probabilities of pair production for a photon colliding with a two-frequency plane electromagnetic wave. In Sec. 5, we discuss the findings and give our conclusions from the derived formulae.

II. THE WAVE FUNCTION

Since the number of photons in the laser beam is very large, we can treat the field as an external, unquantized electromagnetic field. Suppose the two-frequency electromagnetic wave is circularly polarized, then the field of waves can be described by the four-potential:

$$A = A_1 + A_2$$

$$A_1 = a_1 cos\phi_1 + a_2 sin\phi_1 \qquad \phi_1 = (k_1 x)$$

$$A_2 = a_3 cos\phi_2 + a_4 sin\phi_2 \qquad \phi_2 = (k_2 x) \tag{2.1}$$

where $k_{1\mu}$ and $k_{2\mu}$ represent the four-vector propagation (for convenience, we suppose $k_1 > k_2$), while $a_{i\mu}$, $i = 1,4$ are the amplitudes of the potential. The gauge are chosen as $(k_1 A_1) = (k_2 A_2) = 0$. We suppose $\vec{k_1}$ is parallel to $\vec{k_2}$, so $\phi_2/\phi_1 = \eta = const. < 1$ and $k_2 = \eta k_1$, the orthogonal conditions are $(a_1 a_2) = (a_3 a_4) = (k_i a_j) = 0$, $i = 1,2$, $j = 1,4$, while their lengths satisfy ${a_1}^2 = {a_2}^2, {a_3}^2 = {a_4}^2$, and $(k_i k_j) = 0$, $i,j = 1,2$. In this case, the second-order equation is expressed by:

$$\{-\partial^2 - 2ieA\partial + e^2 A^2 - m^2 - ie[(\gamma k_1)(\gamma A_1') + (\gamma k_2)(\gamma A_2')]\}\psi = 0 \tag{2.2}$$

We first suppose $\psi_p = e^{-i(px)} F_1(\phi_1) F_2(\phi_2)$ and put this into equation (2.2), after a similar derivation, as described in section 40 of ref. [12], and use $\int_0^{\phi_1} F_2(\phi_2) d\phi_1 = \int_0^{\phi_2} \frac{F_2(\phi_2)}{\eta} d\phi_2$, where F_1 and F_2 are any kind of function. Then, we find that the exact solution of Dirac's equation has the form:

$$\psi_p = \frac{u(p)}{\sqrt{2q_0}}[1 + \frac{e(\gamma k_1)(\gamma A_1)}{2(pk_1)} + \frac{e(\gamma k_2)(\gamma A_2)}{2(pk_2)}]exp\{-i[S + (qx)]\} \quad (2.3)$$

$$(2.4)$$

where,

$$S = \frac{e(a_1 p)}{(pk_1)} sin\phi_1 - \frac{e(a_2 p)}{(pk_1)} cos\phi_1 + \frac{e(a_3 p)}{(pk_2)} sin\phi_2 - \frac{e(a_4 p)}{(pk_2)} cos\phi_2 - \\ -\frac{e^2}{2(pk_1)}\{\frac{[b_1{}^2 sin(\phi_1 - \phi_2) - b_2{}^2 cos(\phi_1 - \phi_2)]}{1-\eta} + \frac{[b_3{}^2 sin(\phi_1 + \phi_2) - b_4{}^2 sin(\phi_1 + \phi_2)]}{1+\eta}\}$$

where b_i is defined as:

$$b_1{}^2 = (a_1 a_3) + (a_2 a_4) \qquad b_2{}^2 = (a_2 a_3) - (a_1 a_4)$$

$$b_3{}^2 = (a_1 a_3) - (a_2 a_4) \qquad b_4{}^2 = (a_1 a_4) + (a_2 a_3)$$

where $u(p)$ is bispinor, p_μ is a constant four-vector determining the state, $p^2 = m^2$, $q_\mu = p_\mu - \frac{a_1{}^2 e^2}{2(pk_1)} k_1 - \frac{a_3{}^2 e^2}{2(pk_2)} k_2$; suppose $\xi_1{}^2 = -\frac{a_1{}^2 e^2}{m^2}$ and $\xi_3{}^2 = -\frac{a_3{}^2 e^2}{m^2}$, then $q^2 = m^2(1 + \xi_1{}^2 + \xi_3{}^2) = m_*{}^2$, m_* is the "effective" mass of the particle in the field. The normalization condition of wave function (2.3) is:

$$\int \psi_{p'}{}^* \psi_p d^3x = (2\pi)^3 \delta(\vec{q'} - \vec{q}) \quad (2.5)$$

and all the other general relations investigated in Sec.2 of ref. [2] are valid in our case.

III. THE PROBABILITIES OF PHOTON EMISSION BY AN ELECTRON

The S-matrix for an electron emitting a photon with momentum k' and polarization e', see equation (73.19) in ref. [12], is equal to:

$$S_{fi} = -ie \int \overline{\psi}_{p'} (\gamma e'^*) \psi_p \frac{\sqrt{4\pi} e^{ik'x}}{\sqrt{2\omega'}} d^4x \quad (3.1)$$

where e'^{*} is a conjugate of e', ω' is the energy of the scattering photon, and ψ_p and ψ'_p are the initial and final states of the electron, respectively. We use equation (2.3), and following the formula (101.7) in ref. [12], then equation (3.1) can be simplified as:

$$S_{fi} = \frac{1}{\sqrt{2q_0 2q_0' 2\omega'}} \sum_{n_1,n_2} M_{fi}{}^{(n_1 n_2)} (2\pi)^4 \delta^4 [q + n_1 k_1 + n_2 k_2 - q' - k'] \quad (3.2)$$

As argued in ref. [2], each term in equation (3.2) describes absorption from the waves, or emission into the waves, of a definite number, n_1 and n_2, of photons with momentum k_1 and k_2, respectively; if n_i is a positive integer then n_i number of k_i photons are absorbed; otherwise, a multiphoton is emitted. From the representation of the δ-function in equation (3.2), we obtain the following kinematic equation:

$$q + n_1 k_1 + n_2 k_2 = q' + k' \quad (3.3)$$

If we sum the polarizations of inital and final electrons and the emission photons for the square of matrix element $M_{fi}{}^{(n_1 n_2)}$ in equation (3.2), and use the expansion formula eq. (101.7) in ref. [12], then after a very complicated derivation we find:

$$\sum_{polar} |M_{fi}{}^{(n_1 n_2)}|^2 = 4\pi e^2 m^2 \sum_{s_1,s_2} J_{s_1}{}^2(z_3) J_{s_2}{}^2(z_4) \{-4 J_{n_1-s_1-s_2}{}^2(z_1) J_{n_2+s_1-s_2}{}^2(z_2) + [2 + \frac{(k_1 k')^2}{(pk_1)(p'k_1)}] C_{s_1 s_2}\} \quad (3.4)$$

where $J_s(z)$ is the Bessel function, and

$$\begin{aligned} C_{s_1 s_2} = & \frac{4 J_{n_1-s_1-s_2}{}^2(z_1) J_{n_2+s_1-s_2}{}^2(z_2)}{m^2} \frac{(pk_1)(p'k_1)}{(k_1 k')} [(1-\eta)s_1 + (1+\eta)s_2] - \xi_1{}^2 J_{n_2+s_1-s_2}{}^2(z_2)[2 J_{n_1-s_1-s_2}{}^2(z_1) \\ & - J_{n_1-s_1-s_2+1}{}^2(z_1) - J_{n_1-s_1-s_2-1}{}^2(z_1)] - \xi_3{}^2 J_{n_1-s_1-s_2}{}^2(z_1)[2 J_{n_2+s_1-s_2}{}^2(z_2) - J_{n_2+s_1-s_2+1}{}^2(z_2) \\ & - J_{n_2+s_1-s_2-1}{}^2(z_2)] - \frac{e^2}{m^2} J_{n_1-s_1-s_2}(z_1) J_{n_2+s_1-s_2}(z_2) [J_{n_1-s_1-s_2+1}(z_1) J_{n_2+s_1-s_2+1}(z_2) + J_{n_1-s_1-s_2-1}(z_1) \\ & \times J_{n_2+s_1-s_2-1}(z_2)][b_1{}^2 cos(\phi_{10} - \phi_{20}) + b_2{}^2 sin(\phi_{10} - \phi_{20})] - \frac{e^2}{m^2} J_{n_1-s_1-s_2}(z_1) J_{n_2+s_1-s_2}(z_2) [J_{n_1-s_1-s_2+1}(z_1) \\ & \times J_{n_2+s_1-s_2-1}(z_2) + J_{n_1-s_1-s_2-1}(z_1) J_{n_2+s_1-s_2+1}(z_2)][b_3{}^2 cos(\phi_{10} + \phi_{20}) + b_4{}^2 sin(\phi_{10} + \phi_{20})]\} \end{aligned} \quad (3.5)$$

where,

$$tan\phi_{10} = \frac{\alpha_2}{\alpha_1} \qquad tan\phi_{20} = \frac{\alpha_4}{\alpha_3}$$
$$z_1 = \sqrt{\alpha_1^2 + \alpha_2^2} \qquad z_2 = \frac{\sqrt{\alpha_3^2 + \alpha_4^2}}{\eta}$$
$$z_3 = \frac{\sqrt{\beta_1^2 + \beta_2^2}}{1-\eta} \qquad z_4 = \frac{\sqrt{\beta_3^2 + \beta_4^2}}{1+\eta}$$
$$\alpha_i = e[\frac{(a_i p)}{(pk_1)} - \frac{(a_i p')}{(p'k_1)}]$$
$$\beta_i = \frac{e^2}{2}[\frac{1}{(pk_1)} - \frac{1}{(p'k_1)}]{b_i}^2 \qquad i = 1,4 \qquad (3.6)$$

The probability of the emission of a photon by an electron per unit time is:

$$W = \sum_{n_1,n_2} \int |{M_{fi}}^{(n_1 n_2)}|^2 \frac{d^3k'd^3q'}{(2\pi)^6 2q_0 2{q_0}' 2\omega'} (2\pi)^4 \delta^4[q + n_1 k_1 + n_2 k_2 - q' - k'] \quad (3.7)$$

Let us consider two kinds of cases; first is that of two circularly polarized electromagnetic waves which have the same rotary direction, that is, they are both left-circular-polarized or right-circular-polarized; the other case is one where these two waves have an opposite rotary direction. In this case, the field of waves can be expressed as:

$$A = A_1 + A_2$$
$$A_1 = a_1 cos\phi_1 + a_2 sin\phi_1 \qquad \phi_1 = (k_1 x) \qquad (3.8)$$
$$A_2 = \zeta[a_1 cos(\phi_2 + \varphi) \pm a_2 sin(\phi_2 + \varphi)] \qquad \phi_2 = (k_2 x)$$

where φ is a certain phase-difference between these two plane waves; the plus in $\pm$ in the above equation corresponds to first case, and the minus corresponds to the second case. The factor ζ defines the ratio of intensity between the two laser beams, so if the intensity of the wave k_1 is ${\xi_1}^2 = \xi^2$, then the intensity of the other wave is $\xi_3{}^2 = \zeta^2\xi^2$. If we take the invariant $u = \frac{(k_1 p)}{(k_1 p')}$, then the probabilities in equation (3.7) are:

$$W_\pm = \sum_{n_1,n_2} w_\pm{}^{\{n_1,n_2\}} \qquad (3.9)$$

where,

$$w_\pm{}^{\{n_1,n_2\}} = \frac{e^2m^2}{4q_0}\sum_{s_l}\int_0^{u_v}\frac{du}{(1+u)^2}J_{n_1-s_l}{}^2(z_1)J_{n_2\pm s_l}{}^2(z_2)J_{s_l}{}^2(z_l)\{-4+\xi^2[2+\frac{u^2}{(1+u)}][-2(1+\zeta^2)$$
$$+2\zeta\frac{J_{s_l+1}(z_l)+J_{s_l-1}(z_l)}{J_{s_l}(z_l)}+(\frac{J_{n_1-s_l+1}(z_1)}{J_{n_1-s_l}(z_1)}+\zeta\frac{J_{n_2\pm s_l+1}(z_2)}{J_{n_2\pm s_l}(z_2)})^2+(\zeta\frac{J_{n_2\pm s_l-1}(z_2)}{J_{n_2\pm s_l}(z_2)}+\frac{J_{n_1-s_l-1}(z_1)}{J_{n_1-s_l}(z_1)})^2]\} \quad (3.10)$$

where n_1, n_2, and s_l are integers, and

$$u_v = \frac{2v(k_1p)}{m_*{}^2} \quad ; \quad v = n_1 + \eta n_2$$
$$z_1 = 2v\frac{\xi}{\sqrt{1+\xi^2+\zeta^2\xi^2}}\sqrt{\frac{u}{u_v}(1-\frac{u}{u_v})}$$
$$z_2 = \frac{\zeta z_1}{\eta} \quad ; \quad z_l = \frac{2v}{1\mp\eta}\cdot\frac{\zeta\xi^2}{1+\xi^2+\zeta^2\xi^2}\cdot\frac{u}{u_v} \qquad (3.11)$$

where z_i is calculated by using the formulae in ref. [12] of Sec. 101, and the upper sign of $\pm$ or $\mp$ in equation (3.10) and (3.11) denotes that the two waves with the same rotary direction, while the lower sign denotes that they have opposite rotary direction. The results for both cases are independent of the phase φ in equation (3.8); a interesting point is that the two waves can correlated by external electrons, no matter what the phase difference is between them.

Formula (3.10) should include the case of a monochromatic wave; this can easily be checked by putting $\zeta = 0$ into equation (3.10), which then can be reduced to represent the case of a monochromatic wave, and we obtain exactly the same formula as that in ref. [3].

IV. THE PROBABILITIES OF A PAIR PRODUCTION BY A PHOTON

If we substitute $p \rightarrow -p$, $k' \rightarrow -l$ and $d^3k' \rightarrow d^3q$, in equations (3.7) and (3.10), and reverse the common sign of the expression, we can obtain the probabilities of a pair production by a photon of momentum l in two electromagnetic waves per unit time [3]:

$$W_{\pm} = \sum_{n_1,n_2} w_{\pm}{}^{\{n_1,n_2\}} \tag{4.1}$$

where,

$$w_{\pm}{}^{\{n_1,n_2\}} = \frac{e^2m^2}{4l_0}\sum_{s_l}\int_1^{u_v}\frac{du}{u\sqrt{u(u-1)}}J_{n_1-s_l}{}^2(z_1)J_{n_2\pm s_l}{}^2(z_2)J_{s_l}{}^2(z_l)\{2+\xi^2(2u-1)[-2(1+\zeta^2)$$
$$+2\zeta\frac{J_{s_l+1}(z_l)+J_{s_l-1}(z_l)}{J_{s_l}(z_l)}+(\frac{J_{n_1-s_l+1}(z_1)}{J_{n_1-s_l}(z_1)}+\zeta\frac{J_{n_2\pm s_l+1}(z_2)}{J_{n_2\pm s_l}(z_2)})^2+(\zeta\frac{J_{n_2\pm s_l-1}(z_2)}{J_{n_2\pm s_l}(z_2)}+\frac{J_{n_1-s_l-1}(z_1)}{J_{n_1-s_l}(z_1)})^2]\} \tag{4.2}$$

and,

$$u = \frac{(kl)^2}{4(kq)(kq')} \quad ; \quad u_v = \frac{v(kl)}{2m_*{}^2} \quad ; \quad v > v_s = \frac{2m_*{}^2}{(kl)} \tag{4.3}$$

z_1, z_2, z_l and v have the same expression as in equation (3.11). In this case, the kinematic equation is:

$$l + vk_1 = q + q' \tag{4.4}$$

From condition (4.3), we know the higher energy photon will more easily produce electron pairs because as the photon's energy increases v_s becomes smaller; then, lower-order pair production processes(corresponding to a small v) occur. On the other hand, when an incident photon's energy is lower than some energy limit, from the perturbation theory of QED, we believe that generally pair production will cease. However, in our case, pair production still occur by multiphoton processes(corresponding to a large v) for intense waves, and though these probabilities are higher-order terms, it is expected that the effects will be observable.

V. DISCUSSION AND CONCLUSIONS

In the framework in which the electron is at rest, on average, ($\vec{q} = 0$, $q_0 = m_*$), $(qk_1) = m_*\omega_1$, $(qk') = m_*\omega'$, $(k_1k') = \omega_1\omega'(1 - cos\theta)$, and θ is the scattering angle between $\vec{k}_1$ and $\vec{k}'$. With these expressions and the kinematic equation (3.3), we can assess emitted harmonic-photon frequency:

$$\omega' = \frac{n_1\omega_1 + n_2\omega_2}{1 + \frac{v\omega_1}{m_*}(1 - cos\theta)} \tag{5.1}$$

where ω_1 and ω_2 are the energies of two incident photons in this framework. From equation (5.1), we see ω' should be > 0 which correspondes to $v > 0$ $(n_1\omega_1 + n_2\omega_2 = v\omega_1)$; this condition restricts the case of the monochromatic wave($\omega' \approx n\omega$, so $n > 0$), therefore only multiphoton absorption is allowed, while, in our case, n_1 and n_2 can be any integers as long as $v > 0$ is maintained. This is a very important point, which means there are various mixtured harmonics that can be generated, rather than only simple harmonics. On the other hand, when the electron is emitting the roughly harmonic-photon of $n_1k_1 + n_2k_2$, there are multiphoton absorption and emission processes occuring at the same time, as argued in section 3. For example, if $n_1 = 4$ and $n_2 = -3$, this means that when the electron emission of harmonic-photon $\approx 4\omega_1 - 3\omega_2$ is operating within the electron at-rest reference system, the electron absorbs four ω_1 photons and emits three ω_2

photons at same time. This is a multiphoton emission and absorption process, and the multiphoton emission effects here are some kind of stimulated processes [13]; it seems that laser photons (four ω_1 photons in above example) are absorbed by the electron and the electron stimulated by other laser photons(ω_2), so there is emission of multiphoton(three ω_2 photons). Similar effects of multiphoton absorption and stimulated multiphoton emission processes may also be possible for the case of pair production.

Another interesting nonlinear effect is that of interference between two waves affecting the scattering rates. The probabilities $w_{\pm}{}^{\{n_1 n_2\}}$ in equation (3.10) include the sum over s_l, as following way, we can easily assess that s_l connects to interference effects; suppose we put $n_1 - s_l = s_1$ and $n_2 \pm s_l = s_2$ into the equation (3.10), then the harmonic-photon emission probabilities are dependent on the quantities s_1, s_2 and s_l, the kinematic condition in (3.3) becomes $q + s_1 k_1 + s_2 k_2 + s_l(k_1 \mp k_2) = q' + k'$, the harmonics in the framework of the electron at-rest, on average, is $\omega' \approx s_1 k_1 + s_2 k_2 + s_l(k_1 \mp k_2)$; we can see s_1 and s_2 are absorbed, or emitted, photon number of k_1 and k_2, respectively, while s_l is some kind of quantity for describing interference part; for example, when $s_l > 0$ the term $s_l(k_1 - k_2)$ means that the absorption of s_l number of k_1 photons and the emission of s_l number of k_2 photons are taken place at same time, like absorption of s_l number of $k_1 - k_2$ "photons" processes occur, and there is also harmonis of $(k_1 - k_2)$, it seems $k_1 - k_2$ behaves like individual photon; so does k_1+k_2. So s_l connected terms in equation (3.10) are interference terms. The same argument is also true for the case of pair production.

The effects discussed above are based on results derived from Dirac's equation where the field of a laser beam is described in an unquantized classical way. When a two-frequency laser beam is scattered by electrons, there are various composite harmonic-photons with energy $n_1\omega_1 + n_2\omega_2$ emission processes which follow multiphoton absorption and stimulated multiphoton emission processes; the scattering probabilities are affected by the effects of interference between two laser beams. Similar effects may be involved in pair production processes when a γ photon is scattered by a two-frequency laser beam.

VI. ACKNOWLEDGMENTS

The authors acknowledge to Doc. Woodhead for her editoral work for this paper and this work was performed under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

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