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# Need for Higher Order Polynomial Basis for Polynomial Nodal Methods Employed in LWR Calculations*

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# NEED FOR HIGHER ORDER POLYNOMIAL BASIS FOR POLYNOMIAL NODAL METHODS EMPLOYED IN LWR CALCULATIONS

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Advanced nodal method codes are routinely used in the nuclear industry for calculating LWR core power distributions and reactivities. Comprehensive numerical testing of various nodal schemes have demonstrated their accuracy for typical LWR applications.[1,2] Results of recent tests of the ANL nodal codes DIF3D-NODAL[3,4] and VARIANT[5] indicated, however, that the fourth-order polynomial expansion basis employed for Cartesian geometry calculations may not produce sufficiently accurate solutions for problems involving large flux tilts and local flux gradients if one radial node per assembly is employed in the calculations. This fourth order polynomial representation of the intranodal flux is widely used in polynomial nodal methods.[6,7] Of course, accurate solutions can be obtained in Cartesian geometry applications by refining the nodal mesh structure. This approach however is inconsistent with the objective of reducing the execution time of 3-D reactor simulations, e.g. for real-time dynamic applications in which supernodal[8] and more efficient numerical approaches are being explored.

In order to produce accurate nodal solutions for the range of problems of interest, we have found it necessary to use a higher order expansion basis. For this reason, the highest order of the polynomial used in VARIANT for Cartesian calculations, has been increased to sixth order (same as originally used for Hex-Z geometry[5,9]), to ensure that accurate solutions are obtained when one radial node per assembly is employed. Since this higher polynomial order requires only slightly more computational time, there is substantial saving in computational time over that required for a calculation using four or more nodes per assembly. The observed need for a higher order polynomial representation of the intranodal flux (to capture the severe thermal flux gradients near MOX-$UO_2$ assembly interfaces) similarly motivated the adoption of the semi-analytic approach as an option in the SIMULATE-3 code.[10]

To illustrate the accuracy and efficiency effects of adopting sixth order expansions, we present results for a hot zero power PWR benchmark problem in which a control assembly (CA) along one of the core axes is assumed out of the core while the other CAs in the rod bank are in the core. This static problem was developed from the NEACRP 3-D PWR Core Transient Benchmark case C1,[11] and corresponds to the final asymptotic state of the ejected rod configuration. The power distribution for this final state indicates a severe flux and power tilt with a large fraction of the power in the core half containing the ejected rod. Of the 157 assemblies in the core, only 41 assemblies have power greater than the average core power, and all these assemblies are in the sector containing the ejected rod. Results computed with the DIF3D-NODAL, VARIANT and DIF3D-finite-difference codes have been compared to a reference solution generated with DIF3D-NODAL employing 64 radial nodes per assembly. Results are summarized in Table 1. The VARIANT diffusion solutions are displayed for various expansions of the within-node flux and source, and node-surface currents. While VARIANT is capable of providing transport solutions, we have only presented the diffusion theory equivalent results; transport effects on $k_{eff}$ and maximum assembly power are about 0.12% and -1.12% respectively.

The VARIANT results indicate that 2$^{nd}$ order expansion of the within-node source and linear representation of the node-surface currents, are adequate for this problem. The results also demonstrate the improvement in VARIANT solution when the order of the polynomial expansion of the within-node flux is increased from 4$^{th}$ to 6$^{th}$ order (V-241 versus V-261 solutions). For example, the error in $k_{eff}$ is reduced from -0.07% to -0.01%, and the maximum error in assembly power is decreased from 5.42% to 1.78%. The V-261 solution is also more accurate than the DIF3D-FD solution employing 256 planar cells per assembly, which requires over a factor of 100 more computational time. The total computational time for the 6$^{th}$ order expansion case (V-261) is actually less than that for the 4$^{th}$ order expansion case (V-241) because the number of outer iterations is greater in the latter case. Note from the results that the VARIANT 4th order polynomial solution is more accurate than the one-radial-node per assembly DIF3D-NODAL solution, which uses a fourth order expansion of the transversely integrated flux and a second order representation of the

transverse leakage term. It is likely that the DIF3D-NODAL solution accuracy would also benefit from the application of a higher order flux expansion, but this extension has not yet been performed.

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## Table 1. $k_{eff}$ and Assembly-Power Errors for the Ejected Rod State of NEACRP LWR Benchmark Case C-1

| Method | $\varepsilon_k$ (%)[a] | $\varepsilon_{p,max}$ (%)[a] | $\varepsilon_{avg,p}$ (%)[a] | $\varepsilon_{peak}$ (%)[a] | Total cpu Time (s)[b] |
|---|---|---|---|---|---|
| **DIF3D-Nodal**[c] | | | | | |
| 1 node/assembly | 0.20 | -10.41 | 4.47 | 2.82 | 21.85 |
| 4 nodes/assembly | 0.05 | -3.16 | 1.37 | 0.68 | 72.00 |
| 16 nodes/assembly | 0.02 | -0.53 | 0.22 | 0.10 | 379.62 |
| **VARIANT**[d] | | | | | |
| V-241, 1 node/assembly | -0.07 | 5.42 | 2.17 | -0.93 | 82.17 |
| V-441, 1 node/assembly | -0.05 | 4.43 | 1.75 | -0.91 | 151.48 |
| V-442, 1 node/assembly | -0.08 | 5.26 | 2.49 | -1.22 | 261.87 |
| V-261, 1 node/assembly | -0.01 | 1.78 | 0.54 | -0.10 | 79.78 |
| **DIF3D-FD** | | | | | |
| 16 cells/assembly | 0.44 | -17.45 | 8.04 | 5.03 | 124.09 |
| 64 cells/assembly | 0.20 | -8.73 | 3.87 | 2.36 | 1098.11 |
| 256 cells/assembly | 0.08 | -3.31 | 1.50 | 0.90 | 8883.97 |
| Extrapolated[e] | 0.04 | -1.53 | 0.71 | 0.41 | |

[a] $\varepsilon_k$, $\varepsilon_{p,max}$, $\varepsilon_{avg,p}$, and $\varepsilon_{peak}$ are the error in eigenvalue, maximum error in assembly power, average absolute error in assembly power, and error in the maximum assembly power, respectively, relative to the fine-mesh nodal reference.

[b] CPU time on the SUNSPARC-20 workstation, with 166 MHz processors.

[c] The DIF3D-NODAL solutions were obtained using a fourth order expansion of the transversely integrated flux and a second order representation of the transverse leakage term.

[d] The VARIANT diffusion theory solution designated as V-ijk employs polynomials of order i for the within-node source, j for the within node flux, and order k for the node-surface currents.

[e] Solution from extrapolation of 64 and 256 cells/assembly finite difference solutions.