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Decay constants in the heavy quark limit in models à la Bakamjian and Thomas

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are computed,

Abstract

In quark models à la Bakamjian and Thomas, that yield covariance and Isgur-Wise scaling of form factors in the heavy quark limit, ~~we compute~~ the decay constants $f^{(n)}$ and $f^{(n)}_{1/2}$ of S-wave and P-wave mesons composed of heavy and light quarks. Heavy quark limit scaling $\sqrt{M}\ f = $ Cst. is obtained, and it is shown that this class of models satisfies the sum rules involving decay constants and Isgur-Wise functions recently formulated by us in the heavy quark limit of QCD. Moreover, the model also satisfies the selection rules of the type $f^{(n)}_{3/2} = 0$ that must hold in this limit. We discuss Different Ansätze for the dynamics of the mass operator at rest. ~~For non-relativistic~~ are discussed. For non-relativistic kinetic energies $\frac{p^2}{2m}$ the decay constants are finite even if the potential $V(r)$ has a Coulomb part. For the relativistic form $\sqrt{p^2+m^2}$, the S-wave decay constants diverge if there is a Coulomb singularity. Using phenomenological models of the spectrum with relativistic kinetic energy and regularized short distance part (Godfrey-Isgur model or Richardson potential of Colangelo et al.), that yield $\rho^2 \cong 1$ for the elastic Isgur-Wise function, we compute the decay constants in the heavy quark limit and obtain $f_B \cong 300$ MeV, of the same order although slightly smaller than in the static limit of lattice QCD. We find the decay constants of D^{**} with $j = \frac{1}{2}$ of the same order of magnitude. The convergence of the heavy quark limit sum rules is also studied.

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1 Introduction

In a recent paper [1] we have shown that quark models of hadrons with a fixed number of constituents, based on the Bakamjian-Thomas (BT) formalism [2], yield form factors that are covariant and satisfy Isgur-Wise (IW) scaling [3] in the heavy mass limit for one of the quarks. In this class of models a lower bound is predicted for the slope of the heavy meson elastic IW function $\rho^2 > \frac{3}{4}$. Moreover, the model satisfies the Bjorken-Isgur-Wise sum rule [4] that relates the slope of the IW function to the P-wave form factors $\tau_{1/2}(w)$, $\tau_{3/2}(w)$ at zero recoil [5]. We have also explicitly computed the P-wave meson wave functions and the corresponding inelastic IW functions [6] within the model, we have made a numerical study of ρ^2 in a wide class of models of the meson spectrum (each of them characterized by an Ansatz for the mass operator M, i.e. the dynamics of the system at rest) [7], and a phenomenological study [8] of the elastic and inelastic IW functions and the corresponding $\frac{d\Gamma}{dw}$ for $B \to D$, D^*, $D^{**}\ell\nu$.

The purpose of the present paper is to study the decay constants of heavy mesons within the same approach.

2 Decay constants in the B-T scheme : heavy quark scaling

We want to compute in the model the decay constants of mesons $Q\bar{q}$, where Q and q are a heavy and a light quark, defined by

$$\begin{aligned} &< P(0^-)(v)|A_\mu^{qQ}|0>= N\ f_P\ \sqrt{M}\ v_\mu \\ &< V(1^-)(v,\varepsilon)|V_\mu^{qQ}|0>= N\ f_V\ \sqrt{M}\ \varepsilon^*_\mu \end{aligned} \qquad (1)$$

for S states, where $N = \frac{1}{\sqrt{2v^0}}$ and M is the mass of the corresponding bound state ($M = M_P$ or $M = M_V$), and similarly :

$$\begin{aligned} &<^{1/2}\ 0^+(v)|V_\mu^{qQ}|0>= N\ f^{(1/2)}\sqrt{M}\ v_\mu \\ &<^{1/2}\ 1^+(v,\varepsilon)|A_\mu^{qQ}|0>= N\ g^{(1/2)}\sqrt{M}\ \varepsilon^*_\mu \\ &<^{3/2}\ 1^+(v,\varepsilon)|A_\mu^{qQ}|0>= N\ g^{(3/2)}\sqrt{M}\ \varepsilon^*_\mu \end{aligned} \qquad (2)$$

for P states. The model satisfies the heavy quark limit relations (we use the phase convention for states of ref. [6])

$$f_P = f_V = f \tag{3}$$

$$g^{(1/2)} = -f^{(1/2)} \qquad g^{(3/2)} = 0 \quad . \tag{4}$$

In terms of the internal wave function we get (the superindex (n) means any radial excitation), for S-wave mesons :

$$\sqrt{M} f^{(n)} = \sqrt{N_c}\sqrt{2} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \sqrt{(p_2 \cdot v)(p_2 \cdot v + m^2)}\ \varphi^{(n)}(\mathbf{k}_2) \tag{5}$$

where the wave functions at rest of 0^- mesons are given by

$$\begin{aligned} \varphi^{(n)}_{s_1,s_2}(\mathbf{k}_2) &= \frac{i}{\sqrt{2}} (\sigma^2)_{s_1,s_2} \varphi^{(n)}(\mathbf{k}_2) \\ \int \frac{d\mathbf{p}}{(2\pi)^3} |\varphi^{(n)}(\mathbf{p})|^2 &= 1 \end{aligned} \tag{6}$$

and for P wave mesons :

$$\sqrt{M} f^{(n)}_{1/2} = \sqrt{\frac{2}{3}} \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi^{(n)}_{1/2}(\mathbf{k}_2) \left[(p_2 \cdot v)^2 - m_2^2 \right] \tag{7}$$

where the wave function at rest of 0^+ mesons is given by [5]

$$\begin{aligned} \varphi^{(n)}_{s_1,s_2}(\mathbf{k}_2) &= -\frac{i}{\sqrt{6}} \left[(\boldsymbol{\sigma} \cdot \mathbf{k}_2)\, \sigma^2 \right]_{s_1,s_2} \varphi^{(n)}_{1/2}(\mathbf{k}_2) \\ & \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} |\varphi^{(n)}_{1/2}(\mathbf{p})|^2 = 1 \quad . \end{aligned} \tag{8}$$

In these expressions,

$$p_2 = B_v\ k_2 \tag{9}$$

where B_v is the boost operator. The expressions (5) and (7) are Lorentz invariant since, due to the rotational invariance of $\varphi^{(n)}(\mathbf{k}_2)$, $\varphi^{(n)}_{1/2}(\mathbf{k}_2)$, they are functions of $v^2 = 1$. One can take simply the rest frame $v = (1, \mathbf{0})$.

Let us give a brief outline of the calculation. Within the model [1], the internal wave function in motion of a S-wave meson with center-of-mass momentum $\mathbf{P}$ (n denotes the radial excitation) is given in terms of :

$$\Psi^{(n)}_{s_1 s_2}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) = \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,n} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sum_{\{s_i'\}} \prod_{i=1,2} [D_i(\mathbf{R}_i)]_{s_i s_i'} \varphi^{(n)}_{s_1' s_2'}(\mathbf{k}_2) \qquad (10)$$

where $\varphi^{(n)}_{s_1 s_2}(\mathbf{k}_2)$ is the internal wave function at rest (5), and $\mathbf{R}_i$ are the Wigner rotations :

$$\mathbf{R}_i = \mathbf{B}_{p_i}^{-1}\, \mathbf{B}_{\Sigma p_j}\, \mathbf{B}_{k_i} \quad . \qquad (11)$$

The $q\bar{Q}$ to vacuum matrix element of a current, e.g. $J = \gamma_\mu \gamma_5$, reads

$$O_{s_1 s_2}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) =< 0|J|\mathbf{P}-\mathbf{p}_2 s_1; \mathbf{p}_2 s_2 >= [u_{s_1}(\mathbf{P}-\mathbf{p}_2)]^t\, i\gamma^2\gamma^0\gamma_\mu\gamma_5 u_{s_2}(\mathbf{p}_2) \quad (12)$$

The matrix element of interest to us

$$< 0|A^\mu|P^{(n)} >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sum_{\{s_i\}} O_{s_1 s_2}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) \Psi^{(n)}_{s_2 s_1}(\mathbf{P}-\mathbf{p}_2,\mathbf{p}_2) \qquad (13)$$

can be written, after some algebra, using (10)-(12) :

$$< 0|A^\mu|P^{(n)} >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sqrt{\frac{m_1}{p_1^0}} \sqrt{\frac{m_2}{p_2^0}} \varphi^{(n)}(\mathbf{k}_2)$$
$$\frac{1}{\sqrt{2}} \mathrm{Tr} \left\{ \frac{1+\gamma^0}{2} \mathbf{B}_{p_1}^{-1} \gamma^\mu \mathbf{B}_{p_2} \frac{1+\gamma^0}{2} \mathbf{B}_{p_2}^{-1} \mathbf{B}_{\Sigma p_j} \mathbf{B}_{k_2} \mathbf{B}_{k_1}^{-1} \mathbf{B}_{\Sigma p_j}^{-1} \mathbf{B}_{p_1} \right\} \qquad (14)$$

and since the projector $\frac{1+\gamma^0}{2}$ commutes with the Wigner rotations (11), one can rewrite :

$$< 0|A^\mu|P^{(n)} >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sqrt{\frac{m_1}{p_1^0}} \sqrt{\frac{m_2}{p_2^0}} \varphi^{(n)}(\mathbf{k}_2)$$
$$\frac{1}{\sqrt{2}} \mathrm{Tr} \left\{ \gamma^\mu \mathbf{B}_u \mathbf{B}_{k_2} \frac{1+\gamma^0}{2} \mathbf{B}_u^{-1} \mathbf{B}_u \frac{1+\gamma^0}{2} \mathbf{B}_{k_1}^{-1} \mathbf{B}_u^{-1} \right\} \qquad (15)$$

where $u = \frac{p_1 + p_2}{\sqrt{(p_1+p_2)^2}}$. Using the identities :

$$\mathbf{B}_u\, \mathbf{B}_{k_2} \frac{1+\gamma^0}{2} \mathbf{B}_u^{-1} = \frac{m_2 + \not{p}_2}{\sqrt{2m_2\,(k_2^0 + m_2)}} \frac{1+\not{u}}{2}$$
$$\mathbf{B}_u \frac{1+\gamma^0}{2} \mathbf{B}_{k_1}^{-1} \mathbf{B}_u^{-1} = \frac{1+\not{u}}{2} \frac{m_1 + \not{p}_1}{\sqrt{2m_1\,(k_1^0 + m_1)}} \qquad (16)$$

one obtains

$$< 0|A^\mu|P^{(n)} >= \frac{\sqrt{N_c}}{8} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} F(\mathbf{p}_2, \mathbf{P})$$
$$Tr\left[\gamma^\mu \left(m_2 + \not{p}_2\right)\left(1 + \not{u}\right)\left(m_1 + \not{p}_1\right)\right] \varphi^{(n)}(\mathbf{k}_2) \tag{17}$$

with $\mathbf{k}^2 = \mathbf{B}_u^{-1} p^2$ and

$$F(\mathbf{p}_2, \mathbf{P}) = \sqrt{2} \frac{\sqrt{u^0}}{p_1^0} \frac{\sqrt{k_1^0}}{\sqrt{k_1^0 + m_1}} \frac{\sqrt{k_2^0}}{\sqrt{k_2^0 + m_2}} \quad . \tag{18}$$

Finally, in the heavy mass limit [1] :

$$\begin{aligned} u &\to v & \frac{p_1}{m_1} &\to v \\ \frac{k_1^0}{m_1} &\to 1 & k_2^0 &\to \left(\mathbf{B}_v^{-1} p_2\right)^0 = p_2 \cdot v \end{aligned} \tag{19}$$

one obtains :

$$< 0|A^\mu|P^{(n)} >= \frac{1}{\sqrt{2v^0}} \frac{\sqrt{N_c}}{2\sqrt{2}} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}}$$
$$Tr\left[\gamma^\mu \left(m_2 + \not{p}_2\right)\left(1 + \not{v}\right)\right] \varphi^{(n)}(\mathbf{k}_2) \tag{20}$$

where $k_2 = \mathbf{B}_v^{-1} p_2$ and $\varphi^{(n)}(\mathbf{k}_2)$ depends on $\mathbf{k}_2^2 = (p_2 \cdot v)^2 - m_2^2$ because of rotational invariance.

Thus, the integrand (20) must be proportional to v^μ, yielding equation (5) for the decay constant. This expression exhibits the expected heavy quark limit scaling. Moreover, varying the current, one can easily show relations (3). It is also worth noticing that expression (5) becomes the well-known non-relativistic expression [9] if we assume that the light quark is also non-relativistic.

Let us give some details of the calculation of (6) and the corresponding relations (4). Mutatis mutandis, we consider the matrix element $< 0|V^\mu|0^+ >$ with the 0^+ wave functions given by (8). After some algebra, the matrix element equivalent to (15) is in the present case :

$$< 0|V^\mu|0^+ >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sqrt{\frac{m_1}{p_1^0}} \sqrt{\frac{m_2}{p_2^0}} \varphi_{1/2)}^{(n)}(\mathbf{k}_2)$$
$$\frac{1}{\sqrt{6}} \mathrm{Tr} \left\{ \gamma^\mu \gamma_5 \mathbf{B}_u \mathbf{B}_{k_2} \frac{1+\gamma^0}{2} \mathbf{B}_u^{-1} \mathbf{B}_u (\sigma \cdot \mathbf{k}_2) \mathbf{B}_u^{-1} \mathbf{B}_u \frac{1+\gamma^0}{2} \mathbf{B}_{k_1}^{-1} \mathbf{B}_u^{-1} \right\} \quad . \tag{21}$$

Using (16) and taking the limit (19), we obtain

$$< 0|V^\mu|0^+ >= \frac{\sqrt{N_c}}{4\sqrt{6}} \frac{1}{\sqrt{2v^0}} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi_{1/2}^{(n)}(\mathbf{k}_2)$$
$$\mathrm{Tr}\left[-\gamma^\mu \left(m_2 + \not{p}_2\right)\left(1 + \not{v}\right)\left(-\not{v}\not{p}_2 + p_2 \cdot v\right)\left(1 - \not{v}\right)\right] \quad (22)$$

and using the fact that this expression must be proportional to v^μ, we obtain expression (7), scaling invariant in the heavy quark limit. Moreover, varying the current and considering instead the matrix element $< 0|A^\mu|1^+_{1/2} >$ one can easily verify the first relation (4).

The result $g^{(3/2)} = 0$, that must hold in the heavy quark limit [10] deserves a few details. The matrix element to be considered is (m denotes the polarization of the state 1^+, and n the radial excitation) :

$$< 0|A^\mu|1^+_{3/2}, m >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sqrt{\frac{m_1}{p_1^0}} \sqrt{\frac{m_2}{p_2^0}}$$
$$\mathrm{Tr}\left\{\left(\frac{1+\gamma^0}{2}\sigma_2 \mathbf{B}_{p_1}^{-1} i\gamma^0\gamma_5\gamma^0\gamma^\mu\gamma_5 \mathbf{B}_{p_2}\frac{1+\gamma^0}{2}\right)\left[D(\mathbf{R}_2)\right]\left[\varphi^{(n)}(\mathbf{k}_2, m)\right]^t \left[D^t(\mathbf{R}_1)\right]\right\} \quad (23)$$

where the wave function is, for the $1^+ j = \dfrac{3}{2}$ states [6] :

$$\varphi_{s_1,s_2}^{(n)}(\mathbf{k}_2, m) = \frac{i}{\sqrt{2}} \mathbf{e}^{(m)} \cdot \left[\sqrt{\frac{2}{3}}\mathbf{k}_2 - \frac{i}{\sqrt{6}}\left(\mathbf{k}_2 \times \boldsymbol{\sigma}\right)\right] \sigma_2 \; \varphi_{3/2}^{(n)}(\mathbf{k}_2) \quad (24)$$

where $\mathbf{e}^{(m)}$ is a unit vector, and the rotational invariant function $\varphi_{3/2}^{(n)}(\mathbf{k}_2)$ is normalized according to (8). The equivalent expression to (21) will be

$$< 0|A^\mu|1^+_{3/2}, m >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sqrt{\frac{m_1}{p_1^0}} \sqrt{\frac{m_2}{p_2^0}} \frac{\varphi_{3/2}^{(n)}(\mathbf{k}_2)}{\sqrt{2}}$$
$$\mathrm{Tr}\left\{\gamma^\mu \mathbf{B}_u \mathbf{B}_{k_2} \frac{1+\gamma^0}{2} \mathbf{e}^{(m)} \cdot \left[\sqrt{\frac{2}{3}}\mathbf{k}_2 + \frac{i}{\sqrt{6}}(\mathbf{k}_2 \times \boldsymbol{\sigma})\right] \frac{1+\gamma^0}{2} \mathbf{B}_{k_1}^{-1} \mathbf{B}_u^{-1}\right\} \quad (25)$$

We need to compute the expression $\mathbf{B}_u \mathbf{e}^{(m)} \cdot \left[\sqrt{\frac{2}{3}}\mathbf{k}_2 + \frac{i}{\sqrt{6}}(\mathbf{k}_2 \times \boldsymbol{\sigma})\right] \mathbf{B}_u^{-1}$ in the heavy quark limit. After some algebra, one gets :

$$\mathbf{B}_u \; \mathbf{e}^{(m)} \cdot \left[\sqrt{\frac{2}{3}}\mathbf{k}_2 + \frac{i}{\sqrt{6}}(\mathbf{k}_2 \times \boldsymbol{\sigma})\right] \mathbf{B}_u^{-1} \to \frac{1}{\sqrt{6}}\left[-\not{\varepsilon}_v^{(m)}\not{p}_2 + \not{\varepsilon}_v^{(m)}\not{v}(p_2 \cdot v) - \varepsilon_v^{(m)} \cdot p_2\right] \quad (26)$$

where $\varepsilon_v^{(m)}$ is the axial meson polarization in the heavy quark limit. Using (16) in the limit (19) we obtain finally,

$$< 0|A^\mu|1^+_{3/2}, m >= \frac{\sqrt{N_c}}{8\sqrt{3}} \frac{1}{\sqrt{2v^0}} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi_{3/2}(\mathbf{k}_2)$$
$$\mathrm{Tr} \left\{ \gamma^\mu \left(m_2 + \not{p}_2 \right) (1 + \not{v}) \left[-\not{\varepsilon}_v^{(m)} \not{p}_2 + \not{\varepsilon}_v^{(m)} \not{v} (p_2 \cdot v) - \varepsilon_v^{(m)} \cdot p_2 \right] (1 + \not{v}) \right\} \quad (27)$$

This expression is covariant and satisfies heavy quark scaling. In the rest frame, particularizing to $m = 0$, one gets, because of rotational invariance of $\varphi_{3/2}(\mathbf{k}_2)$:

$$< 0|A^z|1^+_{3/2}, \mathbf{P} = \mathbf{0}, m = 0 >= \frac{\sqrt{N_c}}{\sqrt{3}} \frac{1}{\sqrt{2v^0}} \int \frac{d\mathbf{k}_2}{(2\pi)^3} \frac{1}{k_2^0} \frac{\sqrt{k_2^0}}{\sqrt{k_2^0 + m_2}} \varphi_{3/2}(\mathbf{k}_2)$$
$$\left\{ - \left[(k_2^x)^2 + (k_2^y)^2 \right] + 2(k_2^z)^2 \right\} = 0 \quad . \quad (28)$$

Therefore the second relation (4), $g^{(3/2)} = 0$, follows. The vanishing of $g^{(3/2)}$ can be seen also from the following covariant argument. Contracting (27) with the four-vector $\varepsilon_v^{(m)}$ we see that $g^{(3/2)}$ is proportional to the integral

$$g^{(3/2)} \sim \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi_{3/2}(\mathbf{k}_2) \left[3(\varepsilon_v^{(m)} \cdot p_2)^2 + (v \cdot p_2)^2 - m_2^2 \right] \quad . \quad (29)$$

Two types of integrals appear :

$$I_{\mu\nu} = \int \frac{d\mathbf{p}_2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi_{3/2}(\mathbf{k}_2)\ p_{2\mu}\ p_{2\nu} \quad (30)$$
$$I = \int \frac{d\mathbf{p}^2}{(2\pi)^3} \frac{1}{p_2^0} \frac{\sqrt{p_2 \cdot v}}{\sqrt{p_2 \cdot v + m_2}} \varphi_{3/2}(\mathbf{k}_2) \quad . \quad (31)$$

From covariance it follows that

$$I_{\mu\nu} = A v_\mu v_\nu + B g_{\mu\nu}$$
$$I = C \quad (32)$$

where A, B, C are constants. Contracting $I_{\mu\nu}$ with $g_{\mu\nu}$ it follows

$$4B + A = m_2^2 C \quad (33)$$

and therefore

$$g^{(3/2)} \sim 4B + A - m_2^2 C = 0 \quad . \quad (34)$$

3 Sum rules

Let us now show that the QCD heavy quark limit sum rules [10]

$$X(w) \equiv \sum_n \frac{f^{(n)}}{f^{(0)}} \xi^{(n)}(w) = 1 \qquad (35)$$

$$T_{1/2}(w) \equiv \sum_n \frac{f_{1/2}^{(n)}}{f^{(0)}} \tau_{1/2}^{(n)}(w) = \frac{1}{2} \qquad (36)$$

are satisfied within the present B-T scheme.

In order to prove Bjorken sum rule, in ref. [5] we used the completeness relation at fixed $\mathbf{P}$ that holds in the B-T formalism for the internal wave functions. We proceed here in the same way. We need to compute expressions of the form :

$$\sum_n < 0|\widetilde{O}|\mathbf{P}', n >< \mathbf{P}', n|O|\mathbf{P}, 0 > \qquad (37)$$

where $< 0|\widetilde{O}|\mathbf{P}', n >$ and $< \mathbf{P}', n|O|\mathbf{P}, 0 >$ will be related respectively to the decay constants and to the Isgur-Wise functions. We need $\mathbf{P}'$ different from $\mathbf{P}$ in order to demonstrate the sum rules (35)-(36) for any value of the scaling variable w.

To obtain the sum rule (35) for S-waves, the simplest way is to take $\mathbf{P}' = 0$ for the intermediate states and choose the currents $O = \gamma^0\gamma^\nu$, and $\widetilde{O} = \gamma^0\gamma^\mu\gamma_5$ with $\mu = 0$. Then, only the 0^- intermediate states contribute, because the 1^+ states do not at $\mathbf{P}' = 0$, since then the time component of the polarization $\varepsilon_0^{(m)} = 0$. Alternatively, to obtain the sum rule (36) for P-waves, we will take $\mathbf{P}' = 0$ and $O = \gamma^0\gamma^\nu\gamma_5$, and $\widetilde{O} = \gamma^0\gamma^\mu$ with $\mu = 0$. Then, only the 0^+ intermediate states contribute, since the 1^- do not at $\mathbf{P}' = 0$.

We start from the completeness relation at fixed $\mathbf{P}'$:

$$\sum_n \Psi_{s_1''s_2''}^{(n)}(\mathbf{P}' - \mathbf{p}_2'', \mathbf{p}_2'')\, \Psi_{s_1's_2'}^{(n)*}(\mathbf{P}' - \mathbf{p}_2', \mathbf{p}_2') = \delta_{s_1's_1''}\, \delta_{s_2's_2''}\, (2\pi)^3\, \delta(\mathbf{p}_2' - \mathbf{p}_2'') \quad . \qquad (38)$$

We need the matrix elements

$$< \mathbf{P}', n|O|\mathbf{P}, 0 >= \int \frac{d\mathbf{p}_2}{(2\pi)^3} \int \frac{d\mathbf{p}_2'}{(2\pi)^3} \sum_{\{s_i\}} \sum_{\{s_i'\}}$$
$$\Psi_{s_1's_2'}^{n(\mathbf{P}')*}(\mathbf{p}_1', \mathbf{p}_2') O_{s_1's_1}(\mathbf{p}_1', \mathbf{p}_1) \Psi_{s_1s_2}^{0(\mathbf{P})}(\mathbf{p}_1, \mathbf{p}_2)(2\pi)^3\delta(\mathbf{p}_2 - \mathbf{p}_2')\delta_{s_2's_2} \qquad (39)$$

$$< 0|\tilde{O}|\mathbf{P}, n >= \sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sum_{\{s_i\}} \tilde{O}_{s_1 s_2}(\mathbf{p}_1, \mathbf{p}_2) \Psi^{n(\mathbf{P})}_{s_2 s_1}(\mathbf{p}_1, \mathbf{p}_2) \tag{40}$$

related respectively to the IW functions and decay constants. In these expressions :

$$O(\mathbf{p}'_1, \mathbf{p}_1) = \frac{\sqrt{m_1 m'_1}}{\sqrt{p_1^0 p_1'^0}} \frac{1+\gamma^0}{2} \mathbf{B}^{-1}_{p'_1} \, O \, \mathbf{B}_{p_1} \frac{1+\gamma^0}{2} \tag{41}$$

$$\tilde{O}(\mathbf{P}' - \mathbf{p}_2, \mathbf{p}_2) = \sqrt{\frac{m_1}{p_1'^0}} \sqrt{\frac{m_2}{p_2^0}} \frac{1+\gamma^0}{2} \sigma_2 \mathbf{B}^{-1}_{p'_1} i\gamma^0 \gamma_5 \tilde{O} \mathbf{B}_{p_2} \frac{1+\gamma^0}{2} \tag{42}$$

and here O and $\tilde{O}$ are the Dirac matrices of the corresponding currents. After some algebra we obtain, using the completeness relation (38) :

$$\sum_n < 0|\tilde{O}|\mathbf{P}', n >< \mathbf{P}', n|O|\mathbf{P}, 0 >= \tag{43}$$

$$\sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sum_{\{s_i\}} \sum_{s'_1} \tilde{O}_{s'_1 s_2} \left(\mathbf{P}' - \mathbf{p}_2, \mathbf{p}_2\right) O_{s'_1 s_1} \left(\mathbf{P}' - \mathbf{p}_2, \mathbf{P} - \mathbf{p}_2\right) \Psi^{(0)}_{s_1 s_2} \left(\mathbf{P} - \mathbf{p}_2, \mathbf{p}_2\right)$$

where the ground state wave function is given by [1]

$$\Psi^{(0)}_{s_1 s_2}(\mathbf{P} - \mathbf{p}_2, \mathbf{p}_2) = \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \sum_{\{s'_i\}} \left[D(\mathbf{R}_1)\right]_{s_1 s'_1} \left[D(\mathbf{R}_2)\right]_{s_2 s'_2} \varphi^{(0)}_{s'_1 s'_2}(\mathbf{k}_2)$$

$$= \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \left[D(\mathbf{R}_1) \varphi^{(0)}(\mathbf{k}_2) D^t(\mathbf{R}_2)\right]_{s_1 s_2} \tag{44}$$

and using (6), in an obvious notation :

$$\sum_n < 0|\tilde{O}|\mathbf{P}', n >< \mathbf{P}', n|O|\mathbf{P}, 0 >=$$

$$\sqrt{N_c} \int \frac{d\mathbf{p}_2}{(2\pi)^3} \sqrt{\frac{\Sigma p_j^0}{M_0}} \prod_{i=1,2} \frac{\sqrt{k_i^0}}{\sqrt{p_i^0}} \frac{i}{\sqrt{2}} \varphi^{(0)}(\mathbf{k}_2)$$

$$\mathrm{Tr} \left[\tilde{O}^t \left(\mathbf{P}' - \mathbf{p}_2, \mathbf{p}_2\right) O \left(\mathbf{P}' - \mathbf{p}_2, \mathbf{P} - \mathbf{p}_2\right) D(\mathbf{R}_1) \sigma_2 D^t(\mathbf{R}_2)\right] \tag{45}$$

where the matrices $\mathbf{R}_i$ are given by (11).

To isolate the 0^- states, we take, as argued above :

$$\sum_n <0|A^0|\mathbf{0},n><\mathbf{0},n|V^\nu|\mathbf{P},0>=$$

$$\sqrt{N_c}\int\frac{d\mathbf{p}_2}{(2\pi)^3}\sqrt{\frac{\Sigma p_j^0}{M_0}}\prod_{i=1,2}\frac{\sqrt{k_i^0}}{\sqrt{p_i^0}}\sqrt{\frac{m_1m_1'}{p_1^0p_1'^0}}\sqrt{\frac{m_1}{p_1'^0}}\sqrt{\frac{m_2}{p_2^0}}\frac{i}{\sqrt{2}}\varphi^{(0)}(\mathbf{k}_2) \tag{46}$$

$$\mathrm{Tr}\left\{\left[\frac{1+\gamma^0}{2}\sigma_2\mathbf{B}_{p_1'}^{-1}i\gamma^0\mathbf{B}_{p_2}\frac{1+\gamma^0}{2}\right]^t\left[\frac{1+\gamma^0}{2}\mathbf{B}_{p_1'}^{-1}\gamma^0\gamma^\nu\mathbf{B}_{p_1}\frac{1+\gamma^0}{2}\right]D(\mathbf{R}_1)\sigma_2D^t(\mathbf{R}_2)\right\}$$

Using $\sigma_2(\mathbf{B}_p)^t\sigma_2=\mathbf{B}_p^{-1}$, $\sigma_2(\gamma^\mu)^t\sigma_2=\gamma^\mu$, $\mathbf{B}_p\frac{1+\gamma^0}{2}\mathbf{B}_p^{-1}=\frac{1}{2}\left(1+\frac{\not p}{m}\right)$, the fact that $\frac{1}{2}(1+\gamma^0)$ commutes with rotations, and the relations

$$\mathbf{B}_v\mathbf{B}_{k_1}\mathbf{B}_v^{-1}=\frac{m_1+\not p_1\not v}{\sqrt{2m_1\left(k_1^0+m_1\right)}}\qquad \mathbf{B}_v\mathbf{B}_{k_2}^{-1}\mathbf{B}_v^{-1}=\frac{m_2+\not v\not p_2}{\sqrt{2m_2\left(k_2^0+m_2\right)}} \tag{47}$$

one obtains, after some algebra, in the heavy quark limit ($p_1\to m_1v, k_1^0\to m_1$), for the r.h.s. of (46) :

$$\begin{aligned}&\sqrt{N_c}\frac{1}{\sqrt{v^0v'^0}}\frac{1}{\sqrt{v'^0}}\int\frac{d\mathbf{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\sqrt{m_2}\sqrt{k_2^0}\frac{1}{\sqrt{2}}\varphi^{(0)}(\mathbf{k}_2)\\&\mathrm{Tr}\left\{\frac{1}{2}\left(1+\gamma^0\right)\gamma^\nu\frac{1}{2}(1+\not v)\mathbf{B}_v\mathbf{B}_{k_1}\mathbf{B}_{k_2}^{-1}\mathbf{B}_v^{-1}\right\}=\\&\sqrt{N_c}\frac{1}{\sqrt{4v^0v'^0}}\frac{1}{\sqrt{2v'^0}}\int\frac{d\mathbf{p}_2}{(2\pi)^3}\frac{1}{p_2^0}\frac{\sqrt{p_2\cdot v}}{\sqrt{p_2\cdot v+m_2}}\frac{1}{\sqrt{2}}\varphi^{(0)}(\mathbf{k}_2)\frac{1}{2}\\&\mathrm{Tr}\left\{\left(1+\gamma^0\right)\gamma^\nu(1+\not v)\left(m_2+\not p_2\right)\right\}=\frac{1}{\sqrt{4v^0v'^0}}\frac{1}{\sqrt{2v'^0}}\sqrt{M}f^{(0)}\left(g^{0\nu}+v^\nu\right).\end{aligned} \tag{48}$$

On the other hand, the l.h.s. of (46) reads :

$$\begin{aligned}&\sum_n <0|A^0|\mathbf{0},n><\mathbf{0},n|V^\nu|\mathbf{P},0>=\\&\frac{1}{\sqrt{4v^0v'^0}}\frac{1}{\sqrt{2v'^0}}\sum_n\sqrt{M}f_P^{(n)}\xi^{(n)}(w)\left(g^{0\nu}+v^\nu\right)\end{aligned} \tag{49}$$

and therefore the sum rule (35) follows.

The sum rule for the P-states (36) follows straightforwardly in a similar manner by taking $\mathbf{P}'=0$ and $O=\gamma^0\gamma^\nu\gamma_5$, $\widetilde{O}=\gamma^0\gamma^\mu$ with $\mu=0$, since then only the 0^+ intermediate states contribute, as pointed out above. However, to illustrate the methods of calculation in the BT formalism, we will now verify the further sum rule

(36) from the general expressions for $f_{1/2}^{(n)}$ and $\tau_{1/2}^{(n)}(w)$, without appealing to any particular frame.

We want to evaluate the expression

$$T_{1/2}(w)f^{(0)} = \sum_n f_{1/2}^{(n)}\, \tau_{1/2}^{(n)}(w) \quad . \tag{50}$$

From the explicit expressions (7) for $f_{1/2}^{(n)}$ and $\tau_{1/2}^{(n)}(w)$ from ref. [6] :

$$\tau_{1/2}^{(n)}(w) = \frac{1}{2\sqrt{3}} \int \frac{d\mathbf{p}_2'}{(2\pi)^3} \frac{1}{p_2'^0} \frac{\sqrt{(p_2' \cdot v')(p_2' \cdot v)}}{\sqrt{(p_2' \cdot v + m_2)(p_2' \cdot v' + m_2)}} \varphi_{1/2}^{(n)}(\mathbf{k}_2')^* \varphi^{(0)}(\mathbf{k}_2'')$$
$$\times \frac{(p_2' \cdot v')(p_2'.v + m_2) - (p_2' \cdot v)(p_2' \cdot v + wm_2) + (1-w)m_2^2}{1-w} \tag{51}$$

with

$$k_2' = \mathbf{B}_v^{-1} p_2' \qquad\qquad k_2'' = \mathbf{B}_{v'}^{-1} p_2' \quad . \tag{52}$$

The computation of the sum (50) leads to the expression

$$\sum_n \varphi_{1/2}^{(n)}(\mathbf{k}_2')^* \varphi_{1/2}^{(n)}(\mathbf{p}_2) = 6\pi^2 \frac{1}{\mathbf{p}_2^2\, \mathbf{k}_2'^2} \delta(|\mathbf{p}_2| - |\mathbf{k}_2'|) \tag{53}$$

where the r.h.s. follows from ref. [6]. One can then perform the integration over $\mathbf{p}_2$ that amounts to replace all p_2^0 by $(p_2' \cdot v)$. Realizing then that

$$\frac{(p_2' \cdot v')(p_2' \cdot v + m_2) - (p_2' \cdot v)(p_2' \cdot v + wm_2) + (1-w)m_2^2}{1-w} =$$
$$\mathbf{k}_2'^2 \left(-1 + \frac{p_2' \cdot v' - w(p_2' \cdot v)}{(p_2' \cdot v - m_2)(1-w)} \right) \tag{54}$$

one gets

$$2T_{1/2}(w)f^{(0)} = 2\sum_n f_{1/2}^{(n)} \tau_{1/2}^{(n)}(w) =$$
$$f^{(0)} - \sqrt{\frac{2N_c}{\pi}} \int \frac{d\mathbf{p}_2'}{(2\pi)3} \frac{1}{p_2'^0} \frac{\sqrt{(p_2' \cdot v')}}{\sqrt{(p_2' \cdot v' + m_2)}} \varphi^{(0)}(\mathbf{k}_2'') \frac{p_2' \cdot v - w(p_2' \cdot v')}{1-w} \quad . \tag{55}$$

The second term involves the integral

$$\int \frac{d\mathbf{p}_2'}{(2\pi)^3} \frac{1}{p_2'^0} \frac{\sqrt{(p_2' \cdot v')}}{\sqrt{(p_2' \cdot v' + m_2)}} \varphi^{(0)}(\mathbf{k}_2'') p_\mu' = C v_\mu' \tag{56}$$

where C is some constant, because of covariance. Inserting the r.h.s. of (56) into (55) we see that the second term of the r.h.s. of (55) vanishes for $w \neq 1$, and then the sum rule (36) follows. For $w = 1$, one can choose $v = (v^0, \mathbf{v})$, $v' = (1, \mathbf{0})$ and make an expansion of $(w-1)^{-1}$ for $\mathbf{v} \to \mathbf{0}$. Then, the second term in the r.h.s. of (55) vanishes from rotational invariance. The same method allows also to obtain the sum rule (35) along similar lines.

4 Numerical results

As explained at length in ref. [1], the dynamics in the B-T formalism depends on the form of the mass operator M at $\mathbf{P} = 0$. This mass operator can be of any form

$$M = K(\{\mathbf{k}_i\}) + V(\{\mathbf{r}_i, \mathbf{p}_i\}) \tag{57}$$

and one obtains covariance of the form factors and IW scaling with only the very general assumption of rotational invariance of M. We are going now to give the results for the decay constants for various Ansätze of the operator M, not only various forms for the potential $V(\{\mathbf{r}_i, \mathbf{p}_i\})$, but also for the kinetic energy $K(\{\mathbf{k}_i\})$, that can be taken to be of the non-relativistic $\frac{\mathbf{k}_i^2}{2m_i}$ or relativistic $\sqrt{\mathbf{k}_i^2 + m_i^2}$ forms. We choose such models in order to emphasize the physics involved in the decay constants, sensitive to the short distance part of the potential and to the scaling behaviour of the kinetic energy (quadratic or linear in k). Of course, any scheme that we adopt should give a reasonable fit of the whole meson spectrum. The success of quark models in the description of the spectrum with either non-relativistic or relativistic kinetic energies shows that the spectrum by itself does not constrain the form of this kinetic energy. Interestingly, we have shown in ref. [7] that quark models of form factors in the BT formalism show a clear preference for the relativistic form of the kinetic energy, since they give a slope of the elastic IW function $\rho^2 \cong 1$, while models with a non-relativistic kinetic energy give a larger, phenomenologically unacceptable value.

We list here a number of phenomenological quark models of the hadron spectrum that we will use, specifying their interesting features :

1) Isgur, Scora, Grinstein and Wise (ISGW) spectroscopic model [11] (to be distinguished from the ISGW non-relativistic model of form factors) : non-relativistic

kinetic energy $\frac{\mathbf{k}_i^2}{2m_i}$ with linear plus Coulomb potential.

2) Veseli and Dunietz (VD) model [12] : relativistic kinetic energy $\sqrt{\mathbf{k}_i^2 + m_i^2}$ with linear plus Coulomb potential. The Coulomb part is not regularized.

3) Godfrey and Isgur model (GI) [13] : relativistic kinetic energy $\sqrt{\mathbf{k}_i^2 + m_i^2}$ with linear plus a regularized short distance part. This scheme also incorporates the fine structure of the potential.

4) Richardson potential with relativistic kinetic energy, as used by Colangelo, Nardulli and Pietroni (CNP) [14]. This model exhibits the asymptotic freedom behaviour at short distances, but the Coulomb singularity (logarithmically corrected) is regularized by a cut at small r.

These models are solved numerically using a harmonic oscillator basis. In Table 1 we give the masses and decay constants of the states $n = 0$, $\ell = 0$ or 1 as a function of N_{max} in the different models. N_{max} means the number of radial excitations included in the truncated basis, the ground state gaussian plus up to N_{max} radially excitated harmonic oscillator wave functions : $N_{max} + 1$ is then the dimension of the truncated Hilbert space. In Table 2 we give the decay constants of a number of radial excitations in the GI model.

Let us begin our discussion with the models with relativistic kinetic energy. A first important remark to make is that dimensional analysis implies that a Hamiltonian (or mass operator in the B-T formalism) with kinetic energy of the relativistic form $\sqrt{\mathbf{k}_i^2 + m_i^2}$ and a Coulomb potential implies divergent wave functions at the origin $\Psi^{(n)}(0)$ (or decay constants $f^{(n)}$) for S-waves. The reason for this behaviour is that the kinetic and the potential energies exhibit the same scaling properties at large momentum or small distances. The relativistic kinetic energy is not efficient enough in smoothing the r-space wave function. This divergence of the wave function at the origin is the cause of the existence of a critical coupling α^{crit} in this class of models : for $\alpha > \alpha^{crit}$ arises the so-called phenomenon of fall in the center. For a discussion, see for example the paper by Hardekopf and Sucher [15]. On the contrary, the decay constants for P-wave mesons remain finite even in this case. A short distance Coulomb part corrected by asymptotic freedom exhibits the same phenomenon, although $f^{(n)}$ diverges only logarithmically in this latter case, instead of as a power in the former. These general features are exhibited by the model of Veseli and Dunietz [12], as shown in the Table, and have been underlined by these authors. In our Table, the finite values obtained for $\sqrt{M}\ f^{(0)}$ using the singular model for $N_{max} = 10$, 15 or 20 are just an artifact of the truncation method.

The GI model [13], a model of the meson spectrum for all $q\bar{q}$, $Q\bar{q}$ and $Q\bar{Q}$ systems, chooses a short distance part with a fully regularized Coulomb singularity. Within this model it is possible to compute both types of decay constants $f^{(n)}$ and $f^{(n)}_{1/2}$. In the Table we give the scaling invariant quantities $\sqrt{M}f^{(0)}$ and $\sqrt{M}f^{(0)}_{1/2}$ for S and P-wave $n = 0$ mesons, computed in the heavy quark limit.

We have also made the calculation for the CNP model (Richardson potential plus relativistic kinetic energy) [14].

In the case of a kinetic energy of the non-relativistic form $\frac{\mathbf{k}_i^2}{2m_i}$ the S-wave decay constants are finite even in the presence of a Coulomb singularity. This feature is examplified by the ISGW [11] model in Table 1. Moreover, the decay constants are smaller in this case than for the models of relativistic kinetic energy, another manifestation of the singular behavior of the latter.

It is interesting to notice that in the heavy quark limit and for $N_{max} = 20$ one gets, for the models with relativistic kinetic energy and regularized Coulomb singularity :

$$\begin{aligned} \sqrt{M}f^{(0)} &= 0.67 \text{ GeV}^{3/2} \text{ (GI model) ; } 0.83 \text{ GeV}^{3/2} \text{ (CNP model)} \\ \sqrt{M}f^{(0)}_{1/2} &= 0.64 \text{ GeV}^{3/2} \text{ (GI model) ; } 0.70 \text{ GeV}^{3/2} \text{ (CNP model)} \quad . \end{aligned} \tag{58}$$

Applying this asymptotic result to the B meson, one obtains

$$f_B \cong 300 \text{ MeV - } 350 \text{ MeV} \quad . \tag{59}$$

This value is not far away, although slightly larger than the values obtained in lattice QCD in the static limit, to which it should naturally be compared, which ranges between 220 and 290 MeV [16]. However, one should keep in mind that the lattice QCD result includes logarithmic corrections absent in our phenomenological scheme.

It is worth noticing that we obtain the same order of magnitude for the decay constant $f^{(0)}_{1/2}$. This is phenomenologically important because, reasoning within the factorization assumption, this means that the emission of $D^{**}(0^+)$ and $D^{**}(1^+, j = \frac{1}{2})$ is expected to be important in B decays. On the contrary, the emission of $D^{**}(j = \frac{3}{2})$ will be suppressed. Table 2 shows that the decay constants of radial excitations are of the same order of magnitude as in the ground state. However, the error due to the truncation is larger as n increases.

Finally, let us study the convergence of the sum rules (35) and (36), that we have shown formally to hold, in models that give finite results (GI, CNP and ISGW) for which we can compute the decay constants $f^{(n)}$, $f_{1/2}^{(n)}$ and the IW functions $\xi^{(n)}(w)$, $\tau_{1/2}^{(n)}(w)$ [8]. The convergence of the Bjorken-Isgur-Wise sum rule [4] has been studied in ref. [8].

Let us define

$$X^{(N)}(w) = \sum_{n=0}^{N} \frac{f^{(n)}}{f^{(0)}} \xi^{(n)}(w) \qquad\qquad T_{1/2}^{(N)}(w) = \sum_{n=0}^{N} \frac{f_{1/2}^{(n)}}{f^{(0)}} \tau_{1/2}^{(n)}(w) \quad . \tag{60}$$

We compute the sums for $N = N_{max}$ in the different models for various values of w, and see how they compare to the r.h.s. of the sum rules (35) and (36). Let us recall that N_{max} is the maximal number of radial excitations included in the truncation method (the dimension of the variational base is $N_{max}+1$). We show the results for the Godfrey-Isgur model in Figures 1 and 2. The Ox axis represents $1/(N_{max}+1)$. We observe that these sums converge fairly well towards the r.h.s. of the sum rules (respectively 1 and $\frac{1}{2}$) as we increase N_{max}. For fixed N_{max} we can ask how the partial sums $X^{(N)}(w)$ and $T_{1/2}^{(N)}(w)$ ($N \leq N_{max}$) behave as a function of N, i.e. how fast $X^{(N)}(w)$, $T_{1/2}^{(N)}(w)$ approach $X^{(N_{max})}(w)$, $T_{1/2}^{(N_{max})}(w)$ when N increases ($N \geq 0$). Let us give the example $N_{max} = 20$. The convergence is rather fast, but it degrades as w increases. Concerning the X sum rule, for $w = 1$ one has trivially $X^{(0)}(1) = X^{(N_{max})}(1)$, because the ground state saturates the sum, since $\xi^{(n)}(1) = 0$ ($n > 0$). As one increases w, one one needs to sum up to $N = 3$, 4, 5, 6 or 7 respectively for $w = 1.1$, 1.2, 1.3, 1.4 or 1.5 to approach $X^{(N_{max})}(w)$ at the 5 % level. Concerning the $T_{1/2}$ sum rule, one needs $N = 3$, 4, 5, 6, 7 or 8 respectively for $w = 1.1$, 1.2, 1.3, 1.4 or 1.5 to approach $T_{1/2}^{(N_{max})}(w)$ at the 10 % level.

For the CNP model and the non-relativistic ISGW model the convergences both in N_{max} and in N for fixed N_{max} are not as good. For the case of the VD model one gets finite $f_{1/2}^{(n)}$ and $\tau_{1/2}^{(n)}(w)$, but the convergence toward the r.h.s. of the sum rule (30) does not improve as one increases N_{max}. This results from the divergence of the denominator $f^{(0)}$ in (35), (36) when $N_{max} \to \infty$.

In conclusion, we have studied the decay constants of heavy-light mesons in the heavy mass limit in a class of models à la Bakamjian and Thomas, with different Ansätze for the dynamics at rest, with non-relativistic or relativistic kinetic energies. Each particular model gives an acceptable phenomenological description of the spectrum. The models with relativistic kinetic energy, that yield a slope of the elastic IW function $\rho^2 \cong 1$ (as shown in ref. [7]), give finite decay constants if the

Coulomb singularity of the potential is regularized, as in the GI model. At the B mass, one finds f_B slightly larger than in the static limit of lattice QCD. The decay constants of D^{**} with $j = \frac{1}{2}$ are of the same order of magnitude. Moreover, we have shown that heavy quark limit sum rules involving decay constants [10] are satisfied by these class of quark models à la Bakamjian and Thomas, and in the case of the Godfrey-Isgur model the convergence of the sum rules is quite fast.

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Table captions

Table 1. Decay constants of $n = 0$, $\ell = 0$ and $n = 0$, $\ell = 1$, $j = \frac{1}{2}$ mesons in the various models. The ISGW is non-relativistic. In the VD, GI and CNP models, the kinetic energy is relativistic. In the VD model the S-wave decay constants diverge due to the Coulomb singularity. In the GI and CNP models the Coulomb singularity is regularized and the decay constants are finite. N_{max} stands for the number of radial excitations included in the truncated variational harmonic oscillator basis.

Table 2. Decay constants $f^{(n)}$ and $f^{(n)}_{1/2}$ for the first radial excitations of $\ell = 0$ and $\ell = 1$, $j = \frac{1}{2}$ mesons. The error in parenthesis is estimated by comparing the number of radial excitations included in the truncated variational harmonic oscillator basis $N_{max} = 20$ and $N_{max} = 10$. For $n \gtrsim 5$ the error becomes larger than 20 %.

Figure captions

Figure 1. Convergence of the heavy quark limit S-wave sum rule (35) in the GI model [13] for different values of the scaling variable w as one increases N_{max}, the number of radial levels included in the truncated variational harmonic oscillator basis. The Ox axis represents $(N_{max}+1)^{-1}$ and the Oy-axis $\sum_{n=0}^{N_{max}} \frac{f^{(n)}}{f^{(0)}}\xi^{(n)}(w)$, that the sum rule predicts to be equal to 1 for $N_{max} \to \infty$. The different lines correspond to $w = 1.0$, 1.1, 1.2, 1.3, 1.4, 1.5, from up to down.

Figure 2. Convergence of the heavy quark limit S-wave sum rule (36) in the GI model [13] for different values of the scaling variable w as one increases N_{max}, the number of radial levels included in the truncated variational harmonic oscillator basis. The Ox axis represents $(N_{max}+1)^{-1}$ and the Oy-axis $\sum_{n=0}^{N_{max}} \frac{f^{(n)}_{1/2}}{f^{(0)}}\tau^{(n)}_{1/2}(w)$, that the sum rule predicts to be equal to $\frac{1}{2}$ for $N_{max} \to \infty$. The different lines correspond to $w = 1.0$, 1.1, 1.2, 1.3, 1.4, 1.5, from up to down.

MODEL	M_Q (GeV)	N_{max}	$M^{(0)}$-M_Q (GeV)	$M^{(0)}_{1/2}$-M_Q (GeV)	$\sqrt{M}f^{(0)}$ ($GeV^{3/2}$)	$\sqrt{M}f^{(0)}_{1/2}$ ($GeV^{3/2}$)
ISGW [11]	10^4	10	0.0438	0.5467	0.422	0.235
	10^4	15	0.0436	0.5467	0.428	0.235
	10^4	20	0.0435	0.5467	0.431	0.236
		infinite			finite	finite
VD [12]	10^4	10	0.119	0.620	1.36	0.603
	10^4	15	0.108	0.620	1.58	0.617
	10^4	20	0.108	0.620	1.76	0.631
		infinite			infinite	finite
GI [13]	10^4	10	0.386	0.792	0.649	0.620
	10^4	15	0.386	0.792	0.662	0.632
	10^4	20	0.386	0.792	0.667	0.640
	10^4	infinite			finite	finite
CNP [14]	10^4	10	0.389	0.859	0.747	0.669
	10^4	15	0.387	0.858	0.798	0.691
	10^4	20	0.386	0.858	0.828	0.704
		infinite			finite	finite

Table 1

radial excitation	$\sqrt{M}f^{(n)}$ ($GeV^{3/2}$)	$\sqrt{M}f^{(n)}_{1/2}$ ($GeV^{3/2}$)
n = 0	0.67(2)	0.64(2)
n = 1	0.73(4)	0.66(4)
n = 2	0.76(5)	0.71(5)
n = 3	0.78(9)	0.73(8)
n = 4	0.80(10)	0.76(11)
n = 5	0.81(17)	0.77(17)
n = 6	0.82(15)	0.78(15)
n = 7	0.82(28)	0.78(27)
n = 8	0.83(25)	0.79(25)
n = 9	0.80(40)	0.76(40)
n = 10	0.83(42)	0.79(40)

Table 2

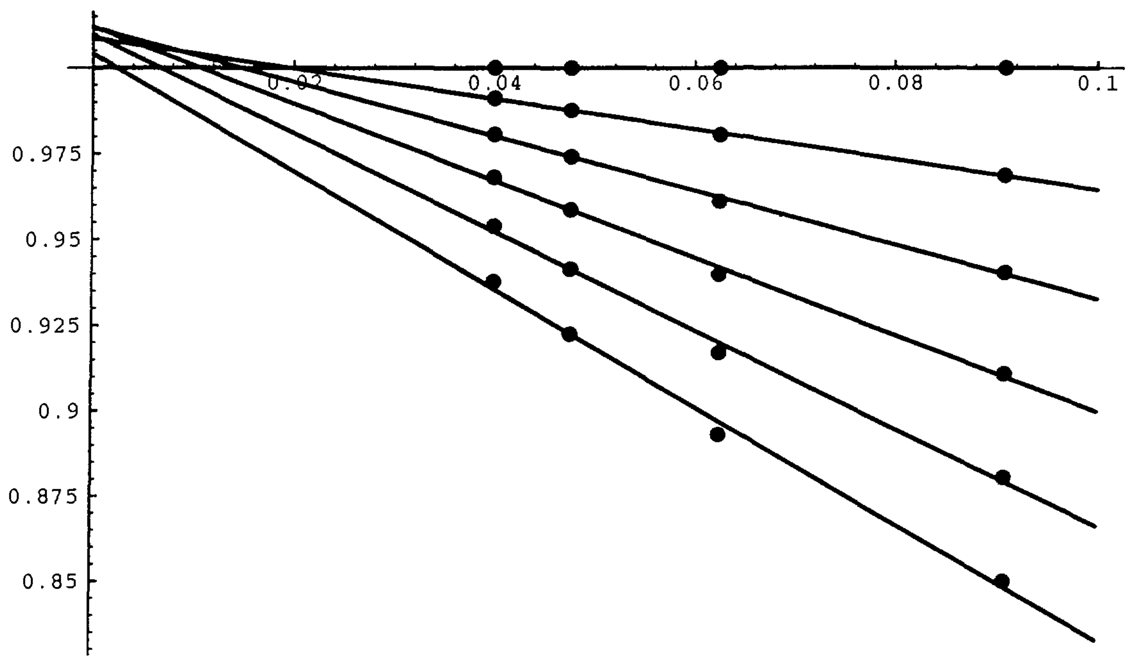

Figure 1:

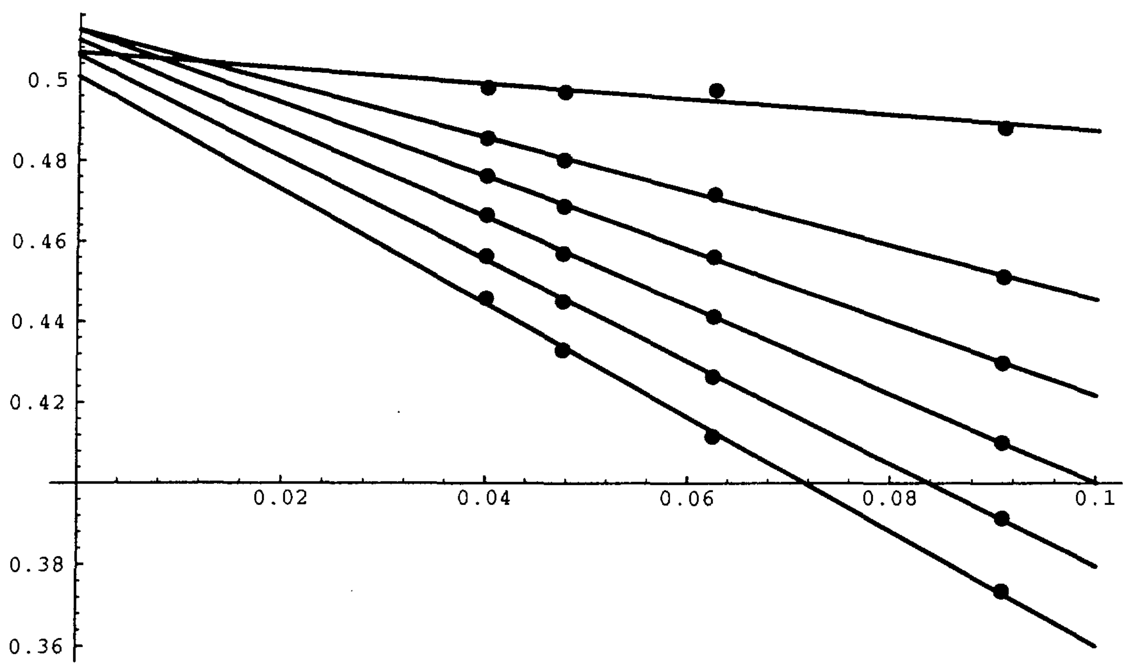

Figure 2: