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PROPOSITIONS POUR MODELISER L'ACCELERATION D'UNE PARTICULE FLUIDE A L'AIDE D'UN MODELE PDF

PROPOSITIONS FOR A PDF MODEL BASED ON FLUID PARTICLE ACCELERATION

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SYNTHÈSE :

Cette note décrit des propositions théoriques pour modéliser l'accélération d'une particle fluide dans un écoulement turbulent. Un tel modèle est d'intérêt pour la modélisation PDF des écoulements turbulents réactifs ainsi que pour la modélisation Lagrangienne des écoulements diphasiques.

Le modèle développé ici reprend des idées déjà avancées par Sawford mais qui sont généralisées au cas des écoulements non-homogènes quelconques. Le modèle est construit de façon à retrouver continûment le modèle de Pope, qui utilise une équation de Langevin sur la vitesse des particules, quand le nombre de Reynolds devient très grand. La dérivation est basée sur la technique dite d'élimination des variables rapides.

Cette technique permet d'analyser les relations et l'évolution de modèles qui correspondent à différents niveaux de modélisation. Elle permet aussi d'analyser avec plus de rigueur certains problèmes, comme la modélisation Lagrangienne des écoulements diphasiques. En particulier, l'application de cette technique montre que les modèles utilisés actuellement peuvent traiter des écoulements à bulles avec prise en compte des forces de gradient de pression et de masse ajoutée.

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EXECUTIVE SUMMARY :

This paper describes theoretical propositions to model the acceleration of a fluid particle in a turbulent flow. Such a model is useful for the PDF approach to turbulent reactive flows as well as for the Lagrangian modelling of two-phase flows.

The model developed here draws from ideas already put forward by Sawford but which are generalized to the case of non-homogeneous flows. The model is built so as to revert continuously to Pope's model, which uses a Langevin equation for particle velocities, when the Reynolds number becomes very high. The derivation is based on the technique of fast variable elimination.

This technique allow a careful analysis of the relations between different levels of modelling. It also allows to address certain problems in a more rigorous way. In particuliar, application of this technique shows that models presently used can in principle simulate bubbly flows including the pressure-gradient and added-mass forces.

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Propositions for a PDF Model
Based on Fluid Particle Acceleration

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1 Introduction

Probability Density Function (PDF) methods constitute an interesting approach to turbulence [1, 2]. They treat convection and local source terms (such as chemical reaction rates) exactly while offering detailed information on turbulence quantities. This special feature explains their particular interest for reactive flows. The PDF approach represents also a suitable framework for other fields, such as two-phase flow modelling, where it provides a sound starting point for the so-called Lagrangian (or particle tracking) models [3, 4, 5].

Broadly speaking, work on PDF models progresses along two lines: development of numerical tools and simulation of various turbulent flows on the one hand and development of new models or extension of current ones on the other hand. Various applications have already been described by Pope and his co-workers. Some efforts have also been devoted by the present authors to develop a numerical code and to analyse its performance. Previous results concerned free-shear flows [6] and discussed the question of external intermittency. The method has recently been extended to wall-bounded flows and applied to channel flows [7].

The aim of the paper is more along the second line. Its purpose is to put forward theoretical considerations that can be used to derive a model for the acceleration of fluid particles. The basic ideas of PDF modelling are first briefly recalled and the motivations for a model which includes acceleration are mentioned. The principle of fast variables elimination is illustrated and used to propose a model for the general case. The interest and utility of this technique is then carried to two-phase flow modelling and is applied to propose a general stochastic model even for bubbly flows.

2 Current PDF Modelling

One way to introduce the PDF approach follows classical statistical reasoning. The turbulence problem is then considered as an N-particle problem which interact through the pressure gradient and viscous forces. Depending on the desired level and precision of the description, various multi-point pdfs can be sought. In this article, attention is limited to one-point pdfs. The method appears as a Lagrangian stochastic approach and consists in simulating the instantaneous behaviour of a large number of fluid particles which are defined by a set of variables (positions, velocity, dissipation rate, scalar ...). These variables follow time evolution equations which are usually Stochastic Differential Equations (SDE) and are often modelled by diffusion processes. However, apart from the precise form of the models, a key question is the choice of the variables which are retained in the state vector to describe the system (here turbulent flows).

For dynamical variables, the present 'standard' model includes positions and velocities and is based on a Langevin equation for the time evolution equation of fluid particle velocities [2, 8]. The simplest version writes for the state vector $\mathbf{X} = (\mathbf{x}, \mathbf{U})$ where k is the turbulent kinetic energy and $\langle\epsilon\rangle$ its mean dissipation rate

$$dx_i = U_i\,dt \tag{1}$$

$$dU_i = -\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\,dt - \left(\frac{1}{2}+\frac{3}{4}C_0\right)\frac{\langle\epsilon\rangle}{k}\left(U_i - \langle U_i\rangle\right)dt + \sqrt{C_0\langle\epsilon\rangle}\,dW_i. \tag{2}$$

This model is mainly justified by Kolmogorov's hypotheses. At high Reynolds numbers, there is a large separation between the characteristic timescales of the small turbulent scales, say τ_η, and of the large ones, say T_L. This allows to consider variations over time steps such that $\tau_\eta \ll dt \ll T_L$. With respect to the timestep, fluid particle accelerations, which are governed by the small scales, can be seen as rapidly varying or fast variables. The leading idea is then to keep only the slow modes, namely here $(\mathbf{x}, \mathbf{U})$, in the state vector while acceleration is replaced by a model based on Gaussian white noise.

As mentioned in the introduction, there is a closed relation (although often overlooked) between PDF and particle tracking approaches [3, 4, 5] for turbulent two-phase flow modelling. The problem is to simulate the instantaneous behaviour of particles (referred to as 'solid particles' to avoid confusion with fluid particles though they can be solid particles, sediments, bubbles, ...) embedded in a turbulent flow. A general form of the particle equations is (keeping only drag, pressure gradient, added-mass and gravity forces)

$$\frac{dx_{p,i}}{dt} = V_{p,i}, \tag{3}$$

$$\frac{dV_{p,i}}{dt} = \frac{U_{fs,i} - V_{p,i}}{\tau_p} + \frac{\rho_f}{\rho_p}\frac{DU_{fs,i}}{Dt} + \frac{1}{2}C_a\frac{\rho_f}{\rho_p}\left(\frac{DU_{fs,i}}{Dt} - \frac{dV_{p,i}}{dt}\right) + \left(1 - \frac{\rho_f}{\rho_p}\right)g_i. \tag{4}$$

For particles denser than the fluid, $\rho_p \gg \rho_f$, the pressure gradient and added-mass forces can be neglected. The model is then closed by writing a time evolution equation for the fluid velocity seen, say $\mathbf{U}_{fs}$, along solid particle trajectories. An attractive approach is to use a Langevin equation similar to the one of the standard PDF model with a different timescale, say T_L^*, to account for particle inertia and crossing-trajectory effects. The precise expression of this equation remains a subject of current research but is beyond the scope of the present paper. Only the form of the equation is of interest here. Among other proposals [9], a simple model writes [3, 4, 5]

$$dU_{fs,i} = -\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\,dt - \frac{U_{fs,i} - \langle U_{fs,i}\rangle}{T_L^*}\,dt + \sqrt{C_0\langle\epsilon\rangle}\,dW_i. \tag{5}$$

Csanady's expression [3] can be used for T_L^* when a mean drift (due for example to an external field such as gravity) is present. The model distinguishes between directions parallel to the mean drift and perpendicular to it

$$T^*_{L,\parallel} = \frac{T_L}{\sqrt{1+\ \beta\frac{|\langle\mathbf{U}\rangle - \langle\mathbf{V}_p\rangle|^2}{2k/3}}} \quad T^*_{L,\perp} = \frac{T_L}{\sqrt{1+\ 4\beta\frac{|\langle\mathbf{U}\rangle - \langle\mathbf{V}_p\rangle|^2}{2k/3}}}. \tag{6}$$

T_L stands for the timescale of equation (2) $T_L = \left[(1/2 + 3/4C_0)k/\langle\epsilon\rangle\right]^{-1}$.

2.1 Motivations for a model for fluid acceleration

In low-Reynolds flows, when the above-mentioned criterion $\tau_\eta \ll T_L$ is not met, particle accelerations are no longer fast variables with respect to the other ones and have to be included in the state vector. Modelling is then shifted to their time evolution equation. Such a proposal has already been made by Sawford [10] using a coloured noise in the velocity equation. This amounts to replacing the white-noise term by a Ornstein-Uhlenbeck process, say γ, and has been shown to satisfactorily reproduce low-Reynolds effects [10, 2]. The model has been developed for stationary homogeneous turbulence and writes

$$dx_i = U_i\,dt \tag{7}$$

$$dU_i = A_i\,dt = -\frac{U_i}{T_L}dt + \gamma_i dt, \tag{8}$$

$$d\gamma_i = -\frac{\gamma_i}{\tau}dt + \sqrt{K}dW_i. \tag{9}$$

Yet, the expression proposed for the diffusion coefficient is only valid for stationary processes. In that case, knowledge of the process variance and timescale is sufficient to yield $K = 2\langle\gamma_i^2\rangle/\tau$. For general and inhomogeneous turbulence, this reasoning is not valid anymore.

A second motivation comes from two-phase flow modelling. Present models represent the fluid velocity seen by a diffusion process. However, for such processes acceleration does not exist (the acceleration calculated directly from model equation (5) is infinite due to the white noise term). Extension of present modelling to the case of bubbles when the pressure gradient and added-mass forces are not negligible seems therefore to require a model for the fluid instantaneous acceleration. We will come back to this reasoning later on.

3 Fast Variables and Adiabatic Elimination

The principle of the technique of fast variable elimination is perhaps best explained on the simple example of Brownian motion in 1D [11]. We consider the set of equations involving position and velocity of a particle with constant velocity timescale, say T, and diffusion coefficient, say K

$$dx = U\,dt \tag{10}$$

$$dU = -\frac{U}{T}\,dt + \sqrt{K}dW. \tag{11}$$

The question is how the model evolves when the velocity timescale becomes much smaller than the characteristic time of observation (here the timestep dt), that is when $T \to 0$. For the present case, the velocity equation can be integrated (disregarding the initial condition or rejecting it to $-\infty$)

$$U(t) = \sqrt{KT^2}\,\eta(t) \text{ where } \eta(t) = \frac{1}{T}e^{-t/T}\int_{-\infty}^{t} e^{t'/T}dW_{t'}. \tag{12}$$

$\eta(t)$ is a Gaussian process with $\langle\eta(t)\rangle = 0$ and $\langle\eta(t)\eta(t')\rangle = (1/2T)e^{-|t-t'|/T} \xrightarrow{T\to 0} \delta(t-t')$. Therefore in the limit of vanishing timescale, the velocity becomes a white-noise process and writes

$$U(t)\,dt \simeq \sqrt{KT^2}\,dW. \tag{13}$$

This procedure shows how the fast variable can be eliminated from the state vector of the model which is reduced (here to only x). This elimination results in the apparition of a white-noise term in the equation where the fast variable dissapears. Consequently, the position equation which was an ODE turns into a SDE

$$dx = \sqrt{KT^2}\,dW. \tag{14}$$

For the case of stationary processes, $K = 2\langle U^2\rangle/T$ which gives $dx = \sqrt{2\langle U^2\rangle T}\,dW$ and we retrieve the known behaviour for Taylor diffusion $\langle x^2(t)\rangle = 2\langle U^2\rangle T\,t$. Previous results are correct provided that KT^2 remains finite. This indicates hows the limit should be taken for (13) to be correct

$$T \to 0 \text{ and } K \to +\infty \quad \text{with} \quad KT^2 \to \text{finite value}. \tag{15}$$

It should be noted that the elimination procedure illustrated above is made in terms of the trajectories of the process. In that respect, eq. (13) is an equivalence between two stochastic processes. This result is more general and has a stricter sense than simply considering the limits of some statistics of the process such as the variance or the autocorrelation.

4 Derivation of the Model

We now consider the general case of non-stationary and inhomogeneous turbulence. The first step of the model is to propose to replace the white-noise term in the velocity equation (2) by a coloured noise. A simple diffusion process satisfying a linear SDE is assumed as in equation (9) along Sawford's proposal. The equations are therefore

$$dx_i = U_i\,dt \tag{16}$$

$$dU_i = -\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}dt - \left(\frac{1}{2}+\frac{3}{4}C_0\right)\frac{\langle\epsilon\rangle}{k}(U_i - \langle U_i\rangle)\,dt + \gamma_i\,dt, \tag{17}$$

$$d\gamma_i = -\frac{\gamma_i}{\tau}dt + \sqrt{K}dW_i. \tag{18}$$

The timescales and the mean quantities which enter these equations are interpolated at the particle positions. First of all, the fluid particle acceleration timescale τ is taken as the local Kolmogorov timescale τ_η. Direct Numerical Simulations have indeed confirmed that τ is proportional to τ_η [12]. We assume here that

$$\tau = \tau_\eta = \left(\frac{\nu}{\langle\epsilon\rangle}\right)^{1/2}, \tag{19}$$

where ν is the molecular viscosity. The diffusion coefficient entering the SDE for γ is then worked out from the fast variable elimination technique described in the previous section. Indeed, we want the present model to revert to the 'standard' one (1)-(2) at high Reynolds numbers. In this limit $\tau_\eta \to 0$ and fluid particle acceleration (and therefore γ) becomes a white-noise process. The elimination technique shows that

$$\gamma_i \, dt \simeq \sqrt{K\tau^2}\, dW_i. \tag{20}$$

Equivalence of the limit process with the standard one then requires that

$$K = \frac{C_0\langle\epsilon\rangle}{\tau^2} \tag{21}$$

In the present context, the equations are not used to obtain an expression for C_0 but considers it as a given constant. The derivation remains valid even in nonhomogeneous turbulence provided that the timescales and mean quantities, τ and $\langle\epsilon\rangle$, are taken as the local ones at the particle location $\mathbf{x}$. This corresponds to the so-called slaving principle. When $\tau \to 0$, the fast variable relaxes to its limit process whose parameters are defined by the slow modes which have remained constant. In other words, the slow modes (here particle location and velocity) slave the fast ones (particle acceleration) in the high Reynolds limit.

It can be checked that previous results are retrieved. For the simpler case of stationary processes and the set of equations (7)-(9), K can be be worked out directly with the stationarity constraints (all mean quantities are constant)

$$K = \frac{2\langle\gamma_i^2\rangle}{\tau} \quad \langle\gamma_i^2\rangle = \langle U_i\gamma_i\rangle\left(\frac{1}{\tau} + +\frac{1}{T_L}\right) = \langle U_i^2\rangle\frac{1}{T_L}\left(\frac{1}{\tau} + \frac{1}{T_L}\right) \Longrightarrow K \sim \frac{2\langle U_i^2\rangle}{\tau^2 T_L} \tag{22}$$

The timescale T_L and the velocity variance are given by

$$T_L = \frac{4}{3C_0}\frac{k}{\langle\epsilon\rangle} \quad \langle U_i^2\rangle = \frac{2}{3}k \Longrightarrow K = \frac{C_0\langle\epsilon\rangle}{\tau^2} \tag{23}$$

which is indeed the same value as in eq. (21).

5 Marginal Velocity pdf

The acceleration-based model has been developed in terms of the trajectories of the vectorial process $\mathbf{Y} = (\mathbf{x}, \mathbf{U}, \gamma)$. In an equivalent way, this represents a closure expression for the associated pdf $f(t, \mathbf{Y})$ in phase space. We can re-express the model equations in a compact way as the SDE of a diffusion process with a drift vector D and a diffusion matrix B

$$d\mathbf{Y} = D(t, \mathbf{Y})\, dt + B^{1/2}(t, \mathbf{Y}) d\mathbf{W}, \tag{24}$$

and the corresponding pdf $f(t, \mathbf{Y})$ follows then a Fokker-Planck equation written here

$$\frac{\partial f}{\partial t} = -\frac{\partial[D_i f]}{\partial Y_i} + \frac{\partial^2[B_{ij} f]}{\partial Y_i \partial Y_j}. \tag{25}$$

Compared to the 'standard model' which handles the pdf $f_{st}(t, \mathbf{x}, \mathbf{U})$, the present model contains an extra variable. This extra variable can be integrated to give the reduced of marginal pdf, noted f_r

$$f_r(t, \mathbf{x}, \mathbf{U}) = \int f(t, \mathbf{x}, \mathbf{U}, \gamma)\, d\gamma. \tag{26}$$

A question of interest is how the marginal pdf f_r compares with the 'standard one' f_{st}. Since the acceleration-based model has been precisely built so as to retrieve the standard one when $\tau \to 0$, we know that in that limit $f = f_{st}$. Yet, when τ is not negligible, the technique of fast variable elimination cannot be used. That question is perhaps best addressed in terms of the pdf of the process. By performing the integration over γ in the Fokker-Planck satisfied by $f(t, \mathbf{Y})$, we obtain

$$\begin{aligned} \frac{\partial f_r}{\partial t} + U_i \frac{\partial f_r}{\partial x_i} = \frac{1}{\rho_f}\frac{\partial \langle p \rangle}{\partial x_i}\frac{\partial f_r}{\partial U_i} &+ \left(\frac{1}{2} + \frac{3}{4}C_0\right)\frac{\langle \epsilon \rangle}{k}\frac{\partial\left([U_i - \langle U_i \rangle] f_r\right)}{\partial U_i} \\ &- \frac{\partial\left(\langle \gamma_i \,|(\mathbf{x}, \mathbf{U})\rangle f_r\right)}{\partial U_i}. \end{aligned} \tag{27}$$

The resulting pdf for f_r is now unclosed due to the presence of the mean conditional term

$$\langle \gamma_i \,|(\mathbf{x}, \mathbf{U})\rangle f_r = \int \gamma_i f(\mathbf{x}, \mathbf{U}, \gamma)\, d\gamma_i. \tag{28}$$

To derive a closed expression requires new elements of information on f. This can be obtained in some circumstances such as homogeneous turbulence. In that case, the coefficients of the SDE are constant and the SDE (24) has a linear drift term and a constant diffusion matrix. The solution $\mathbf{Y}$ of such an equation is then a Gaussian process, and we can perform Gaussian integration by parts [14]. This result states that for a Gaussian process $\mathbf{Y} = (Y_1, \ldots, Y_{n+1})$, integration over one variable of the process gives in terms of the marginal pdf

$$\int Y_{n+1} f_{n+1}(Y_1, .., Y_{n+1})\, dY_{n+1} = -\sum_{i=1}^{n} < Y_i Y_{n+1} > \frac{\partial f_n}{\partial y_i}. \tag{29}$$

In our case, Gaussian integration by parts gives

$$-\langle \gamma_i \,|(\mathbf{x}, \mathbf{U})\rangle f_r = \langle \gamma_i (U_j - \langle U_j \rangle)\rangle \frac{\partial f_r}{\partial U_j} + \langle \gamma_i (x_j - \langle x_j \rangle)\rangle \frac{\partial f_r}{\partial x_j}. \tag{30}$$

The correlations can be worked out from the SDE

$$\alpha_{ij} = \langle \gamma_i (x_j - \langle x_j \rangle)\rangle = \frac{K\tau^3}{2}\frac{1}{1 + \tau/T}\delta_{ij} \quad \beta_{ij} = \langle \gamma_i (U_j - \langle U_j \rangle)\rangle = \frac{K\tau^2}{2}\frac{1}{1 + \tau/T}\delta_{ij}. \tag{31}$$

With relation (30) the pdf equation satisfied by f_r is now closed and has the form

$$\frac{\partial f_r}{\partial t} + U_i \frac{\partial f_r}{\partial x_i} = \frac{1}{\rho_f}\frac{\partial \langle p \rangle}{\partial x_i}\frac{\partial f_r}{\partial U_i} + \left(\frac{1}{2} + \frac{3}{4}C_0\right)\frac{\langle \epsilon \rangle}{k}\frac{\partial\left([U_i - \langle U_i \rangle] f_r\right)}{\partial U_i} + \frac{\partial^2 [\alpha_{ij} f_r]}{\partial U_i \partial x_j} + \frac{\partial^2 [\beta_{ij} f_r]}{\partial U_i \partial U_j}. \tag{32}$$

Or, in a compact way using $\mathbf{X} = (\mathbf{x}, \mathbf{U})$ for the reduced state vector

$$\frac{\partial f_r}{\partial t} = -\frac{\partial [D_i f_r]}{\partial Y_i} + \frac{1}{2}\frac{\partial^2 [M f_r]}{\partial U_i \partial U_j}, \tag{33}$$

where M is a bloc matrix

$$M = \left(\begin{array}{c|c} 0 & \alpha_{ij} \\ -- & -- \\ \alpha_{ji} & \beta_{ij} + \beta_{ji} \end{array} \right). \tag{34}$$

The pdf equation for f_r looks like a Fokker-Planck equation since it contains a second-order derivative term involving a symmetric matrix. However, it is straightforward to show that $det(M) = -[det(\alpha)]^2$ and, consequently, M has always at least one negative eigenvalue. Eq. (32) is thus not a Fokker-Planck equation and exhibits new behaviour. Since eigenvalues of the matrix M correspond to diffusion coefficients, a negative eigenvalue signifies that the model contains a negative diffusion coefficient (loosely referred to as anti-diffusion behaviour).

If we consider again the limit of vanishing τ, we have

$$\tau \to 0 \text{ and } K \to +\infty \quad \text{with} \quad K\tau^2 \to C_0\langle\epsilon\rangle. \tag{35}$$

Then

$$\alpha_{ij} \to 0 \ \beta_{ij} \to \frac{C_0\langle\epsilon\rangle}{2}\delta_{ij} \tag{36}$$

the pdf equation now becomes a real Fokker-Planck equation with a random term in velocity space

$$\frac{\partial f_r}{\partial t} + U_i \frac{\partial f_r}{\partial x_i} = \frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\frac{\partial f_r}{\partial U_i} + \left(\frac{1}{2} + \frac{3}{4}C_0\right)\frac{\langle\epsilon\rangle}{k}\frac{\partial\left([U_i - \langle U_i\rangle]\, f_r\right)}{\partial U_i} + \frac{1}{2}\frac{\partial^2\left[C_0\langle\epsilon\rangle f_r\right]}{\partial^2 U_i}. \tag{37}$$

This is indeed the pdf equation satisfied by f_{st} (showing again, but now in terms of pdfs, that in this limit $f_r = f_{st}$) and the corresponding trajectory equations are Eqs. (1)-(2).

6 Application for Two-Phase Flow Modelling

As mentioned in section 2.1 the acceleration-based model has direct application for two-phase flow modelling. If we leave out the question of how to account for particle inertia and crossing-trajectory effects and simply use a modified timescale T_L^* for the fluid velocity seen along particle trajectories, we have a complete form

$$\begin{aligned} dx_{p,i} &= V_{p,i}\, dt\,, && (38)\\ \left(1 + \frac{1}{2}C_a\frac{\rho_f}{\rho_p}\right) dV_{p,i} &= \frac{U_{fs,i} - V_{p,i}}{\tau_p}\, dt + (1 + \frac{1}{2}C_a)\frac{\rho_f}{\rho_p}A_{fs,i}\, dt + (1 - \frac{\rho_f}{\rho_p})g_i\, dt\,, && (39)\\ dU_{fs,i} &= A_{fs,i}\, dt = -\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\, dt - \frac{U_{fs,i} - \langle U_{fs,i}\rangle}{T_L^*}\, dt + \gamma_{fs,i}\, dt\,, && (40)\\ d\gamma_i &= -\frac{\gamma_i}{\tau}dt + \sqrt{\frac{C_0\langle\epsilon\rangle}{\tau^2}}dW_i. && (41) \end{aligned}$$

The complete form is useful for low-Reynolds but also for high-Reynolds flows. Indeed when τ tends towards 0, the elimination procedure can be applied. This reveals that the assumed necessity to have a model for the fluid acceleration and a differentiable process for the fluid velocity is in fact incorrect. As $\tau \to 0$, the fluid acceleration tends towards a white-noise process and the limit equations write

$$
\begin{aligned}
dx_{p,i} &= V_{p,i}\,dt\,, && (42)\\
\left(1+\frac{1}{2}C_a\frac{\rho_f}{\rho_p}\right)dV_{p,i} &= \frac{U_{fs,i}-V_{p,i}}{\tau_p}\,dt+(1-\frac{\rho_f}{\rho_p})g_i\,dt+\left(1+\frac{1}{2}C_a\right)\frac{\rho_f}{\rho_p}\times\\
&\quad\left(-\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\,dt-\frac{U_{fs,i}-\langle U_{fs,i}\rangle}{T_L^*}\,dt+\sqrt{C_0\langle\epsilon\rangle}dW_i\right) && (43)\\
dU_{fs,i} &= -\frac{1}{\rho_f}\frac{\partial\langle p\rangle}{\partial x_i}\,dt-\frac{U_{fs,i}-\langle U_{fs,i}\rangle}{T_L^*}\,dt+\sqrt{C_0\langle\epsilon\rangle}dW_i. && (44)
\end{aligned}
$$

Therefore, the limit set of equations *is* realizable. What happens is that as $\tau \to 0$ *both* fluid and solid particle velocity equations become SDEs at the same time. Then *both* processes, namely $\mathbf{V}_p$ and $\mathbf{U}_{fs}$ become diffusion processes with the same white-noise term $d\mathbf{W}$ in both equations. The second line in the 'solid particle' equation represents a model for the pressure gradient and added-mass forces. The mean pressure gradient accounts for the mean acceleration of the fluid particle while the fluctuating part including the white-noise term models the fluctuating acceleration.

7 Conclusion

This article has tried to put forward ideas that relevant to both the PDF approach and Lagrangian modelling of two-phase flows. At the core of both approches is the need to model instantaneous behaviour of fluid particles. An equation for fluid particle acceleration has been proposed which is useful for both domains. The model is built so as to revert to the Langevin model of Pope. Equivalence between the stochastic processes is not limited to low-order statistics but is worked out in terms of the trajectories of the process and alternatively in terms of the pdfs. The form of the model is obtained using the technique of fast variable elimination. This technique is a convenient tool that allows the behaviour of a model to be analyzed and reduced when widely separated timescales enter the equations. Its application has been illustrated for Lagrangian two-phase flow even at high-Reynolds to reveal that present models can handle bubbly flows without theoretical difficulties. It remains to assess how the model behaves in practical simulations.

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