IC/97/192

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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ABSTRACT

General dispersion relation for a self-gravitating magnetized and finite temperature dusty plasma has been derived using the Vlasov-kinetic theory in guiding center technique. Results of earlier studies in unmagnetized situations turn out to be special cases of our general dispersion relation. In addition to the usual dust-acoustic waves in unmagnetized plasmas, we find an ultra-low-frequency mode in the frequency range between cyclotron frequencies of ions and charged dust particles and the Jean's instability of the self-gravitating dusty plasma systems.

MIRAMARE – TRIESTE
November 1997

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I. Introduction

In many astrophysical dusty plasma systems, such as, interstellar spaces, circumstellar disks, dark molecular clouds, nebulae, etc., the charged dust grains are held under the combined influence of electromagnetic and gravitational forces [1–3]. In recent years, a number of studies [4–7] has been made on the instabilities and low-frequency modes in a self-gravitating dusty plasma without magnetic fields using the fluid model of plasmas. However, an ambient magnetic field is invariably present in astrophysical dusty plasma environments where the magnetic field may play a vital role in instabilities and mode modifications leading to dust coagulation forming structures. Therefore, a general framework is necessary for the analysis to include the magnetic field in the formulation of different modes and instabilities. Because of the large mass of the dust particles compared to the ion masses, an appropriate Vlasov-kinetic treatment for the finite-Larmor-radius (FLR) effects is necessary to find the possible modifications, new modes and instabilities. In this Letter, we study the characteristic behavior of a self-gravitating dusty plasma in the presence of a magnetic field by employing the Vlasov equation expressed in the guiding center approach [8,9].

II. General dispersion relation

We consider the propagation of a very low-frequency electrostatic mode $(\omega, \underline{k})$ in a three-component uniformly magnetized self-gravitating dusty plasma where constituents are the electrons, the ions and a significant amount of charged dust grains. At the equilibrium, the gravitational force on the grains is balanced by the gradient of the total plasma pressure and the plasma is assumed to be quasi-neutral, i.e., $en_{io} = en_{eo} + eZ_{do}n_{do}$, where $-e$ is the charge of an electron, Z_{do} is the number of electron charge residing on the dust grains, n_{eo}, n_{io} and n_{do} are the equilibrium number densities of the electrons, ions and the dust grains, respectively. Furthermore, the dust grains are assumed to be negatively charged point particles. The grain size and the intergrain distance are also assumed to be smaller than the Debye length in the three-component plasma. We assume that the characteristic wave frequency is much larger than the attachment/ionization or recombination frequency, so that the latter are not important. Thus, we can assume constant charge on dust grains without fluctuation which can lead to small additional damping of the waves under consideration.

The equation of motion for a charge particle in the presence of the fluctuating potential

$\phi(\omega, \underline{k})$ of the electrostatic wave and the gravitational potential $\psi(\omega, \underline{k})$ can be written as

$$\begin{aligned} \frac{d\underline{v}_\alpha}{dt} &= -\frac{q_\alpha}{m_\alpha}\underline{\nabla}\phi - \underline{\nabla}\psi - \underline{v}_\alpha \times \underline{\omega}_{c\alpha}, \\ &= -\frac{q_\alpha}{m_\alpha}\underline{\nabla}(\phi + \frac{m_\alpha}{q_\alpha}\psi) - \underline{v}_\alpha \times \underline{\omega}_{c\alpha}, \end{aligned} \quad (1)$$

where $\alpha = e, i, d$ and $\omega_{c\alpha} = q_\alpha B_s/m_\alpha c$; B_s is the ambient magnetic field assumed to be constant and uniform, q_α, m_α are the charge and mass of the particle α and c is the velocity of light in a vacuum.

Now, the guiding center method of solving the Vlasov equation for forces other than the electromagnetic type in a plasma was formulated before [8]. Using this modified approach and following Liu and Tripathi [9], we obtain the density perturbation as

$$n_\alpha = -\frac{n_{o\alpha}q_\alpha(\phi + m_\alpha\psi/q_\alpha)}{T_\alpha}\left[1 + \frac{\omega}{k_\parallel v_\alpha}\sum_{n=-\infty}^{+\infty} Z\left(\frac{\omega - n\omega_{c\alpha}}{k_\parallel v_\alpha}\right) I_n(b_\alpha)\exp\left(-b_\alpha\right)\right], \quad (2)$$

where Z is the plasma distribution function of its argument, $\omega_{p\alpha}^2 = 4\pi q_\alpha^2 n_{\alpha o}/m_\alpha$, $v_\alpha^2 = 2T_\alpha/m_\alpha$, $b_\alpha = k_\perp^2 v_\alpha^2/2\omega_{c\alpha}^2$, I_n is the nth order modified Bessel function with its argument b_α and T_α is the temperature of the particle α measured in the units of the Boltzmann constant k_B.

The Poisson's equations satisfied by ϕ and ψ can be written as

$$\nabla^2\phi = -4\pi\sum_\alpha q_\alpha n_\alpha, \quad (3)$$

$$\nabla^2\psi = 4\pi G\sum_\alpha m_\alpha n_\alpha, \quad (4)$$

where G is the gravitational constant.

Substituting Eq.(2) in Eqs.(3) and (4), we obtain

$$(1 + \sum \chi_\alpha) \quad \phi \quad + \sum_\alpha (m_\alpha\chi_\alpha/q_\alpha)\psi = 0, \quad (5)$$

$$(G\sum_\alpha m_\alpha\chi_\alpha/q_\alpha) \quad \phi \quad -(1 - \sum_\alpha \omega_{J\alpha}^2\chi_\alpha/\omega_{p\alpha}^2)\psi = 0, \quad (6)$$

where $\omega_{J\alpha}^2 = 4\pi G m_\alpha n_{\alpha o}$ is the Jeans frequency and χ_α is the susceptibility of particles α, given by

$$\chi_\alpha = \frac{2\omega_{p\alpha}^2}{k^2 v_\alpha^2}\left[1 + \frac{\omega}{k_\parallel v_\alpha}\sum_{n=-\infty}^{\infty} Z\left(\frac{\omega - n\omega_{c\alpha}}{k_\parallel v_\alpha}\right) I_n(b_\alpha)\exp\left(-b_\alpha\right)\right]. \quad (7)$$

Thus, keeping the gravitational effects of the ionic species, we obtain the generalized dispersion relation in a finite temperature dusty plasma as

$$(1 + \chi_e + \chi_i + \chi_d)\left(1 - \sum_j \frac{\omega_{Jj}^2}{\omega_{pj}^2}\chi_j\right) + G\left(\sum_j \frac{n_{oj}q_j\chi_j}{\omega_{pj}^2}\right)^2 = 0, \quad (8)$$

where j denotes ions and dust particles.

III. Analysis and the dust-lower-hybrid mode

We now analyze the general dispersion relation, Eq.(8) for various limiting cases studied earlier and the dispersion relation of a new ultra-low-frequency dust mode, may be called the dust-hybrid-lower mode in a self-gravitating magnetized dusty plasma :

(i) When $B_s = 0$, we obtain the dispersion relation, Eq.(7) of Verheest et al.[5] for an unmagnetized self-gravitating dusty plasma. We obtain the usual dust-acoustic wave when we consider $B_s = 0, G = 0$ and $kv_d \ll \omega \ll kv_i \ll kv_e$ [10]. We also obtain the dispersion relation in the above limits

$$1 - \frac{\omega_{ss}^2}{\omega^2} - \frac{\omega_{da}^2}{\omega^2 + \omega_{Jd}^2} = 0, \tag{9}$$

using the same notations of Avinash and Shukla [4], which is their Eq.(8) for the dust-acoustic waves in a self-gravitating dusty plasma to study the purely growing instability leading to the possible explanation for the strong condensation in many astrophysical situations including circumstellar disks, interplanetary dust, galaxies and planetary rings.

(ii) Considering the gravitational effect through dust grains only and neglecting those of electrons and ions [4,5] in the limit $\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce}$, $k_\parallel v_d \ll \omega$, $k_\perp v_d/\omega_{cd} > 1 > k_\perp v_i/\omega_{ci} > k_\perp v_e/\omega_{ce}$, the dispersion relation, Eq.(7) reduces to

$$\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{k_\perp^2}{k^2} - \frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2} + \frac{\omega_{pi}^2}{\omega_{ci}^2}\frac{k_\perp^2}{k^2} - \frac{\omega_{pi}^2}{\omega^2}\frac{k_\parallel^2}{k^2} - \frac{\omega_{pd}^2}{\omega^2}\right)\left(1 + \frac{\omega_{Jd}^2}{\omega^2}\right) + \frac{\omega_{Jd}^2\omega_{pd}^2}{\omega^4} = 0. \tag{10}$$

One solution of Eq.(10), in both the limits $\omega_{pi}^2 \gg \omega_{ci}^2$ and $\omega_{pi}^2 \ll \omega_{ci}^2$, is

$$\omega^2 \simeq -\omega_{Jd}^2, \tag{11}$$

which is a purely growing mode with frequency ω_{Jd}. The exact solution of Eq.(10) depends on the propagation vector k but it becomes a purely growing mode due to the above mentioned limits. This is due to the inherent presence of the magnetic field taken into account in the guiding center method. Although the magnitude of ω_{Jd} is small the instability can play an important role over a long period of time in the formation of structures in astrophysical dusty plasma environments.

The other solution of Eq.(10) in the limit $\omega_{pi}^2 \gg \omega_{ci}^2$, $k_\perp \gg k_\parallel$, $k_\parallel v_d \ll \omega$ and $k_\perp v_d \gg \omega_{cd}$ is a dust-lower-hybrid type mode given by

$$\omega^2 = \omega_{DLH}^2\left[1 + \frac{k_\parallel^2}{k^2}\frac{1}{Z_d^2}\frac{n_{eo}}{n_{do}}\frac{m_d}{m_e}\left(1 + \frac{n_{nio}}{n_{eo}}\frac{m_e}{m_i}\right)\right], \tag{12}$$

where

$$\omega_{DLH}^2 = \omega_{cd}\omega_{ci}\left(\frac{Z_d n_{do}}{n_{io}}\right)\left(1 + \frac{n_{eo}}{n_{io}}\frac{m_e}{m_i}\right). \tag{13}$$

Equation (12) is the dispersion relation for the electrostatic mode in the frequency range $\omega_{cd} < \omega < \omega_{ci} < \omega_{ce}$ and strong FLR thermal kinetic effects for the massive dust particles. We observe that for $k_{\parallel} = 0$ the mode becomes a static vibration and the frequency becomes directly proportional to $\sqrt{\omega_{ci}\omega_{cd}}$. Therefore, this mode may be called a dust-lower-hybrid mode. We recover the same dust-lower-hybrid frequency, Eq.(3) of Ref.[11] for transverse propagation ($k_{\parallel} \approx 0$) in a cold dusty plasma. For exact transverse propagation with $\omega_{cd} \leq \omega \ll \omega_{ci}$ and $k_{\perp}v_d/\omega_{cd} \geq 1 > k_{\perp}v_i/\omega_{ci}$, the low- frequency mode is given by Eq.(4) of Ref.[11]

With the inclusion of the dust charge fluctuations and following Jana et al.[12], Mahanta et al.[13] have studied the low-frequency electrostatic lower-hybrid-like mode having $kv_{\alpha}, \omega_{ci}, \omega_{cd} \ll \omega \ll \omega_{ce}$ and obtained the modified solution (Eq.(10) of their paper). This was already studied by Shukla[14] without considering charge fluctuations which only lead to the novel mechanism of damping of the mode.[15–19] However, in our study we consider still lower frequency regime ($\omega_{cd} \ll \omega \ll \omega_{ci}$) and obtain the dust-lower-hybrid mode (Eq.(12)) for constant charge on the dust grains. It is anticipated that the dust-charge-fluctuations would lead to an additional damping which is beyond the scope of the present paper.

IV. Discussion

We have studied the general dispersion properties of a self-gravitating dusty plasma in the presence of an ambient magnetic field and finite temperature of the plasma. Our dispersion relation, Eq.(8) reduces to those for the limiting cases studied earlier [4-7]. We find a purely growing mode at the Jeans frequency due to the gravitational effects of the dusty plasma. The second solution of our dispersion relation becomes independent of the purely growing instability and may be called a dust-lower-hybrid mode in the magnetized dusty plasma.

We consider the case when $\omega_{cd} \ll \omega \ll \omega_{ci}$, which was not taken into account in the earlier study by Shukla [14]. In this frequency range, we see that the resulting dispersion relation, Eq.(11) is a hybrid mode, which can propagate almost perpendicular to the external magnetic field. For exactly perpendicular propagation ($k_{\parallel} = 0$), we see from the dispersion relation, Eq.(11) of this mode that the frequency of the mode becomes a constant and it is a static vibration at the ion-dust hybrid frequency $\omega \sim \sqrt{\omega_{cd}\omega_{ci}}$.

These modes may cause enhanced low frequency electrostatic noise from the dust-plasma environments. In the process of dust coagulation and crystallization, the resonant interaction of dust grains and these modes may provide a new mechanism of dust attraction causing the formation of structures.

In the present investigation, the dust grains were taken as a third-component of the homogeneous dusty plasma having constant charge and mass. For simplicity, we have assumed that the charge on the dust grains is not affected by the waves. The variation of the dust charges can lead to additional damping known as the Tromsødamping[15–19] apart from the collisionless Landau damping of the low-frequency electrostatic mode studied here.

As the dust wave frequency is much smaller than the cyclotron frequencies of electrons and ions, the Landau damping of this mode on electrons and ions would be negligible.[20] Moreover, since $\omega > \omega_{cd}$ and $k_{\|} v_d \ll \omega$, the collisionless Landau damping of the mode ($\propto \exp\left(-\omega^2/k_{\|}^2 v_d^2\right)$) to the dust particles is also negligible, consequently, the dust-lower-hybrid mode is a natural mode of the magnetized dusty plasma.

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The low-frequency modes studied here can have many applications in space and astrophysics as well as in laboratory experiments for Coulomb-dust crystallization and dust-coagulation in magnetized dusty plasmas. Various collective effects including dust Coulomb crystallization and the parametric mode coupling interactions through these ultra-low-frequency modes in magnetized dusty plasmas will be an important field of research in the future and the work in these lines is in progress.

Acknowledgments:

This work was done within the framework of the Associateship Scheme of the International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is acknowledged. The authors would like to thank the ICTP for hospitality. The authors are also grateful to Dr. N.N. Rao for fruitful discussions.

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