SKI Report 97:10

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Esko Lehtinen
Urho Pulkkinen
Kurt Pörn

April 1997

ISSN 1104-1374
ISRN SKI-R--97/10--SE

STATENS KÄRNKRAFTINSPEKTION
Swedish Nuclear Power Inspectorate

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Statistical Trend Analysis Methods for Temporal Phenomena

Esko Lehtinen [1]
Urho Pulkkinen [1]
Kurt Pörn [2]

[1] VTT Automation, Industrial Automation,
P.O. Box 1301, FIN-02044 VTT, Finland

[2] Pörn Consulting,
Skivlingvägen 24, SE-611 63 Nyköping, Sweden

April 1997

SKI Project Number 95242

Summary

We consider point events occuring in a random way in time. In many applications the pattern of occurrence is of intrinsic interest as indicating, for example, a trend or some other systematic feature in the rate of occurrence. The purpose of this report is to survey briefly different statistical trend analysis methods and illustrate their applicability to temporal phenomena, in particular.

Before any analytical methods are treated some simple graphical methods are considered. Graphical methods are important e.g. in finding the grosser features of the data and also checking on the assumptions on which the more formal methods of analysis are based. Graphical displays are always helpful if data is wanted to be presented.

The first hypothesis usually is that events are occurring randomly in time. A well-known mathematical model of a completely random series of events is the Poisson prosess. When the intensity with which the events occur in time is constant, there is no increasing or decreasing trend in the event occurences, and the Poisson prosess is said to be a time homogeneous Poisson prosess. We shall deal, however, with a generalization, the non-homogenous Poisson process (NHPP), in the case of which the intensity of the occurence of events is a function of time.

The trend testing of point events is usually seen as the testing of the hypotheses concerning the intensity of the occurrence of events. When the intensity function is parametrized, the testing of trend is a typical parametric testing problem.

The Laplace test is orginally developed as a parametric test for the NHPP obeying a certain intensity function. There are models, for which the Laplace test has rather good statistical properties. From a practical point of view, the Laplace test provides a good indication of the existence of a trend. Together with graphical presentations, the Laplace trend statistics give rather a clear picture about the possible trends. The use of Laplace statistics is practically recommendable to analyse the point phenomena. However, if the phenomenon is not properly described by a point process model, one must not use the Laplace test.

In industrial applications the operational experience generally does not suggest any specified model and method in advance. Therefore, and particularly, if the Poisson-process assumption is very questionable, it is desirable to apply tests that are valid for a wide variety of possible processes. The alternative approach for trend testing is to use some non-parametric procedure. In this report we have presented four non-parametric tests: The Cox-Stuart test (a modification of the sign test), the Wilcoxon signed ranks test, the Mann test, and the exponential ordered scores test.

It is obvious that the fewer or weaker are the assumptions that define a particular model, the less qualifying we need to do our decision arrived at by the statistical test associated with that model. That is, the fewer or weaker are the assumptions, the more general are the conclusions. However, the most powerful tests are those which have the

strongest or most extensive assumptions. The parametric tests have a variety of strong assumptions underlying their use. When we have reason to believe that the conditions for a parametric test are met in the data under analysis, then we should certainly choose a parametric statistical test for analyzing those data. If these conditions are not met, some relevant nonparametric test can be used.

In addition to the classical parametric and non-parametric approaches we have also considered the Bayesian trend analysis. First we discuss a Bayesian model, which is based on a power law intensity model. The Bayesian statistical inferences are based on the analysis of the posterior distribution of the trend parameters, and the probability of trend is immediately seen from these distributions. In principle, it is possible to apply non-parametric Bayesian models.

We applied some of the methods discussed in this report in an example case. The results were not contradictory, and every model detected the trend that was assumed in the Monte Carlo generation of the example data. It is to be noted that this report is a feasibility study rather than a scientific evaluation of the statistical methodologies, and the example analyses can be seen only demonstrations of the methods. Furthermore, there is a lot of other statistical methods relevant to analysis of phenomena varying along with time which have not been considered in this report.

This study was started within the scope of the Nordic joint project NKS / SIK-1 on the initiative of Ralph Nyman, SKI / RA, and has been conducted with financial support from SKI, the Swedish Nuclear Power Inspectorate, which is hereby gratefully acknowledged.

Sammanfattning

Vi betraktar diskreta händelser som inträffar slumpmässigt i tiden. I många tillämpningar finns dock ett mönster i händelserna som är av stort intresse genom att det återger exempelvis en trend eller något annat systematiskt drag i händelsefrekvensen. Avsikten med denna rapport är att översiktligt kartlägga olika metoder för statistisk trendanalys och att illustrera deras tillämplighet speciellt på tidsberoende fenomen.

Kartläggningen börjar med några enkla grafiska metoder. Grafiska metoder är särskilt användbara när det gäller att identifiera grova datastrukturer och att testa de antaganden som ligger till grund för tänkbara analytiska metoder. Dessutom kommer grafiska metoder alltid väl till pass när data skall presenteras.

Ett mycket vanligt förekommande antagande är att de händelser man betraktar inträffar helt slumpmässigt i tiden. Den s.k. Poissonprocessen är en välkänd matematisk modell för en fullständigt slumpmässig serie av händelser. Om intensiteten i processen är konstant förefinns ingen ökande eller avtagande trend i händelsernas inträffande, och processen säges vara en homogen Poissonprocess. Här skall vi emellertid betrakta en utvidgad klass av processer, s.k. inhomogena Poissonprocesser (NHPP), för vilka händelseintensiteten är en funktion av tiden.

Att testa trenden i en punktprocess är vanligtvis liktydigt med att testa hypoteser rörande intensiteten för händelsernas inträffande. När intensiteten beskrivs av en parametrisk funktion är trendtestet ett typiskt parametriskt testproblem.

Det s.k. Laplace-testet utvecklades ursprungligen som ett parametriskt test av NHPP med en viss typ av intensitetsfunktion. Det finns modeller för vilka Laplace-testet har ganska goda statistiska egenskaper. Sett ur praktisk synpunkt ger Laplace-testet en god indikation på förekomsten av trend. Tillsammans med grafiska presentationer ger Laplaces trendstatistika en god bild av tänkbara trender. Användning av Laplace-testet rekommenderas för analys av punktprocesser. Om det aktuella fenomenet inte låter sig adekvat beskrivas av någon modell för punktprocesser bör man inte använda Laplace-testet.

Drifterfarenheter från industriella tillämpningar ger vanligtvis inte någon direkt fingervisning om lämplig modell och metod. Därför, och i synnerhet om antagandet om Poissonprocess är diskutabelt, är det önskvärt att använda tester som är tillämpliga på en vid klass av tänkbara processer. Ett alternativt trendtest är att använda någon icke-parametrisk procedur. I denna rapport presenteras fyra icke-parametriska tester: Cox-Stuart's test (modifierat teckentest), Wilcoxon's teckenrang test, Mann's test och det exponentiella ordnade indextestet (exponential ordered score test).

Det är uppenbart att ju färre eller svagare antaganden som ligger till grund för en modell desto mindre förbehållsamma resultat kan uppnås med det statistiska test som är förenat med modellen. Annorlunda uttryckt, ju färre eller svagare antaganden, desto mera generella slutsatser. Å andra sidan, de starkaste testen är sådana som bygger på de

starkaste eller mest omfattande antagandena. Användningen av parametriska test vilar på en mängd antaganden. När vi har skäl att tro att analysdata uppfyller förutsättningarna för ett parametriskt test kan vi med säkerhet använda ett sådant test för analys av dessa data. Om sådana förutsättningar ej föreligger kan man pröva något icke-parametriskt test.

Utöver de klassiska, parametriska och icke-parametriska tillvägagångssätten har vi också betraktat Bayesiansk trendanalys. Bl.a. behandlar vi en Bayesiansk modell som bygger på den s.k. potensprocessen, en NHPP vars intensitet är en potens av tiden. Bayesiansk statistisk inferens utgår ifrån analys av posteriorifördelningen för trendparametrarna, och sannolikheten för trend kan direkt utläsas ur denna fördelning. I princip är det också möjligt att tillämpa icke-parametriska Bayesianska modeller.

Några av de metoder som diskuteras i rapporten har tillämpats på testfall. De resultat som erhållits är inte motstridiga, och varje modell upptäckte den trend som antogs vid Monte Carlo-genereringen av testdata. Det bör understrykas att denna rapport är en användbarhetsstudie snarare än en vetenskaplig utvärdering av statistiska metoder, och studiens analysexempel bör ses enbart som en demonstration av metoderna. Vidare är vi väl medvetna om att det finns många andra statistiska metoder som är relevanta för analys av tidsberoende fenomen och som vi inte beaktat i denna studie.

Studien påbörjades inom ramen för det nordiska samarbetsprojektet NKS/SIK-1 på initiativ av Ralph Nyman, SKI / RA, och har utförts med finansiellt stöd av SKI, vilket härmed tacksamt noteras.

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1 Introduction

We consider point events occuring in a random way in time. That is, we have a continuum, usually but not nesessarily one-dimensional, and a series of points distributed haphazardly along it. For example, the events may be stops of a machine, as measured in the running-time of the machine, or restorings or replacements of equipment, etc.

In many applications the main interest lies in obtaining an estimate of the mean rate of occurence of events, for example: how many stops per running hour. In other applications the pattern of occurrence is of intrinsic interest as indicating, for example, a trend or some other systematic feature in the rate of occurrence. We shall be concerned mainly with the latter type of application.

The purpose of this report is to survey briefly different statistical trend analysis methods and to illustrate their applicability to temporal phenomena, in particular. Simple graphical methods of presentation will also be considered. Graphical methods are important both in finding the gross features of the data and also checking the assumptions on which the more formal methods of analysis are based.

We denote the observed intervals between successive events by x_1, x_2, ... The series would be given equivalently by the instants of occurrence of events measured from the start of period of observation. These times are obtained by forming cumulative sums of the x_r's, i.e. $t_1=x_1$, $t_2=t_1+x_2,\ldots$, $t_r=t_{r-1}+x_r$, where t_r is the time of occurrence of the rth event.

2 Graphical presentation

It will quite often be required to present this type of data graphically, either as a preliminary to a more detailed analysis or in order to have a simple record which can be brought up to date as fresh information is obtained. When the main interest is in changes in the average rate of occurrence of events, there are two methods of graphical presentation, one based on cumulative numbers and the other based on individual numbers of occurrences.

The simplest cumulative plot is the total number of events to have occurred at or before t, against t. At each event, the plot jumps one step upwards. An important property is that the slope of the line joining any two points on the plot is the average number of events per unit time for that period. It reveals how the average rate of occurrences fluctuates with t.

To obtain a non-cumulative plot we take time as the abscissa and then divide the time scale into convenient equally spaced time periods and count the number of events in each period.

Examples of a cumulative and a non-cumulative plot are given in Figures 1 and 2.

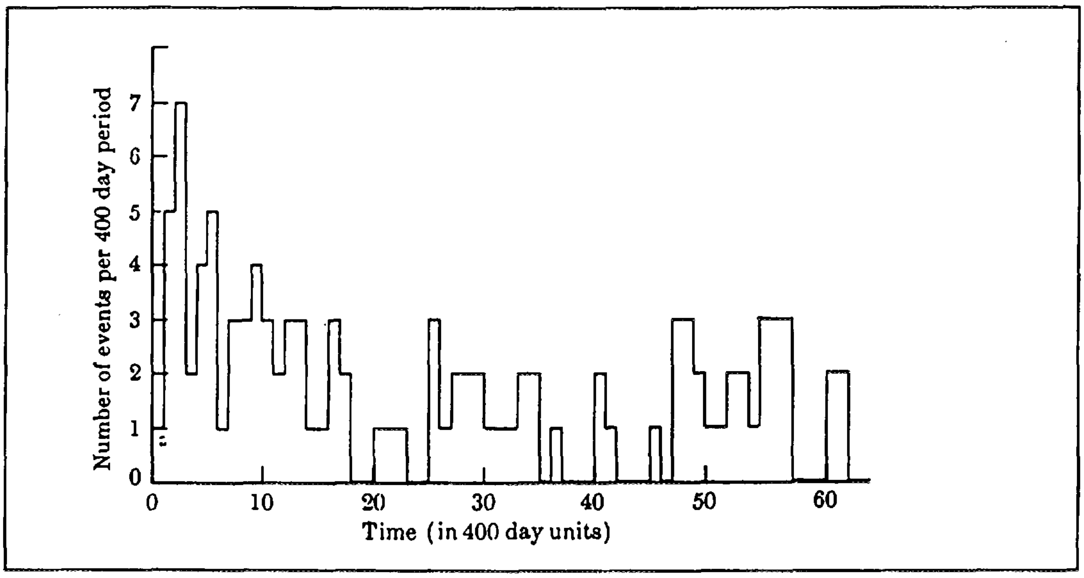

Figure 1: *A non-cumulative plot of a series of events (Coal-mining disasters. Numbers in successive 400-day periods, taken from (Cox and Lewis, 1968)).*

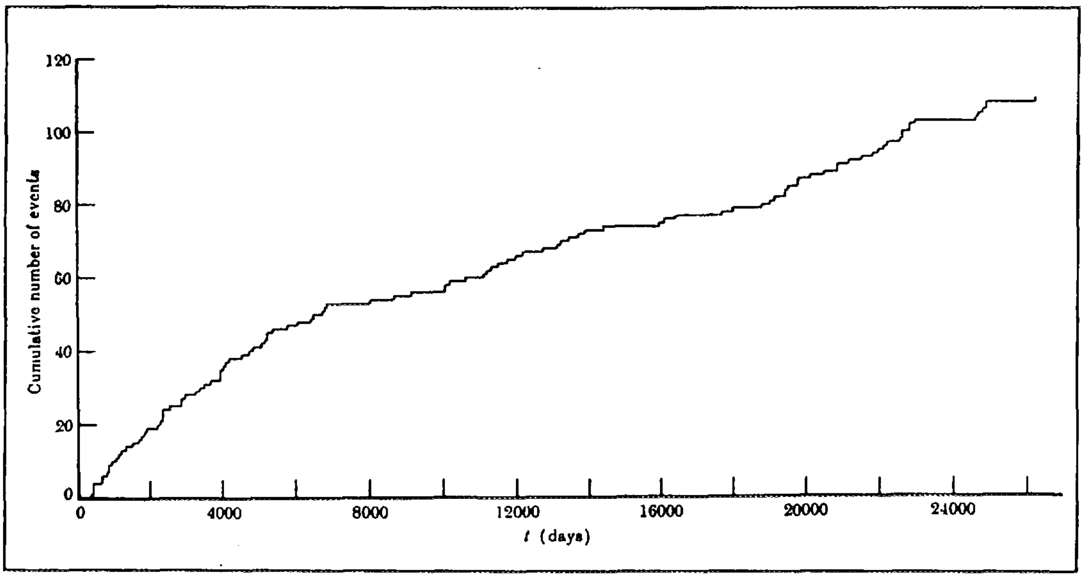

Figure 2: *A cumulative plot of a series of events (Coal-mining disasters. Cumulative number versus t, taken from (Cox and Lewis, 1968)).*

An advantage of the cumulative plot is that it enables small systematic changes in the rate of occurrence to be noticed readily. Advantages of the non-cumulative plot are that the local fluctuations are directly indicated.

3 Point process models and tests

3.1 Homogeneous Poisson process

In applications the first hypothesis usually is that events are occurring completely randomly in time. Thus, if we wish to test, for instance, the reality of the apparent trend in the failure rate, we take a null hypothesis that the failures occur randomly at constant rate. Such a hypothesis can be considered for two reasons. First, before we can have confidence in the reality of an apparent systematic effect in the series, we need to show that the effect is unlikely to have arisen just by chance. Secondly, simple methods for the comparison of the rates of occurrence in different series are available whenever the individual series can be assumed to be comletely random.

As a mathematical model of a completely random series of events we consider the Poisson prosess. The definition and main probabilistic properties of the Poisson process are well-known and are discussed in the literature, for example by Snyder (1976).

Consider events occurring along a line which for definiteness will be called the time axis. Let N_t denote the number of events occurring in an arbitary interval of length t. Then N_t has a Poisson distribution of mean λt, i.e.

$$Pr(N_t = r) = \frac{(\lambda t)^r e^{-\lambda t}}{r!} \quad (1)$$

where λ is a constant with the dimension of the reciprocal of time. It will measure the mean rate of occurence of events over a long period of time and will be called the rate of occurrence or more fully the probability rate of occurrence of events (N_t/t converges in probability to λ as $t \to \infty$, justifying the name rate of occurrence given for the parameter λ of the Poisson process (see Snyder, 1976)).

We denote $\mu = \lambda t$. As μ increases, the Poisson distribution is asymptotically normal with mean and variance μ. For many problems connected with significance tests and confidence intervals, the normal approximation is quite accurate enough even below $\mu = 10$. The main source of error in the approximation arises from the skewness of the Poisson distribution.

A second important group of properties of the Poisson process concern the intervals between events. Let X denote the interval from the time origin to the first event. Using (1) we may show that the probability distribution of X is exponential, with cumulative distribution and the density function

$$F_X(x) = 1 - e^{-\lambda x} \quad (2)$$

and

$$f_X(x) = \lambda e^{-\lambda x}. \tag{3}$$

A very important point is that because the occurrences in any section of a Poisson process are independent of the preceding sections of the process, the origin from which X is measured may be defined in variety of ways. Thus X may be

(a) the time from original time origin to the first event;
(b) the time from any fixed time point to the next event;
(c) the time from any event to the next succeeding event, i.e. the interval between successive events;
(d) the time from any point t' determined by the pattern of events in $(0,t']$ to next event.

Further if X_1, X_2,... are the intervals between the origin and the first event, between the first and second events, and so on, the random variables X_1, X_2,... are mutually independent and each with the probability density function $\lambda e^{-\lambda x}$ (3). In fact the Poisson process can be defined by this property (see Snyder, 1976).

As we can notice from the above, the intensity with which the points occur in time is constant. This kind of Poisson processes are called time homegeneous Poisson processes, and there is no increasing or decreasing trend in the point occurrences. In opposite to this, it is possible to define Poisson processes, the intensity of which is a function of time.

3.2 Non-homogenous Poisson process

It is possible to generalize the Poisson process in many ways, for example by considering a process in several dimensions. We shall deal, however, with only one generalization, the time-dependent or non-homogenous Poisson process (NHPP).

As before, it is still required that occurrences in different time periods are independent, but now the rate of occurrence of events is a function of time: $\lambda(t)$. This function is also called intensity function. The time-dependent Poisson process or NHPP can model many kinds of behaviour for the reliability. For example, a reliability growth (decay, stability) is equivalent to a NHPP with an intensity decay (growth, stability).

One of the main properties of the time-dependent Poisson process is that the number of events occurring in the time interval (s,t) has a Poisson distribution with mean

$$E\left[N_{(s,t)}\right] = \int_s^t \lambda(u)du \tag{4}$$

Further, $N_{(s,t)}$ follows the Poisson distribution

$$P(N_{(s,t)}=n) = \frac{\left(\int_s^t \lambda(u)du\right)^n e^{-\int_s^t \lambda(u)du}}{n!} \tag{5}$$

It is worth noticing that a non-homogeneous Poisson process may be used as a so called minimal repair model for failures of repairable components. According to the minimal repair model, the components are after a repair as good as old, i.e. the failure rate of the component after a repair has the same value as just before the repair. The usual assumption applied in reliability models is the "as good as new - model", in which it is assumed that the repair is comparable to the replacement of the component with a new one.

The choice of the intensity function determines the trend properties of a non-homogeneous Poisson process. Many models have been applied for various purposes. One of the most popular intensity functions is the simple *power law model*, the intensity function of which is

$$\lambda(t) = \alpha\beta t^{\beta-1}, \tag{6}$$

where $\alpha > 0$ and $\beta > 0$ are parameters. The above model is sometimes called *Crow model* or *Weibull process.*

The expected number of occurences within a time interval *(s,t)* is obviously

$$E[N_{(s,t)}] = \alpha(t^\beta - s^\beta) \tag{7}$$

We notice that if $\beta > 1$ then the intensity is increasing. We may easily modify (6)-(7) as

$$\lambda(t) = \alpha\beta t^{\beta-1} + \lambda_0, \tag{8}$$

and

$$E[N_{(s,t)}] = \alpha(t^\beta - s^\beta) + \lambda_0(t-s), \tag{9}$$

which is the model discussed also in section 5.

Another form of intensity often applied is

$$\lambda(t) = e^{\alpha+\beta t}, \tag{10}$$

which is increasing if $\beta > 0$. The *log-linear model* (10) is also known as *Goel-Okumoto model* (Gaudion, 1992).

It is very easy to postulate different intensity functions: one must only make sure that the intensity function is positive and nonexplosive (i.e. $\int_0^T \lambda(t)dt < \infty$, $\forall T < \infty$). The intensity functions may even be discontinuous. For most trend analysis purposes the above intensity models are usually sufficient.

3.3 Generalizations

The Poisson process is an archetype of counting processes. Other models may be derived as modifications from the Poisson process .

One group of new models is obtained by assuming that the intensity is a random variable. The most simple version of this model is a non-homogeneous Poisson process, the parameters of which are random variables. In this case the conditional probability distribution of the number of events within the interval *(s,t)* given the parameter θ is the Poisson distribution

$$P(N_{(s,t)}=n|\theta) = \frac{\left(\int_s^t \lambda(t',\theta)dt'\right)^n e^{-\int_0^T \lambda(t',\theta)dt'}}{n!}. \tag{11}$$

Since θ is a random vector, it is modelled by a probability distribution

$$P(\theta \in d\theta|\xi) = g(\theta|\xi)d\theta, \tag{12}$$

in which ξ is the parameter of the distribution $g(\cdot|\xi)$. Parameter ξ may be known or unknown, in the latter case the uncertainty is again modelled by a probability distribution.

We obtain the marginal distribution of the number of failures by integrating (over) the distribution of θ

$$\begin{aligned} P(N_{(s,t)}=n) &= \int_{\theta\in\Theta} P(N_{(s,t)}=n|\theta)g(\theta|\xi)d\theta \\ &= \int_{\theta\in\Theta} \frac{\left(\int_s^t \lambda(t',\theta)\,dt'\right)^n e^{-\int_s^t \lambda(t',\theta)dt'}}{n!} g(\theta|\xi)d\theta. \end{aligned} \tag{13}$$

The above model is applied in section 5 where we consider Bayesian methods for identifiction of trends. We may call the above model a mixed or doubly stochastic Poisson process. It is possible to generalize the notion of doubly stochastic Poisson process by assuming that the intensity function is a stochastic process. We shall discuss this case shortly in section 5.

Another type of modified Poisson process is the case where the intensity depends on the history of the process N_t. For example we may assume that the times between occurences are random variables, the distribution of which depends on the number of of events occurred earlier. By assumming that this distribution is exponential and that the parameter of the i'th interevent time depends geometrically on the number of earlier occurences, we may postulate the model

$$f_{T_i}(t|\theta) = (\theta_0)\theta_1^{i-1} e^{-\theta_0\theta_1^{i-1}t}, \tag{14}$$

in which θ_0 and θ_1 are positive parameters.

3.4 Laplace test for trend identification

The Laplace test is originally developed as a parametric test for a certain non-homogeneous Poisson process obeying the intensity function

$$\lambda(t|\alpha,\beta) = e^{\alpha+\beta t}, \tag{15}$$

in which α and β are real parameters (see Cox and Lewis, 1968 and Grow, 1992). We notice that if $\beta > 0$ (< 0) then the intensity is increasing (decreasing) which means that there is a trend.

The Laplace test statistic corresponding to the above intensity function can be defined for two cases: it is assumed that the observations on the process are stopped either at the time point τ or at the time point T_n, where n'th point occurs. If the observations are stopped at the time point τ then the test statistic has the form

$$U_\tau = \frac{S_n - \frac{1}{2}n\tau}{\sqrt{\frac{n\tau^2}{12}}} \tag{16}$$

where

$$S_n = \sum_{i=1}^{n} T_i \tag{17}$$

If the observation is stopped at event T_n the statistics is defined by

$$U_n = \frac{S_{n-1} - \frac{1}{2}(n-1)T_n}{\sqrt{\frac{(n-1)T_n^2}{12}}} \tag{18}$$

In practice the above versions of test statistics do not differ significantly. For large values of n the test statistic is approximately normally distributed, and the critical values of the statistic are defined accordingly. The approximation is adequate at the 5% significance level for $n>3$.

The U statistic is used with following hypotheses:

- no-trend vs intensity decline: reject the null hypothesis $\beta \geq 0$ at the significance level γ, if $u<l_\gamma$;
- no-trend vs intensity growth: reject the null hypothesis $\beta \leq 0$ at the significance level γ, if $u>m_\gamma$;
- no-trend vs intensity decline or intensity growth: reject the null hypothesis $\beta=0$ at the significance level γ, if $|u|>n_\gamma$.

For γ = 5%, the critical values are l_γ= -1.645, m_γ= +1.645, n_γ = 1.960.

The Laplace test is originally designed for the log-linear model (10). However, it is possible to modify the test to cover also other intensity models. A simple modification is suitable for the power law model

$$\lambda(t|\alpha,\beta) = \alpha\beta t^{\beta-1}, \tag{19}$$

in which the parameters α and β are positive, The intensity (19) looks like the above exponential intensity model modified by a logarithmic transformation. The growth of $\lambda(t)$ depends on the position of β relative to 1. "β = 1" corresponds to the homogeneous Poisson process.

The modification of the Laplaces test is made by defining the sum of log-failure times as

$$S_k^* = \sum_{i=1}^{k} \ln(T_i/\tau). \tag{20}$$

We notice that S^* is a logarithmic counterpart of S in equations (16)-(18). The modified Laplace test may be constructed for two cases, as earlier.

If the observations are stopped at time τ, then the modified Laplace test statistic is defined by

$$S_{\text{mod}} = -2S_n^* = -2\sum_{i=1}^{n} \ln\left(\frac{T_i}{\tau}\right) \tag{21}$$

If the observations are stopped at n'th event, then the Laplace test statistic has the form

$$S_{\text{mod}} = -2S^{*}_{n-1} = -2\sum_{i=1}^{n} \ln\left(\frac{T_i}{T_n}\right) \tag{22}$$

For large values of n, S_{mod} is approximately normally distributed, and the critical values of the statistic are defined accordingly. The approximation is adequate at the 5% significance level for $n>3$. Generally S_{mod} follows $\chi^2(2k)$ distribution and it can be used with following hypotheses:

- no-trend vs intensity decline: reject the null hypothesis $\beta\geq 1$ at the significance level γ, if $S_{mod}<\chi^2(2k;1-\gamma)$;
- no-trend vs intensity growth: reject the null hypothesis $\beta\leq 1$ at the significance level γ, if $S_{mod}>\chi^2(2k;\gamma)$;
- no-trend vs intensity decline or intensity growth: reject the null hypothesis $\beta=1$ at the significance level γ, if $S_{mod}<\chi^2(2k;1-\gamma/2)$ or $S_{mod}>\chi^2(2k;\gamma/2)$,

where

- $k=n$ in the case when n events are observed between $(0,\tau)$, τ given in advance (stop at time τ);
- $k=n-1$ in the case when the observation ends at n'th event (stop at event n).

From the theoretical point of view, the Laplace test is not completely satisfactory because neither its exact statistical significance level, nor its power are calculable. Further, its should be noticed that the Laplace test in its "standard form" (see equations (16)-(18)) is a parametric test for the intensity given by (15). The power and the exact statistical significance level depend on the functional form of the intensity function.

3.5 Practical aspects

General point process models discussed in this chapter are intented for description of phenomena which occur pointwise in time. Examples of this kind of phenomena are failures of components and transients at nuclear power plants. Point process models can also be used to describe some phenomena connected to continuous processes, e.g. the level crossing phenomena of cumulative processes.

An archetype of point process models is the homogeneous Poisson process, in which the events are assumed to occur with a constant intensity. As discussed above, the Poisson process model can easily be generalized.

The trend testing of point processes is usually seen as the testing of the hypotheses concerning the intensity of the process. When the intensity function is parametrized, the testing of trend is a typical parametric testing problem. Parametric tests can be developed for different intensity functions.

The Laplace test discussed above is originally a parametric test for a specific intensity function. In practice, the Laplace test is often used like a nonparametric trend test. According to Gaudoin (1992), there are models, for which the test has rather good statistical properties. Further, the difference between the options of different stopping rules of the observations is not practically significant.

From a practical point of view, the Laplace test provides a good indication of the existence of a trend. Together with graphical presentations, the Laplace trend statistics give a rather clear picture about the possible trends. The use of Laplace statistics is practically recommendable to analyse the point phenomena. However, if the phenomenon is not properly described by a point process model, one must not use the Laplace test.

In practice, very low values of the statistical significance level are not always actually used, but the test statistics would rather be applied to indicate a possible trend or to compare trends. Since the Laplace statistics have also theoretical foundations, it is very useful for this kind of use.

4 Non-parametric tests

In industrial applications the operational experience generally does not suggest any specified model and method in advance. Therefore, and particularly, if the Poisson-process assumption is very questionable, it is desirable to apply tests that are valid for a wide variety of possible processes. The alternative approach for trend testing is to use a non-parametric procedure.

The Cox-Stuart test (a modification of the sign test), *the Wilcoxon signed ranks test, the Mann test, the exponential ordered scores test, the run test,* and the tests based on *Kendall's* τ *or Spearman's* ρ are examples of non-parametric trend tests, which may be used to study the existence of trend in a series of subsequent observations. We shall shortly present the first four ones mentioned above.

In following we assume that $X_1, X_2, \dots, X_n$ is a sample of mutually independent random variables. The basic task is to verify the hypothesis H_0 against an alternative hypothesis H_1:

H_0: there is not any trend;
H_1: there is an increasing (decreasing) trend.

4.1 Cox-Stuart test

Assume that the data consist of observations on a sequence of mutually independent random variables $X_1, X_2, \dots, X_n$, arranged in the order in which the random variables are observed. The measurement scale of the X_i's is at least ordinal. The X_i's are either identically distributed or there is a trend.

The Cox-Stuart test can be used to detect any specified type of nonrandom pattern, such as a sine wave or other periodic pattern. The idea of the Cox-Stuart test is based on the comparison of the first and the second half of the sample. If there is a downward trend the observations in the second half of the sample should be smaller than in the first half. If they are greater, the presence of an upward trend is suspected. If there is not any trend one should expect only small differences between the first and the second half of the sample due to randomness.

Thus, to perform a trend analysis, the sample of differences is to be calculated: $y_1=x_{1+c}-x_1$, $y_2=x_{2+c}-x_2$, ... , $y_c=x_n-x_{n-c}$, where $c=n/2$, if n is even, $c=(n+1)/2$, if n is odd. The differences equal to zero are not taken into account. For simplicity, let us denote the sample of positive differences by y_1, ... , y_m.

The Cox-Stuart test is a sign test applied to the sample of non-zero differences y_1, ... , y_m. Let $sgn(a)=1$, if $a>0$ and $sgn(a)=-1$, if $a<0$. The test statistic of the Cox-Stuart test is

$$T=\sum_{i=1}^{m} sgn(y_i) \tag{23}$$

Decision rule: At the significance level α, reject the hypothesis H_0 and accept the alternative hypothesis H_1, if $T>t(\alpha)$ (increasing trend), if $T<t(\alpha)$ (decreasing trend), where $t(\alpha)$ is the proper quantile of the binomial distribution. For $m>20$, an approximation

$$t(\alpha) = \frac{1}{2}[m + w(\alpha)]\sqrt{m} \tag{24}$$

where $w(\alpha)$ is the α-quantile of the standard normal distribution, can be applied.

4.2 Wilcoxon signed ranks test

To use the Wilcoxon signed ranks test it is necessary to compute ranks R_1, R_2, ... , R_m for absolute values of differences $|y_1|$, $|y_2|$, ... , $|y_m|$, i.e.

$$R_i = \#\{j \mid |y_j| \le |y_i|,\ j=1,2,...,m\},\ i=1,2,...,m, \tag{25}$$

where $\#\{A\}$ denotes the number of elements in set A. If there are equal values of $|y_i|$, $i = 1, 2, ... , m$, i.e., there are ties in the sample, the average ranks shall be calculated for them.

The test statistics is

$$T = \frac{\sum_{i=1}^{m} sgn(y_i) R_i}{\sqrt{(\sum_{i=1}^{m} R_i^2)}}. \qquad (26)$$

For the case of no ties it is more convenient to use only the positive signed ranks:

$$T^{+} = \sum_{i=1}^{m} R_i \cdot I(y_i), \qquad (27)$$

in which $I(y_i) = 1$, if $y_i > 0$ and $I(y_i) = 0$, otherwise.

<u>Decision rule</u>: At the significance level α, reject the hypothesis H_0 and accept the alternative hypothesis H_1, if $T^{+} > w_{1-\alpha}$ (or $T > w_{1-\alpha}$) for increasing trend, if $T^{+} < w_{\alpha}$ ($T < w_{\alpha}$) for decreasing trend, where

(1) in the case of small and untied samples: w_p is the pth quantile, that should be found in the proper table of Wilcoxon signed rank test;

(2) in the case of large ($m > 20$) and tied samples: w_p is the pth quantile of the standard normal distribution.

5.3 The Mann test

The Mann test is based on paired comparison of the X_i's. It compares, for example, each failure interarrival time with each earlier one. If there is no trend then the expected number of comparisons where the later interarrival time is greater than the earlier one (a so-called inversion, denoted by W) is equal to the number with the reverse situation, i.e., the expected number of inversions for n failures is

$$E[W|H_0] = \frac{n(n-1)}{4}. \qquad (28)$$

The further the number of inversions is below this the greater is the probability that there is a decreasing trend in the times between failures. The variance of the inversions is

$$D^2[W|H_0] = \frac{n(n-1)(2n+5)}{72}. \qquad (29)$$

The test statistic is

$$\zeta = \frac{W + \frac{1}{2} - E[W|H_0]}{\sqrt{D^2[W|H_0]}}, \tag{30}$$

and it converges to the standard normal distribution for large n (generally $n>10$ required) (Tibor,1993).

4.4 Exponential ordered scores test

The test is valid for all distributions of the X_i's as far as they are independent, because the test is based on the ranks of the X_i's. To the observed value X_i which is the rth largest in magnitude, we attach the score

$$s_{r,n} = \frac{1}{n} + ... + \frac{1}{n-r+1}, \quad r = 1,...,n. \tag{31}$$

Since we are interested in the trend of X_i on serial numbers, we take the independent variables z_i's to be the linear orthogonal polynomial $z_i=-(n+1)+2i$. Consequently,

$$S = \sum_{i=1}^{n} s_{r(i),n} z_i, \tag{32}$$

in which $r(i)$ = rank(X_i), has the expected value equal to zero and its variance is

$$V = \sum_{i=1}^{n} z_i^2 K_{2,n}, \tag{33}$$

where $K_{2,n}$ is the second semi-invariant of the finite population of scores, i.e.

$$K_{2,n} = 1 - \frac{\frac{1}{n} + \frac{1}{n-1} + ... + \frac{1}{2}}{n-1}. \tag{34}$$

The standard test statistic is

$$\zeta = \frac{S}{\sqrt{V}}, \tag{35}$$

which can be approximated by the standard normal distribution.

5 Bayesian methods

5.1 Parametric inference

Bayesian statistical analyses may, similarly as the conventional methods, be interpreted as parametric and non-parametric. In parametric Bayesian inference, the posterior distributrions are given for finite dimensional parameter vectors, while in non-parametric inference, the posterior analysis may have reference to infinite dimensional variables, such as whole probability distributions or failure intensity functions.

Very often statistical records subjected to trend analysis are made up of recorded number of events, y_i, of a certain type under corresponding operating time periods t_i. An alternative form of statistical records could be observed times between successive events. Let us first restrict the description to the former case and assume that a recorded time series, $\{y_i, t_i\}$; $i = 1,..., n$, is a trajectory of a nonhomogeneous Poisson process, which implies, among other things, that the intensity is a time dependent function $\lambda(t|\theta)$, and that the number of events in an arbitrary interval $(s, t]$ is Poisson distributed with mean

$$E[Y_{(s,t]}] = \int_s^t \lambda(t'|\theta)dt'. \tag{36}$$

The intensity $\lambda(t|\theta)$, in which θ is a parameter vector, is defined as the probability in unit time that at least one event occurs in an infinitesimal interval $(t,t+\Delta t]$. As the time dependent function $\lambda(t)$, the following expression is chosen:

$$\lambda(t) = C \cdot \lambda_1 \cdot t^{C-1} + \lambda_0, \tag{37}$$

where $C>0, \lambda_1>0$ and $\lambda_0>0$ are fictive model parameters. Due to this intensity function, the process can be called an *Extended Power Law process*, i.e. a Power Law process extended with the constant intensity parameter λ_0 in order to achieve an arbitrary asymptotic level. The parameter C is the central trend parameter since $C<1$ leads to decreasing intensity, $C=1$ to constant intensity and $C>1$ to increasing intensity. Thus this trend model can handle both decreasing and increasing trends as well as concave and convex tendencies.

The parameters $C>0$, $\lambda_1>0$ and $\lambda_0>0$ are estimated, applying a Bayesian methodology (see Pörn (1990)), by means of the computer program BayTREND (1996). The methodology is based on Bayes' theorem, which can be presented briefly in the form

$$p(\theta|y) \sim p(y|\theta) \cdot p(\theta). \tag{38}$$

Here the a priori distribution $p(\theta)$ describes the knowledge about the model parameter θ ($\theta = (C,\lambda_1,\lambda_0)$ in the application of this study) that exists before any observations have been made. When observations y are available the a priori distribution can be updated to the posterior distribution, $p(\theta|y)$, through the probability model, $p(y|\theta)$, expressing the likelihood of getting the observations y if the parameter θ is known.

Thus, in the specific application of this study, the estimation of the parameters C,λ_1,λ_0 starts from an a priori distribution $p(C,\lambda_1,\lambda_0)$, which is then modified through the Poisson based likelihood function to the posterior distribution $p(C,\lambda_1,\lambda_0|y_1,...,y_n)$, conditioned by the available observations $y_1,...,y_n$. A priori, the parameters are assumed to be mutually independent and to have the distributions

$$p(C) \sim e^{C}, \quad p(\lambda_1) \sim \lambda_1^{-\frac{1}{2}}, \quad p(\lambda_0) \sim \lambda_0^{-\frac{1}{2}}. \tag{39}$$

These distributions, of which $p(\lambda_1)$ and $p(\lambda_0)$ are improper, are so called "non-informative" distributions. They have been derived according to the principle of *data translated likelihood* (see Box & Tiao (1973)).

This means that each feasible combination of the three parameters is assigned a probability weight, first a prior weight according to the non-informative ditributions above and second a posterior weight, where the latter is determined to a great extent by the probability (likelihood) for the actual recorded events, conditioned by the given parameter combination. Thus each candidate in the given class of deterministic intensity functions is tried in this way and is assigned a posterior weight based on the prescribed prior distribution and the likelihood of the given records.

A graphical presentation of the results of the above Bayesian model is given in Figure 3. For each accumulated operating time t we have thus a distribution which describes the uncertainty about the intensity value $\lambda(t)$ in question. The time dependent mean value of this distribution may be considered as a trend curve for the time series considered. Uncertainty about this mean value is shown only for the total operating time. Further, the marginal distribution of the trend parameter C is presented, from which one can find the strength of trend, especially in the beginning of observation period. The graphic presentations include also a predictive distribution, which gives the probability of various outcomes of events during the next period of observation, after the record period. This predictive distribution is based on the assumption that the trend model used to describe the recorded events is valid also for the future behaviour. Moreover, in the predictive distribution all parametric uncertainties of the trend model have been taken into account through integration.

In above, the Bayesian parametric inference was described for a specific form of the intensity function. However, it is rather easy to apply the same methodology for any form of the intensity. As in above, the analytic determination of the posterior distributions is not possible and one must apply numerical methods. Rather straightforward methods for this purpose can be developed on the basis of Monte-Carlo sampling (see e.g. Tanner, 1991).

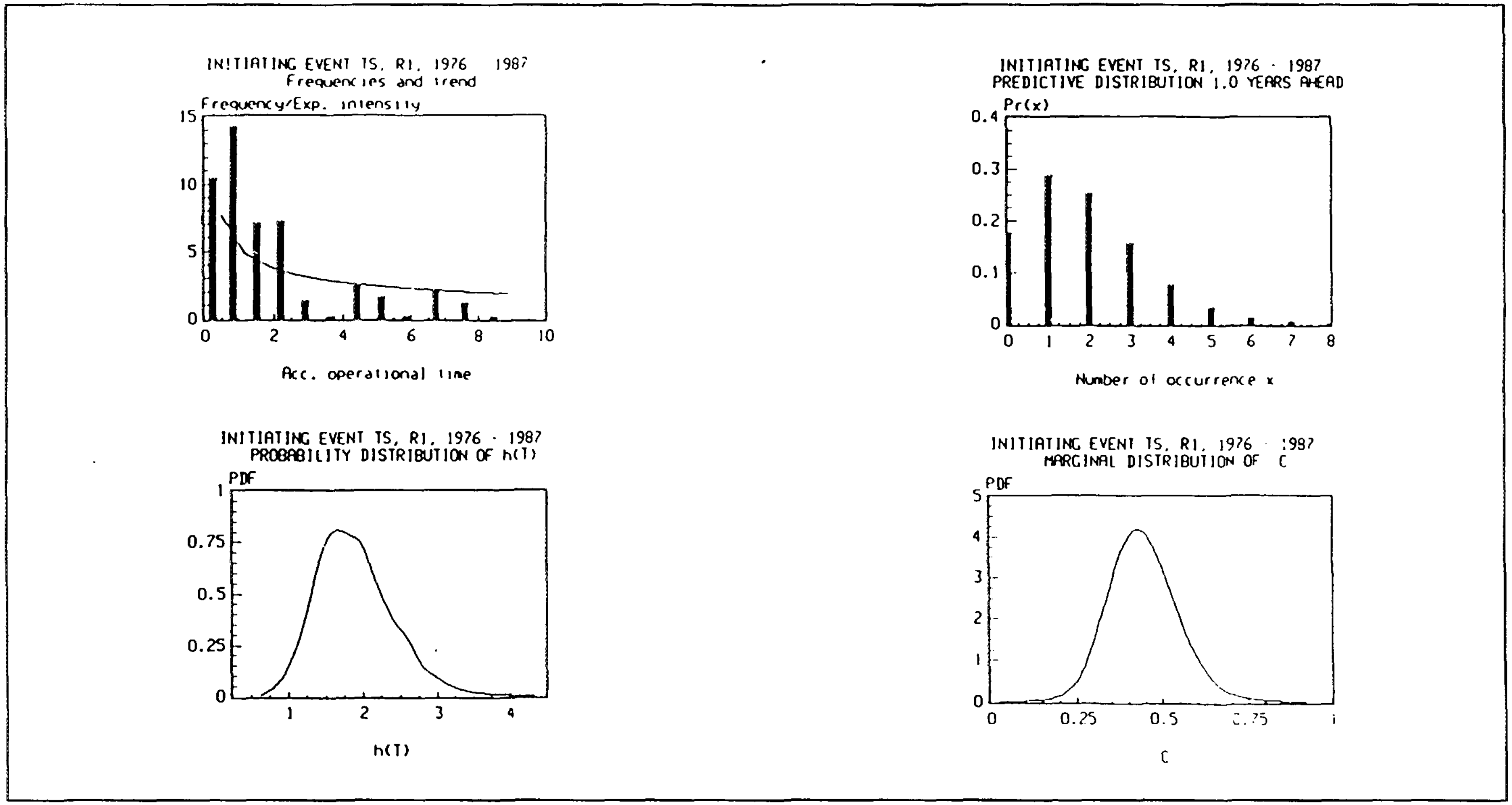

Figure 3: *A sample case diagram from the I-book (Pörn et al, 1993).*

5.2 The interpretation of the results

This chapter is intended to be a guide for the reader to interpret and use the results correctly. The arbitrarily chosen sample case describes the occurrence of Initiating Events within the group T_S for a Swedish reactor unit during the operating years 1975-86. The accumulated operating time during these years is shown on the x-axis of the first diagram in Figure 3. From the results obtained for this sample case we have chosen to present four diagrams for illustrative purposes. This set of diagrams is supplemented with a diagram showing the observed number of events as a cumulative function of time and the corresponding expected number of events. This diagram is supplied when the input option for "times between events" is used in the code.

The first diagram (left upper corner) presents the frequency of events versus the expected intensity, which can be called the trend curve. The frequency of events in each year has been normalized with respect to the actual annual exposure time. This will give a more realistic and comparable basis in relation to the computed expected intensity. Thus heights of columns of some events may be noninteger. In order to indicate that we did have an observation in a certain year when no events occurred, a tiny column is used. The trend curve represents the expected (mean) intensity of $\lambda(t)$ during the record period. At each time instant t, the intensity $\lambda(t)$ is associated with an uncertainty that is described by a distribution. We choose to present the mean values of these distributions as our trend curve. Visually examining this curve is usually enough to study the trend but in some uncertain cases, consulting the distribution of C (see "marginal distribution of trend parameter C" below) will be needed.

The second diagram (left lower corner) shows the probability distribution of $\lambda(T)$ (noted by $h(T)$ in Figure), i.e. the distribution of the intensity at the end of record period. This particular distribution is of greatest interest since it presents our uncertainty about $\lambda(T)$ at the moment. Knowing this will be helpful in making predictions of the development of trend in the next observation period and the occurrence of events.

The third diagram (right upper corner) presents the predictive distribution of number of occurrences during an operating year ahead, that is , it gives the estimated probabilities of having 0, 1, 2, ... occurrences of some event during the prediction period. From this diagram one can also obtain roughly the probability of having, e.g., less than two events or more than three events.

The forth diagram (right lower corner) presents the marginal distribution of parameter C and this distribution is obtained from the joint distribution of λ_1, λ_0 and C. Since the ranges $C \leq 1$ or $C \geq 1$ determine whether we have a decreasing or an increasing intensity in time respectively, one can compare the area to the left of 1 and the area to the right of 1 under the distribution curve of C. This comparison will indicate the kind of trend (decreasing or increasing) as well as the strength (how strongly we are convinced of such a trend). In case of a strong trend, this comparison gives overwhelmingly support to the trend curve in the first diagram, but when the areas differ little one has to examine the results together, both numerical and graphical, to draw conclusions about the trend.

It should be noted, however, that the parameter C has impact on the intensity only through the time dependent term $C\lambda_1 t^{C-1}$. That is, one may have a clear decreasing trend e g in the beginning of the record period, while the rest of the period is characterized by a rather weak trend around the "asymptotic" level λ_0.

The Bayesian trend analysis approach described above seems both promising and useful. The basic model covers nonhomogeneous Poisson processes (NHPP), of which homogeneous Poisson processes (HPP) are special cases. Because of the specific class of intensity functions that are used here we have named the model "expanded Power Law Process. Thus the model can be expected to be strictly applicable only to cases where the trend is non-stochastic and monotone.

Further, the statistical treatment is fully Bayesian, an approach which is in accordance with the statistical analysis of component failure rates applied in Sweden. Thanks to the Bayesian approach it is rather easy to compute the uncertainty of the primary parameter, the degree of tendency and the predictive probability distribution of future events.

The Bayesian method of statistical inference outlined above have been used in the recently issued I-Book (see Pörn et al, 1993), a handbook for the treatment of initiating events in Nordic nuclear power plants. The Bayesian trend analysis approach has also been applied to accident records of commercial air taxi in the Nordic countries (see Pörn & Shen, 1993).

A substantial, but relatively easy, extension of the applicability of the model above would be obtained by generalizing the model to include what is called trend-renewal processes, a class of processes that contain HPP, NHPP and renewal processes as specific cases (Lindqvist, 1993). Such an extended trend analysis tool could be applied also to non-monotone and stochastic process intensities, features that are expected to be valid for many maintenance procedures.

5.3 Non-parametric Bayesian inference

The trend analysis of point processes can also be formulated as a problem of non-parametric Bayesian inference. The basic principle is to model the intensity function as a stochastic process, which may have also increasing or decreasing realizations. Beeing a stochastic process, a realization of the intensity cannot be modelled by using a finite number of parameters but as an element of an infinite dimensional function space. The objective of Bayesian inference is then to determine the posterior distribution of such functions, which is not actually possible. However, it is possible to generate samples of of the intensity functions, if the form of the intensity process is simple.

One rather simple but still rich form of such processes is an intensity function which is piecewise constant, and which has jumps down or up at random time epochs. It is possible to write the likelihood function given any realization of the intesity process, and thus it is possible to generate samples from the "posterior intensity process". Methods like Gibbs sampling or any Markov-chain Monte-Carlo methods are applicaple for this purpose (see, Arjas & Gasparra, 1993, 1994 and Tanner, 1991).

6 Examples

As an example, a data set consisting of 45 times between events has been analysed by applying Cox-Stuart and Wilcoxon tests, Laplace test, a power law non-homogenous Poisson process model and the Bayesian model described in section 5. The data set is given in Table 1. The data has been generated by Monte Carlo simulation. The simulated data consists of four data sets, each of which follow a gamma distribution with its own expected values. The simulated values can be interpreted here as times between events.

Table 1. *The example data.*

n:o	Observed value	n:o	Observed value	n:o	Observed value
1	21	16	190	31	502
2	100	17	372	32	536
3	261	18	100	33	235
4	3	19	97	34	937
5	80	20	194	35	352
6	119	21	230	36	1104
7	79	22	1	37	293
8	97	23	49	38	465
9	36	24	60	39	967
10	186	25	1	40	691
11	133	26	84	41	195
12	552	27	5	42	652
13	141	28	15	43	110
14	173	29	0.5	44	640
15	190	30	40	45	386

The simulated data is presented also in Figure 4, from which we can see that there is an increasing trend in the times between events. In the following it is studied whether the methods discussed earlier detect this trend.

According to the Cox-Stuart test there is an increasing trend in times between events. the value of Cox-Stuart test statistic is 16.0, and the corresponding p-value is 0.0165, which means that at 5% signifigance level the hypothesis H_0: "no trend" must be rejected. The Wilcoxon test results in the same conclusion at 5% signifigance level (test statistics T^{+}=209.0, p-value p=0.0369).

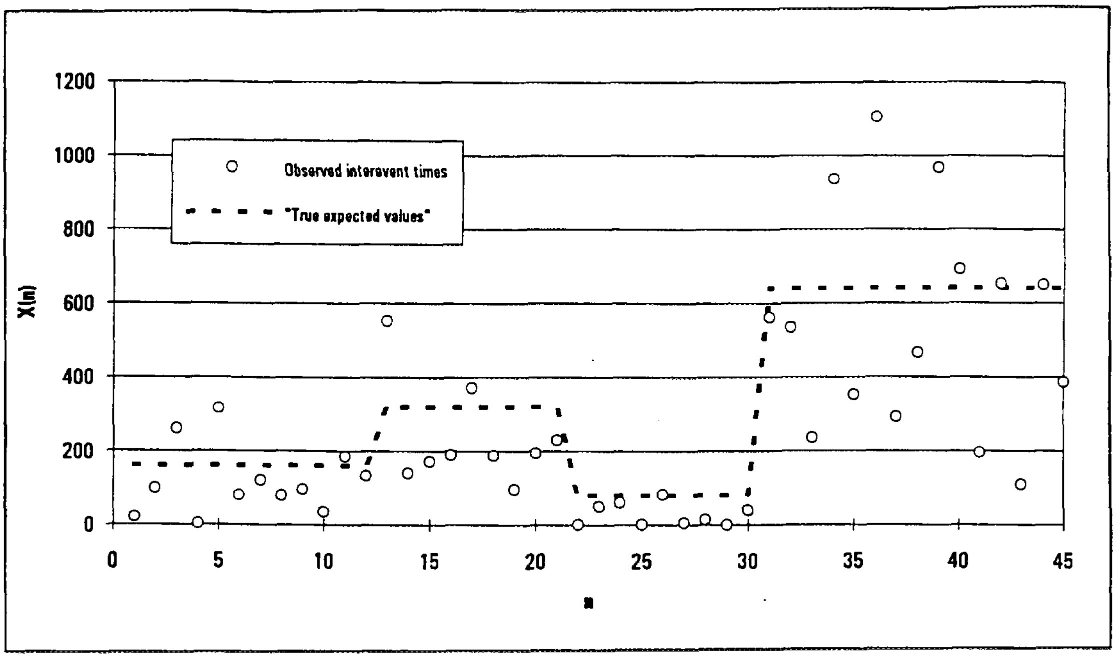

Figure 4. *The simulated example data.*

The Laplace test is presented in Figure 5, where the value of the test statistic is presented after each observation point (number of failures). The test detects a decreasing (negative) event rate trend after the 35th observation, and even more certainly after the later observations, at the 5% one-sided signifigance level; the value of Laplace test statistic $u<-1.96$, the critical value given by the standardized normal distribution.

The non-homogenous Poisson process with power law intensity (Weibull) estimated from the example data set is presented in Figure 6. The trend parameter has point estimate $\beta=0.666<1$, which indicates a decreasing event rate. It may be noticed that the temporary increasing trend of event rate between 20th-30th observations is not directly reflected in the parameter estimates, if the whole data set is used in estimation.

Finally, the example data set is analysed by using the Bayesian model. The results are presented in Figures 7 - 10. The marginal posterior distribution of the trend parameter, *C*, is given in Figure 7, from which it is seen that values of *C* are concentrated to the left of value $C=1$, indicating a decreasing trend in event intensity. The expected event intensity and frequency of events are presented in Figure 8, which also shows a decreasing trend. The posteriori distribution of event intensity at the end of the observation period is presented in Figure 9 and the predictive distribution of number of events at 100 time units ahead is given in Figure 10.

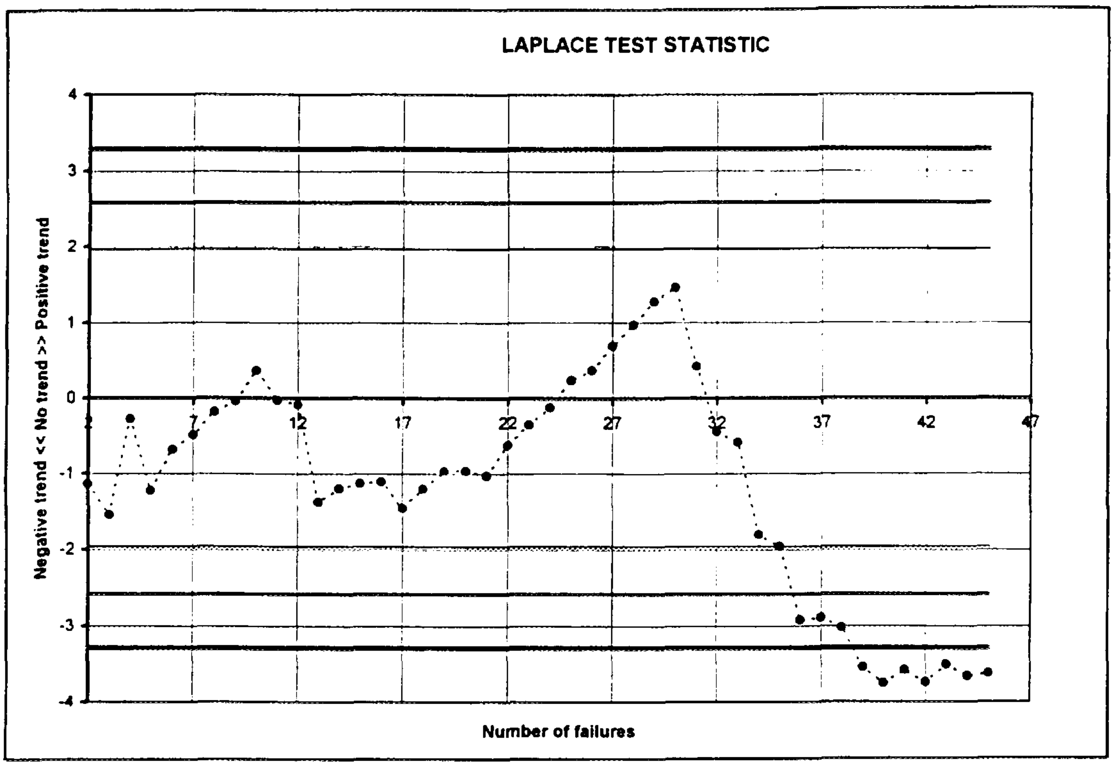

Figure 5. *Laplace test.*

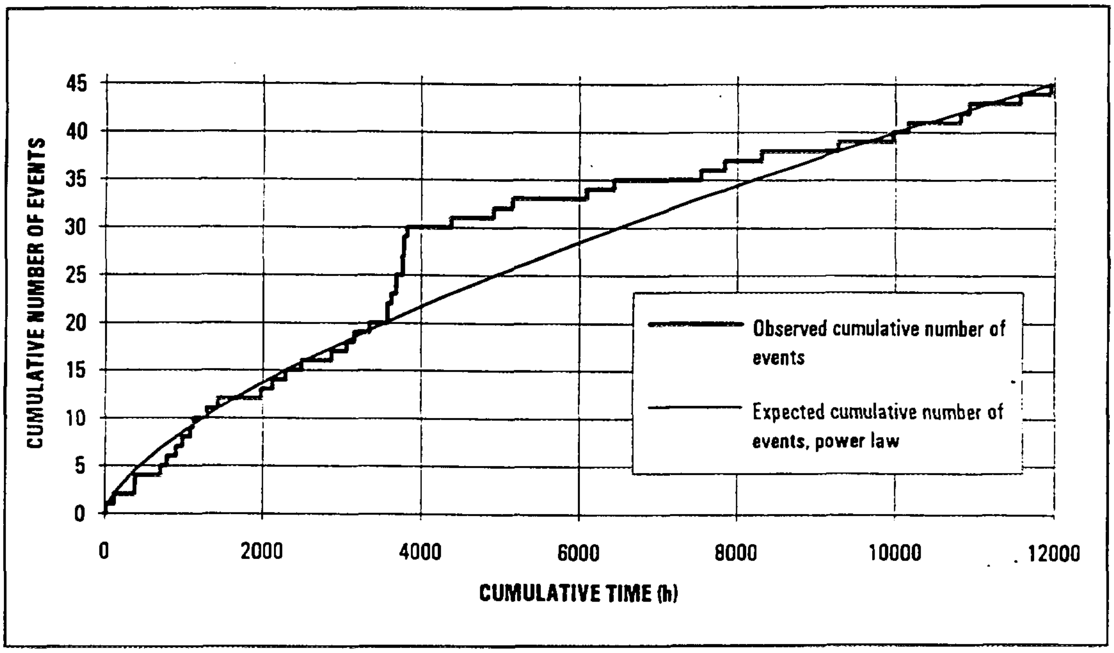

Figure 6. *Estimation of a non-homogenous Poisson process model.*

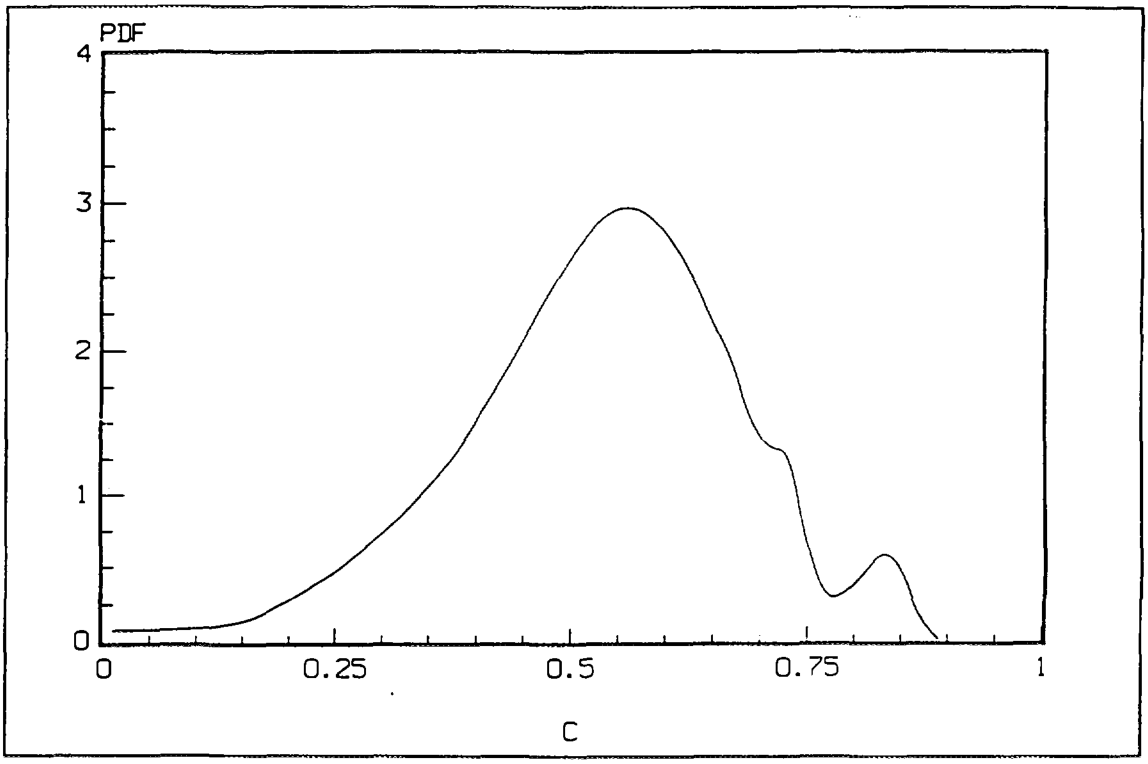

Figure 7. *Posterior distribution of the trend parameter.*

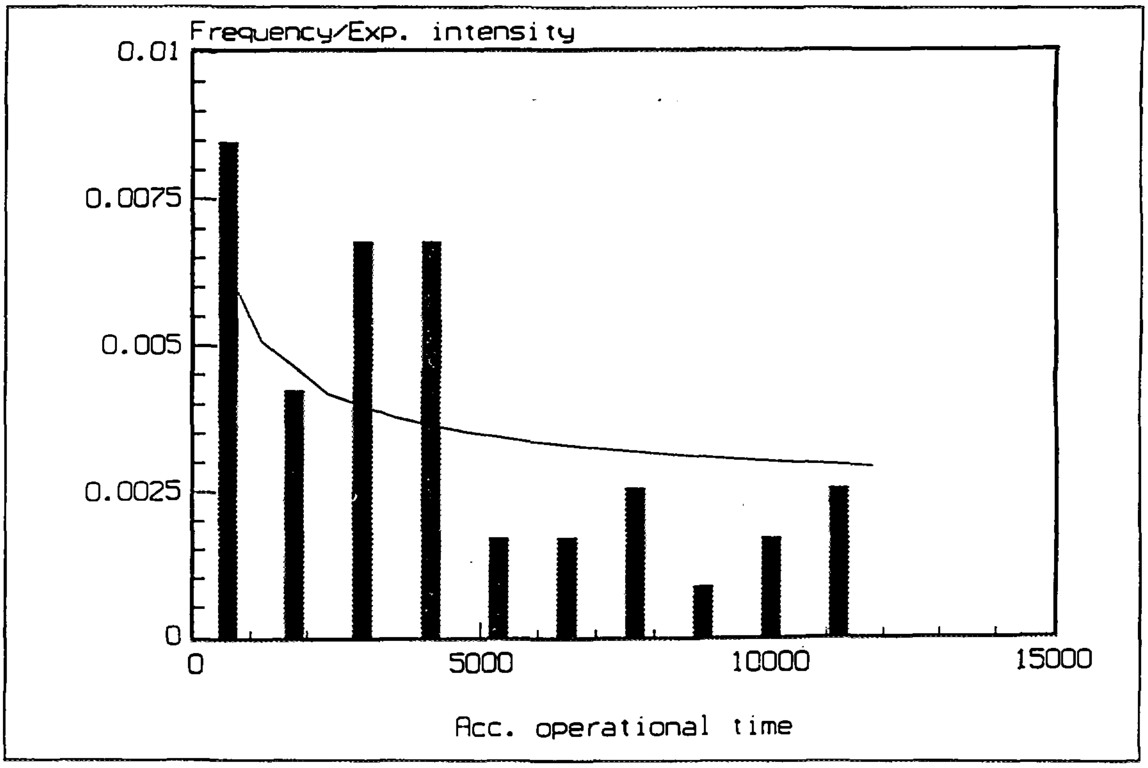

Figure 8. *Event frequency and expected intensity.*

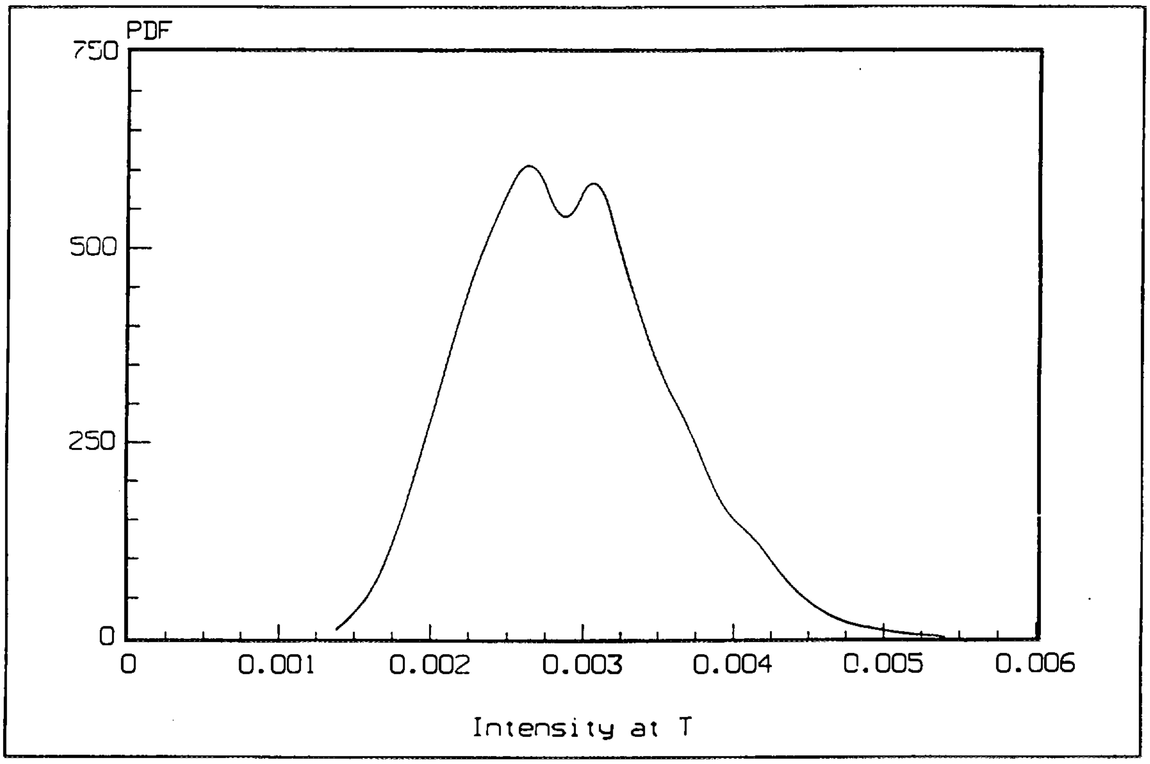

Figure 9. *Posterior distribution of event intensity at the end of observation period.*

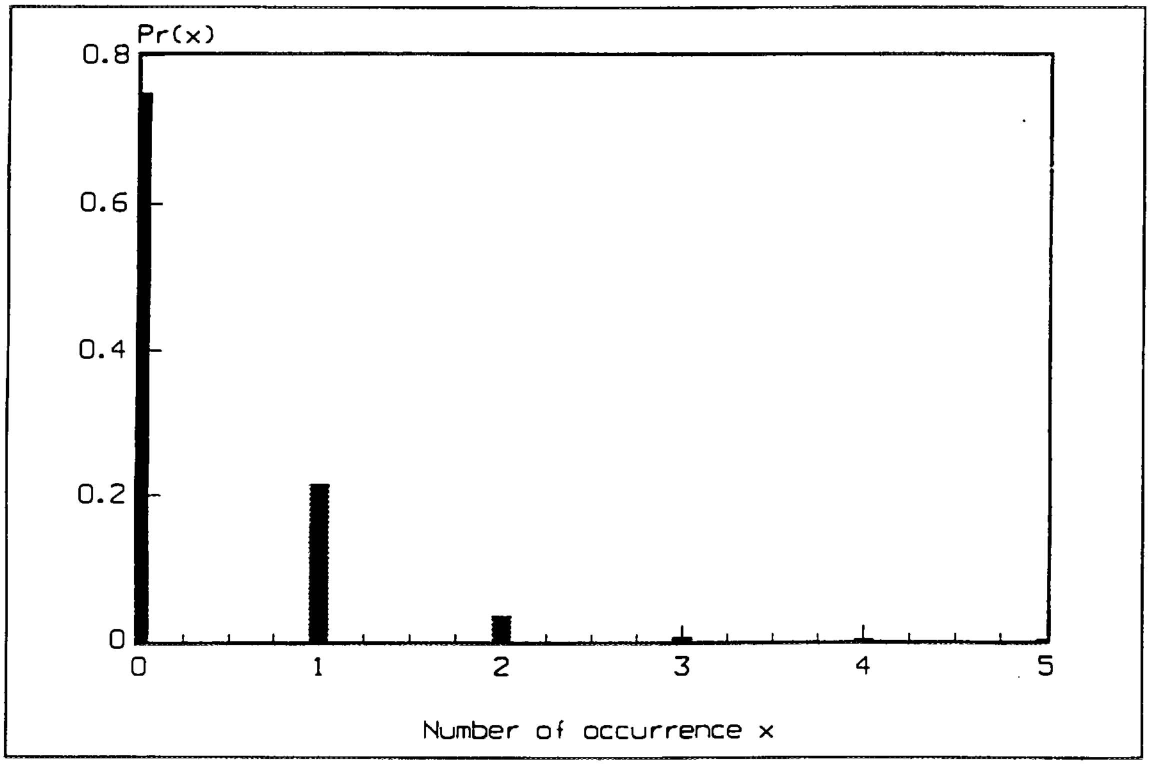

Figure 10. *Predictive distribution of the number of events 100 time units ahead.*

7 Conclusions

In this report we have discussed methods for analysing trends in event occurence. We have considered graphical methods, both parametric and non-parametric statistical tests, point process models and Bayesian model techniques.

Graphical methods are necessary giving an exploratory view over the observations, and identifying the need for deeper trend analyses. Often, the graphical methods are enough to show that there is no trend. However, the graphical tools are not sufficient for the evaluation of the statistical signifigance and for making statistical conclusions on the trend phenomena. Therefore one must apply formal statistical tests or models. On the other hand, although the graphical methods could indicate a trend, the sample size of statistical observations may be so small that it is not possible to accept statistically any hypothesis of the trend.

With every statistical test is associated a model and a measurement requirement; the test is valid under certain conditions, and the model and the measurement requirement specify those conditions. Sometimes we are able to test whether the conditions of a particular statistical model are met, but more often we have to assume that they are met. Thus the conditions of the statistical model of a test are often the assumptions of the test.

It is obvious that the fewer or weaker are the assumptions that define a particular model, the less qualifying we need to do our decision arrived at by the statistical test associated with that model. That is, the fewer or weaker are the assumptions, the more general are the conclusions.

However, the most powerful tests are those which have the strongest or most extensive assumptions. The parametric tests have a variety of strong assumptions underlying their use. When those assumptions are valid, these tests are the most likely of all tests to reject the hypothesis of nonexistence of a trend, for instance, when it is false. That is, when data may appropriately be analyzed by a parametric test, that test will be more powerful than any other in resulting in an acceptance of the trend.

When we have reason to believe that the conditions for a parametric test are met in the data under analysis, then we should certainly choose a parametric statistical test for analyzing those data. If these conditions are not met, some relevant non-parametric test can be used. Non-parametric statistical methods often involve less computational work and therefore are easier and quicker to apply than other statistical methods.

The parametric tests are based in this case on some point process models. An archetype of point process models is the Poisson process model, the parametric tests of which can be applied in trend detection. In this report, we consider the Laplace test, which is originally intended for a certain intensity model. There are models, for which the Laplace test has rather good statistical properties. However, if the Poisson process model (or any other model) is questionable, one should choose a simpler model and apply non-parametric trend tests. Here we have presented a lot of non-parametric tests.

In addition to the classical parametric and nonparametric approaches we have also considered the Bayesian trend analysis. First we discuss a Bayesian model, which is based on a power law intensity model. The Bayesian statistical inferences are based on the analysis of the posterior distribution of the trend parameters, and the probability of trend is immediately seen from these distributions. In principle, is is possible to apply non-parametric Bayesian models.

We applied some of the methods discussed in this report to an example case. The results were not contradictory, and every model detected the trend that was assumed in Monte Carlo generation of the example data. However, this report is a feasibility study rather than a scientific evaluation of the statistical methodologies, and the example analyses can be seen only as demonstrations of the methods. Furthermore, it is to be noted that there is a lot of other statistical methods relevant to analysis of phenomena varying along with time which have not been considered in this report.

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STATENS KÄRNKRAFTINSPEKTION
Swedish Nuclear Power Inspectorate

Postadress/Postal address	Telefon/Telephone	Telefax	Telex
SKI	Nat 08-698 84 00	Nat 08-661 90 86	11961 SWEATOM S
S-106 58 STOCKHOLM	Int +46 8 698 84 00	Int +46 8 661 90 86	