A Test of Wigner's Spin-Isospin Symmetry from Double Binding Energy Differences

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In the supermultiplet model of nuclei it is assumed that nuclear forces are independent of isospin as well as spin [1, 2, 3]. Nuclear states can then be characterized by the quantum numbers of the spin-isospin or SU(4) symmetry, giving rise to simple predictions concerning nuclear β-decay rates and masses. The former arise because the Fermi as well as Gamow-Teller operators are generators of SU(4) and as such β transitions can only occur between states belonging to the same supermultiplet; predictions of nuclear binding energies are obtained in a lowest-order approximation from the permutational symmetry of the *orbital* part of the many-body wavefunction which determines the degree of spatial overlap between the nucleons. Since the original work by Wigner [1] and Hund [2] it has become clear that SU(4) symmetry is badly broken in the majority of nuclei because of the increasing importance with mass of the spin-orbit term in the nuclear mean-field potential. Nevertheless, it remains a useful *ansatz* for studying global properties of p- and sd-shell nuclei from a simple perspective. Moreover, as will be shown in this Letter, it may have a particular and renewed relevance in the study of the heavier $N \simeq Z$ nuclei from ^{56}Ni to ^{100}Sn, a declared experimental goal of many of the current proposals for new facilities based on accelerated radioactive beams [4].

The most conclusive test of SU(4) symmetry is through a comparison with realistic shell-model calculations which can be readily performed for nuclei up to ^{40}Ca. The goodness of SU(4) symmetry in the ground state is then obtained by taking the overlap between the shell-model wavefunction and the favored SU(4) representation. This approach is followed, for example, for sd- and pf-shell nuclei by Vogel and Ormand [5]. The overall conclusion of such studies is that in nuclei heavier than ^{16}O significant departures from SU(4) symmetry occur.

To obtain a test of the goodness of SU(4) symmetry directly from masses is more difficult. Franzini and Radicati [6] suggested the use of a ratio $R(T_z)$ of ground-state energy differences involving four isobaric nuclei with different isospin projections T_z and showed that the values agree rather well with the SU(4) predictions for nuclei with masses up to $A \approx 110$. However, it was demonstrated subsequently [7] that this ratio $R(T_z)$ is not very sensitive to SU(4) symmetry mixing.

In [8] it is pointed out that a sensitive test of SU(4) symmetry can be made by using double binding energy differences which also provide information concerning the strength of the neutron-proton (np) interaction which is known to play a pivotal role in the structure of nuclei [9]. Recently, the quantity

$$\delta V_{\rm np}(N,Z) \equiv \tfrac{1}{4}\Big([B(N,Z)-B(N-2,Z)]-[B(N,Z-2)-B(N-2,Z-2)]\Big), \qquad (1)$$

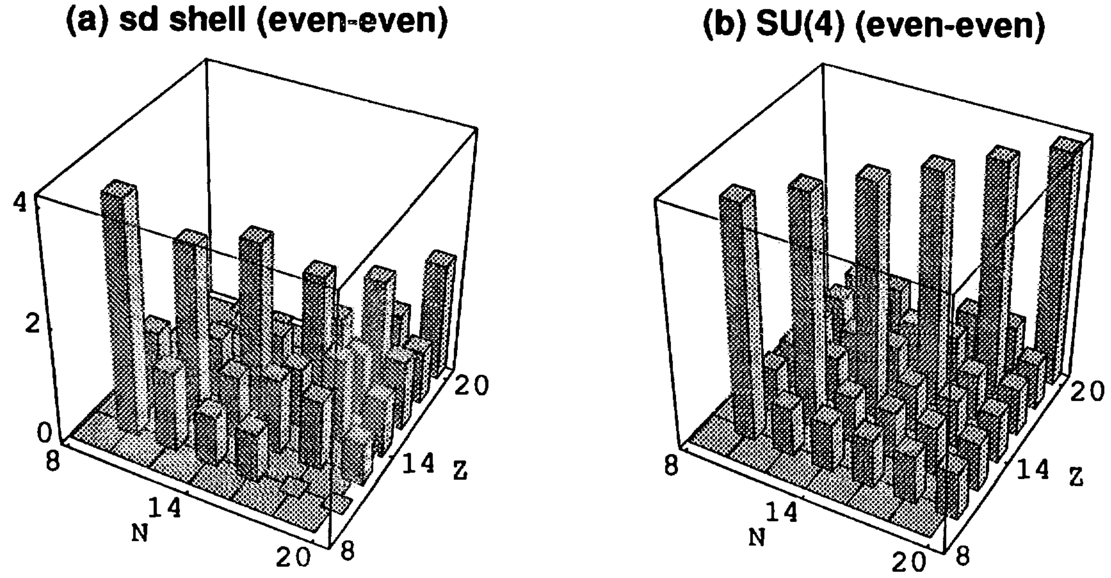

Figure 1: Barchart representation of double binding energy differences (a) as observed in even–even sd-shell nuclei and (b) as predicted by Wigner SU(4). The data are taken from [11]; an empty square indicates that data are lacking. The x and y coordinates of the centre of a cuboid define N and Z and its height z defines $-\delta V_{\rm np}(N, Z)$.

The spin-isospin or SU(4) symmetry is investigated.

where $B(N, Z)$ is the (negative) binding energy of an even-even nucleus with N neutrons and Z protons, was used by Brenner *et al.* [10] to extract the empirical interaction strength of the last neutron with the last proton. A notable outcome of this analysis was the occurrence of particularly large interaction strengths for $N = Z$ nuclei. Although this feature is consistent with both schematic and realistic shell-model calculations [10], a simple interpretation of this result is still lacking. We have shown that the $N = Z$ enhancements of $|\delta V_{\rm np}|$ are an unavoidable consequence of Wigner's SU(4) symmetry and that the degree of the enhancement provides a sensitive test of the quality of the symmetry itself.

A representative sample of the data is shown in Fig. 1(a) which gives $-\delta V_{\rm np}(N, Z)$ (where known) for the *sd* shell.

While for $N \neq Z$ the np interaction strength is roughly constant and of the order of -1 MeV, the dramatic enhancement of $|\delta V_{\rm np}|$ occuring for $N = Z$ is clearly evident. This prominent feature can be understood from the simple perspective of Wigner's supermultiplet theory. Wigner's scheme in a harmonic-oscillator shell with degeneracy $\omega = \sum(2l+1)$ implies the classification

$$\mathrm{U}(4\omega) \supset \left(\mathrm{U}_{\rm orb}(\omega) \supset \cdots \supset \mathrm{O}_{\rm orb}(3)\right) \otimes \left(\mathrm{U}_{ST}(4) \supset \mathrm{SU}_{ST}(4) \supset \mathrm{SU}_S(2) \otimes \mathrm{SU}_T(2)\right). \quad (2)$$

The dots refer to an appropriate labelling scheme for the orbital part of the fermion wavefunction, such as Elliott's SU(3) scheme [12]. The total M-fermion wavefunction transforms antisymmetrically under $\mathrm{U}(4\omega)$ and is decomposed into an orbital part, behaving as $[M_1, M_2, M_3, M_4]$ under $\mathrm{U}_{\rm orb}(\omega)$, and a spin-isospin part. To ensure overall antisymmetry the latter by necessity transforms under $\mathrm{U}_{ST}(4)$ as the conjugate representation $[\widetilde{M}_1, \widetilde{M}_2, \widetilde{M}_3, \widetilde{M}_4]$ (i.e., rows and columns of the Young tableau interchanged) and determines the supermultiplet $SU_{ST}(4)$ representation $(\lambda\mu\nu)$ ($\lambda = \widetilde{M}_1 - \widetilde{M}_2$, $\mu = \widetilde{M}_2 - \widetilde{M}_3$, and $\nu = \widetilde{M}_3 - \widetilde{M}_4$). From the $SU_{ST}(4) \supset SU_S(2) \otimes SU_T(2)$ reduction the possible values of S and T follow.

The short-range character of the residual nuclear interaction favors maximal spatial overlap between the fermions which is achieved in the most symmetric $U_{orb}(\omega)$ representation. Antisymmetry of the overall wave function then requires the least symmetric $U_{ST}(4)$ representation or, equivalently, the one where the eigenvalue of the quadratic Casimir operator of $SU_{ST}(4)$,

$$g(\lambda\mu\nu) = 3\lambda(\lambda+4) + 3\nu(\nu+4) + 4\mu(\mu+4) + 4\mu(\lambda+\nu) + 2\lambda\nu, \tag{3}$$

is minimal.

For even-even nuclei the favored SU(4) representation is $(0T0)$, where T is the isospin of the ground state. In lowest order (i.e., assuming unbroken SU(4) symmetry and neglecting orbital contributions) the binding energy is then $a + bg(0T0)$ with b positive. The coefficients a and b depend smoothly on mass number [6]. Assuming constant coefficients for the four nuclei in (1), a simple expression is found for δV_{np} that depends on b only. (In fact, the analysis presented below remains valid if a and b depend *linearly* on mass number.) The result is

$$\delta V_{np}(N,Z)/b = \begin{cases} \frac{1}{4}[g(000) - g(010) - g(010) + g(000)] = -10, & N = Z \\ \frac{1}{4}[g(0T0) - g(0,T-1,0) - g(0,T+1,0) + g(0T0)] = -2, & N \neq Z \end{cases}. \tag{4}$$

Wigner's supermultiplet theory in its *simplest* form (i.e., without symmetry breaking—dynamical or otherwise—in spin and/or isospin) therefore predicts $|\delta V_{np}|$ to be five times bigger for $N = Z$ than for $N \neq Z$. This result is displayed in Fig. 1(b).

Odd-mass nuclei can be treated in an identical way and give rise to similar conclusions [8].

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