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Prediction of the Number of 14 MeV Neutron Elastically Scattered from Large Sample of Aluminium Using Monte Carlo Simulation Method

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Abstract

In neutron scattering processes, the effect of multiple scattering is to cause an effective increase in the measured cross-sections due to increase on the probability of neutron scattering interactions in the sample. Analysis of how the effective cross-section varies with thickness is very complicated due to complicated sample geometry and the variations of scattering cross-section with energy. Monte Carlo method is one of the possible method for treating the multiple scattering processes in the extended sample. In this method a lot of approximations have to be made and the accurate data of microscopic cross-sections are needed at various angles. In the present work, a Monte Carlo simulation programme suitable for a small computer was developed. The programme was capable of predicting the number of neutrons scattered from various thicknesses of aluminium samples at all possible angles between 0° to 180° with 10° increments. In order to make the programme not to complicated and capable of being run on microcomputer with reasonable time, the calculation was done in two dimension co-ordinate system. The number of neutrons predicted from this model show in good agreement with previous experimental results.

Introduction

Information on the angular distribution of neutron elastic scattering is of considerable interest and can be obtained by the study of scattering of neutron in large sample. As the thickness of the sample will be extensive, multiple scattering will play an important role in the scattering of fast neutron in the materials. The effect of multiple scattering is to cause an effective increase on the probability of neutron scattering interaction in the sample. Analysis of how the effective cross-section varies with thickness is very complicated due to the complicated sample geometry and the variations of scattering cross-section with energy. Monte Carlo method is one of the possible method for treating the multiple scattering processes in the extended sample. However, in using this method a lot of approximations have to be made and accurate data of microscopic cross-sections are needed at various neutron energies.

Monte Carlo method requires the used of large computing facilities with the capability of random number generation and large virtual storage space. In this method, a number of source neutrons are considered to travel through a system and the individual neutron histories are traced. Random numbers are used to determine the squence of events in each neutron's life history in a particular manner. In the present work, a Monte Carlo simulation programme suitable for a small computer was developed. The programme was capable of predicting the number of neutrons scattered from various thicknesses of the sample at all possible angle between 0° to 360° with 10° increments. In order to make the programme not to complicated and capable of being run on microcomputer with reasonable time, the calculation was done in two dimension co-ordinate system. The programme was written in Turbo Pascal and run on the IBM PC.

Description of the Model

Monte Carlo method which has been widely employed in neutron transport problem, is based on the simulation of the physical events which take place inside the material because an incident neutron. Random numbers are used to determined the sequence of events in each neutron life history in the material. In reaction between nuclei and neutron the sequence of events are determined by means of effective cross-sections.

Consider a mono-energetic neutrons with energy E and intensity N_0 incident upon a sample of thickness x as shown in Figure 1. The problem of the neutron histories was traced by the following subsections.

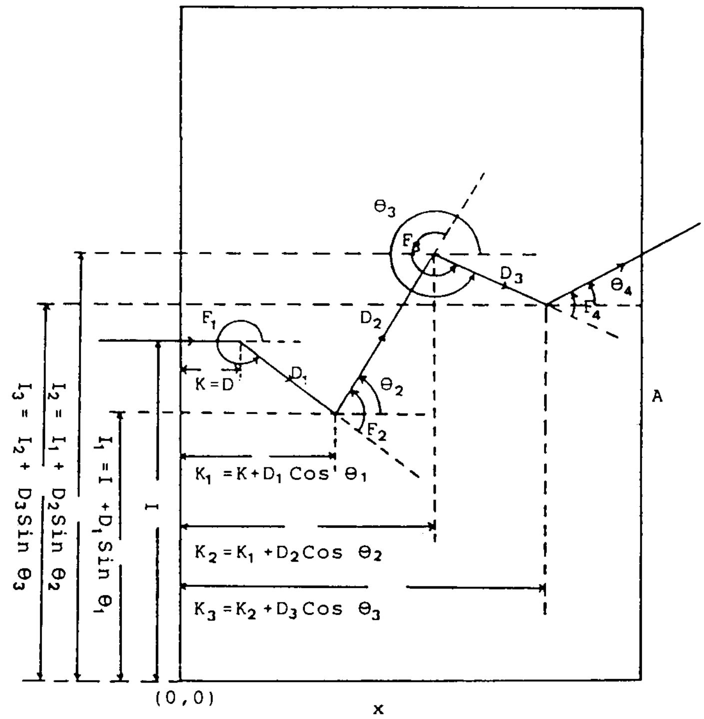


Figure 1: Block diagram to calculate the passage of neutrons using Monte Carlo method.

1. *Choose point of interaction*

Assuming that neutrons enter the material with 0° before first interaction occurs. The point of interaction was chosen based on the width of the sample irradiated by neutron beam. In order to make the results to be compared with the previous experimental results [1], the width of the sample irradiated by neutrons beam at horizontal direction was chosen as 4.2 cm. Therefore, the point of interaction is given by the following equation,

$$I = 4R + 8 \tag{1}$$

where, R is the random number generated (between 0 to 1)

2. *Calculation of distance travel*

Consider n_0 neutrons incident into a block of material. The number of neutrons, N remaining after passed through a thickness x is given by,

$$N = n_0 e^{-\Sigma x}$$

or

$$N = n_0 e^{-x/\lambda} \tag{2}$$

where $\Sigma = 1/\lambda$ and λ is the mean free path of the neutron in the material.

The probability of a neutron being absorbed is $(1-N/n_0)$ and hence the probability of it not being absorbed is N/n_0. Therefore, by generating a random number P where $P = e^{-x/\lambda}$, the distance of the neutron travel in the material can be calculated by the following equation,

$$x = \log_e\left(\frac{1}{P}\right) \tag{3}$$

However, $0 < P < 1$, therefore, if $P = 0$, then x is infinite. Hence the expression,

$$x = \log_e\left(\frac{1}{1-P}\right) \qquad (4)$$

is used because P is never 1 and x can never infinite. The value of x is then tested by the following conditions,

i) If $x \geq a$, then the neutron has escaped from the material.
ii) If $x < a$, then the neutron has escaped from the material,

where a is the distance to edge of the sample from the point of interaction. If condition (i) is obeyed then the neutron history is terminated. If the condition (ii) is obeyed, then the history continues into the next section.

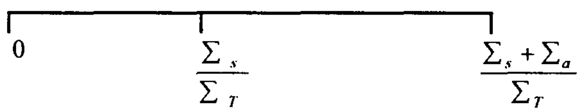
3 *Type of interaction.*

In neutron physics, reactions between nuclei and neutrons are ordinarily described by means of partial cross-sections. Since the cross-sections are of the form,

$$\Sigma_T = \Sigma_s + \Sigma_a \qquad (5)$$

then the probability of scattering and absorption are Σ_s / Σ_T and Σ_a / Σ_T respectively, where, Σ_T is the macroscopic total cross-section Σ_s the macroscopic elastic scattering cross-section and Σ_a the macroscopic absorption cross-section.

These probabilities can be represented as lengths of a line of the following figure,



In order to determine the type of interaction, another random number, v is generated and tested against the following conditions

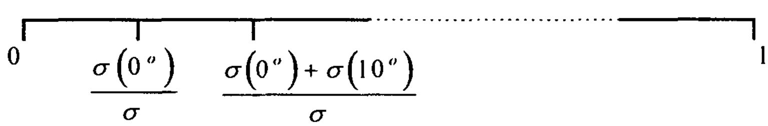
i) If $v \geq \frac{\Sigma_a}{\Sigma_T}$, then absorption occurs.

ii) If $v \geq \frac{\Sigma_s}{\Sigma_T}$, the scattering occurs

If condition (i) is observed the neutron history is terminated and the next neutron is allowed to pass through the sample. If condition (ii) is chosen the problem proceeds to the next section of the programme.

4. *Choice of scattering angle*

This Monte Carlo programme was capable to predict the number of neutrons scattered at all possible angles between 0° to 350° with 10° increments. A similar approach to the method described in section 3 was used. The probabilities of the neutron being scattered through each angle can be considered as the length of each segment of the following figure,



where,

$$\sigma(\theta) = \frac{d\sigma}{d\Omega}(\theta)\Delta\Omega \qquad (6)$$

$d\sigma/d\Omega$ is the differential elastic scattering cross section, $\Delta\Omega$ the solid angle and $\sigma = \sigma(0°) + \sigma(10°) + \sigma(200°) + \sigma(30°) + \ldots\ldots\ldots\ldots + \sigma(350°)$ is the integrated elastic scattering cross-section.

The computer then generate a random number R, and tests by the following conditions.

If $R < \sigma(0°)$, then $\theta = 0°$, else

If $R < \sigma(0°) + \sigma(10°)/\sigma$, then $\theta = 10°$, else

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If $R < \sigma(0°) + \sigma(10°) + ... + \sigma(340°) + \sigma(350°)/\sigma$, then $\theta = 350°$

The computer goes through each comparison in order to choose the scattering angle for each interaction.

5. *Calculation of energy and mean free path.*

As we are concerned only on the elastic scattering process, the fraction of energy retained at each interaction can be calculated from the conservation of momentum and energy in the center of mass system and laboratory system. If gives [2],

$$Q(\varphi) = \frac{E'}{E_0} = \frac{A^2 + 2ACos\varphi + 1}{(A+1)^2}$$

and

$$Cos\theta = \frac{ACos\varphi + 1}{(A + 2ACos\varphi + 1)^{1/2}} \quad (7)$$

where,

A = the mass of the nuclei,

φ = center of mass scattering angle,

θ = laboratory scattering angle.

In term of laboratory scattering angle, equation (7) can be written as

$$Q(\theta) = \left[\frac{(Cos\theta + A - 1)^{1/2}}{1 + A}\right]^2 \quad (8)$$

The new mean free path of scattered neutron was calculated by the following simplification. As the probability of an interaction occurring is proportional to the time the neutron spends near the nucleus, therefore,

$$\sigma \ \alpha \ \frac{1}{v} \quad (9)$$

where, v is the velocity of neutron.

The kinetic energy of neutron is given by,

$$E = \frac{1}{2}mv^2 \quad (10)$$

therefore, equation (10) can be written as

$$\sigma \ \alpha \ \frac{1}{\sqrt{E}}$$

The mean free path of neutron is,

$$\lambda = \frac{1}{\Sigma} = \frac{1}{n\sigma}$$

therefore,

$$\lambda \ \alpha \ \sqrt{E}$$

Hence,

$$\lambda_n = \lambda_0 Q(\theta)$$

where λ_n is the mean free path after interaction, λ_0 mean free path before interaction and $Q(\theta)$ the fraction of energy retained.

Result and discussion

The aim of the Monte Carlo calculation is to predict the angular distributions of elastic scattered neutrons from various thicknesses of aluminium samples. The thicknesses up to 2 mean free path were investigated. The differential elastic scattering cross-sections used in this calculation were taken from ENDF B-IV [3]. In order to obtain greater accuracy, the programme was run for 10^6 neutrons for each thickness of the sample. The programme was written in Turbo Pascal and run in IBM PC.

Figure 2 shows the angular distributions of the number of elastic scattered neutrons at various thicknesses of aluminium sample. The figure shows that the number of elastic scattered neutrons predicted by this model are in reasonable agreement with the previous experimental results [1]

In the present calculation, neutrons are assumed to enter the material at 0° direction before first interaction occur. From the calculation, the results showed that the number of neutrons scattered at 0° decrease as the thickness of the sample increase. This is because the thicker the sample, the greater the chance of neutron is deflected from straight ahead direction. Because of this, the number of neutrons scattered at angles between 10° to 180° increase as the thickness increase especially at higher angles.

From the calculation it was shown that the number of neutrons scattered at angles greater than 180° have a similar behaviour. Figure 3 shows an example of the symmetric behaviour of the angular distribution of the scattered neutrons at angles from 0° to 350° from 3.81 cm thickness of aluminium sample.

The validation of the model

In order to check the validation of the model, the differential scattering cross-sections were calculated from the number of neutrons predicted by this model. The differential elastic scattering cross-sections were calculated from the following equation[2],

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{N_e(\theta)\sigma}{n_0\left[1 - \exp(-n\sigma x)\right]\Delta\Omega} \qquad (10)$$

where

$N_e(\theta)$ = the number of scattered neutrons predicted from the model,
$\Delta\Omega$ = the solid angle,
σ = the total cross-section,
x = the thickness of the sample,
n = the number of nuclei per cm^3.

The differential elastic scattering cross-sections calculated by above equation were compared with the input data of ENDF-BIV [3]. As for example, Figure 4 shows the result for aluminium of 3.81 cm thickness. The solid line that appears in the figure is the differential elastic scattering cross-sections from ENDF BIV. The result of differential cross-sections predicted by this model are in reasonable agreement with the input data, However, the results predicted at angles between 80° to 100° are much lower than ENDF BIV data. The reason for this is that, at these angles neutrons have maximum thickness to penetrate into the sample because of the rectangular shape of the sample geometry. Because of this, the chance of neutrons escape from the sample at these angles is less compares to other angles.

References

[1] Husin Wagiran, Ph.D Thesis, Universiti of Aston in Birmingham, 1987
[2] Knoll G.F, Radiation Detection and Measurement, 2nd Edition, John Waley and Sons, 1989.
[3] Neutron- Aluminium cross section, ENDF BIV, 1974

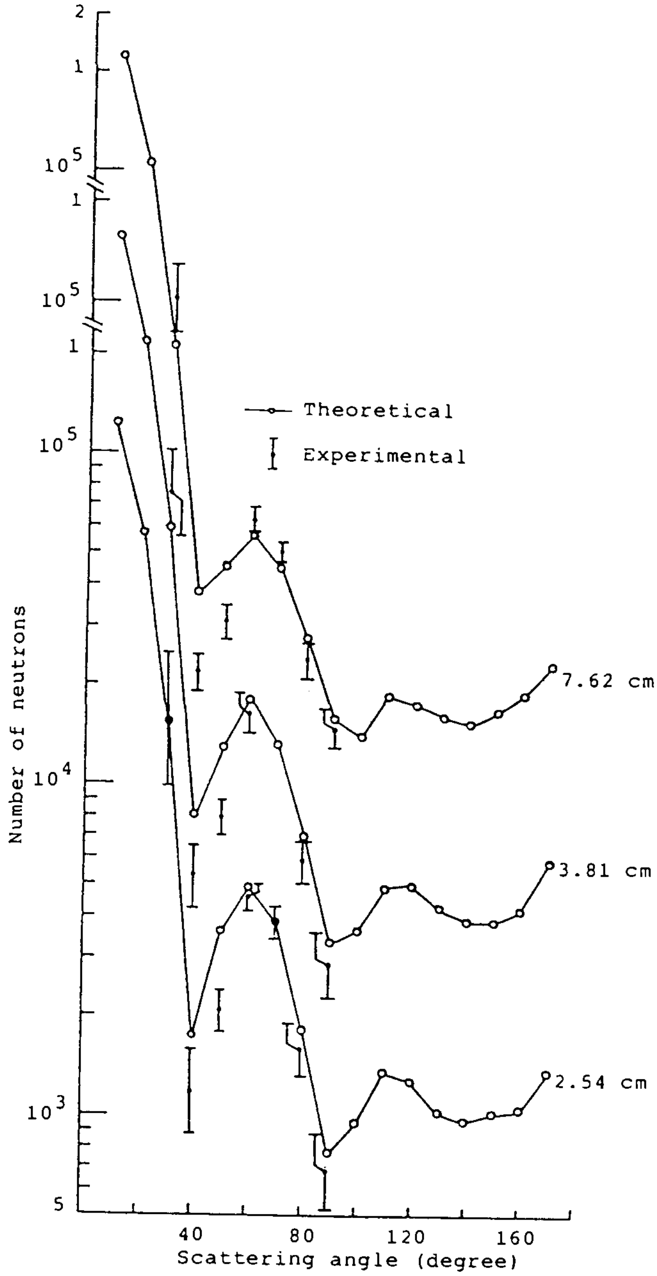

Figure 2: The angular distributions of the number of scattered neutrons from various thicknesses of aluminium calculated from Monte Carlo method

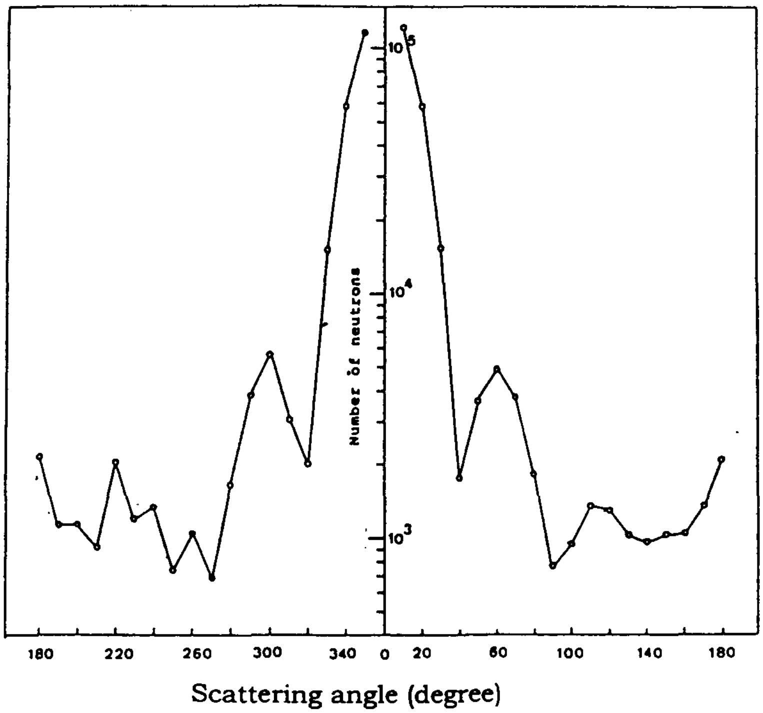

Figure 3: The symetric behaviour of the angular distribution of the scattered neutrons at angles between 0° to 350° from 3.81 cm thickness of aluminium.

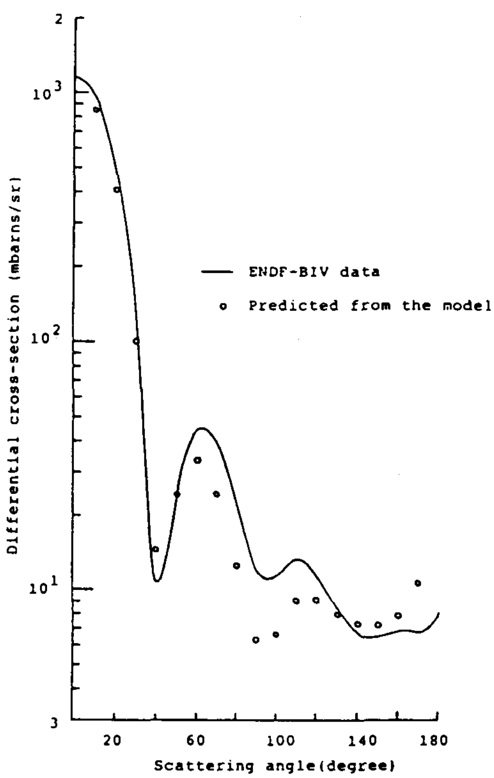

Figure 4: The angular distribution of the differential elastic scattering cross-section calculated from Monte Carlo model for 3.081 cm thickness of aluminium.