THE ELECTRON-PHONON INTERACTION IN STRONGLY CORRELATED SYSTEMS

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ABSTRACT

We analyze the effect of strong electron-electron repulsion on the electron-phonon interaction from a Fermi-liquid point of view and show that the electron-electron interaction is responsible for vertex corrections, which generically lead to a strong suppression of the electron-phonon coupling in the $v_F q/\omega \gg 1$ region, while such effect is not present when $v_F q/\omega \ll 1$. Here v_F is the Fermi velocity and q and ω are the transferred momentum and frequency respectively. In particular the e-ph scattering is suppressed in transport properties which are dominated by low-energy-high-momentum processes. On the other hand, analyzing the stability criterion for the compressibility, which involves the effective interactions in the dynamical limit, we show that a sizable electron-phonon interaction can push the system towards a phase-separation instability. Finally a detailed analysis of these ideas is carried out using a slave-boson approach for the infinite-U three-band Hubbard model in the presence of a coupling between the local hole density and a dispersionless optical phonon.

1. Introduction

The interplay between the electron-phonon (e-ph) coupling and the electron-electron (e-e) interaction is an open problem, which still lacks a complete understanding. At present this topic is very hot since various groups[1] claim that there are experimental evidences supporting a prominent role of the lattice in both cuprates and fullerenes. In this paper we shall report some recent results which are relevant in systems where a strong local repulsion results into a strong band narrowing and a large mass enhancement[2]. These systems include heavy fermions and correlated systems near a metal-insulator transition (like copper oxides).

We shall address two main questions, which are relevant both on a general ground as well as in the framework of high-temperature superconductivity.

i) The first question concerns the possibility of having a large phonon-mediated effective e-e coupling due to the strong mass enhancement ($m^*/m \gg 1$) occurring in a strongly correlated Fermi liquid: After all the dimensionless coupling $\lambda = \gamma^2 \nu_0$ (γ is the usual bare electron-phonon coupling and ν_0 is the free-electron density of states at the Fermi level) could grow very large because of the density of states renormalization arising from the mass enhancement $\lambda \to \lambda = \gamma^2 \nu^*$, with $\nu^* = (m^*/m)\,\nu_0$.

ii) The second question regards the possible occurrence of instabilities in the electronic gas. Previous studies of single- and multi-band Hubbard models in the strong-coupling ($U \gg t$) limit have revealed a strong tendency of these systems to undergo Phase Separation (PS) and Charge-Density-Wave (CDW) instabilities as soon as short-range interactions are introduced. This occurs irrespectively to the magnetic (e.g. nearest-neighbor Heisenberg coupling)[3,4] or the Coulombic (e.g. nearest-neighbor repulsion)[5,6] nature of the short-range interaction. In the framework of interest here one can ask whether also an e-ph coupling can destabilize the electron gas.

We outline here the answers to the above questions, which will be analyzed in more detail in the rest of the paper.

As far as the first problem is concerned, a general analysis performed within a standard Fermi-liquid scheme with phonons coupled to the electron density reveals a dependence of the effective phonon-mediated e-e interaction on the ratio between the transferred momentum q and the frequency ω, because of the vertex corrections generated by the e-e interaction. This dependence can be particularly strong in the presence of a large quasiparticle mass enhancement. As a result, when $v_F q/\omega \gg 1$, we find that the vertex corrections suppress the effective interaction so that the resulting static coupling λ is small. On the contrary, in the opposite limit $v_F q/\omega \ll 1$ the vertex corrections are ineffective and a large effective e-ph interaction results.

On the other hand an affirmative answer can be given to the second question concerning the occurrence of instabilities in the electron gas. This reflects the fact that the stability criterion for the symmetric Landau parameter, $F_0^s > -1$, required for a positive compressibility,* involves the total (e-e and e-ph mediated) interaction in the dynamical limit $v_F q/\omega \to 0$.

2. Phonon-mediated effective interaction: a Fermi-liquid discussion

Without a significant loss of generality we address the above questions discussing the case of an optical phonon coupled to the local density of electrons by a constant coupling γ. In the presence of a (possibly large) e-e interaction one has to worry about the e-ph vertex corrections involving the e-e interaction, for which no Migdal theorem can be applied. For the problem we are considering this e-ph vertex is a density vertex. The problem is then conveniently cast in the language of the standard Fermi-liquid theory[7] to exploit the two relations connecting the density vertex $\Lambda(q,\omega)$ and the wavefunction renormalization z_ω in the dynamic and static limits

$$z_\omega \Lambda\left(q=0, \omega \to 0\right) = 1; \qquad z_\omega \Lambda\left(q \to 0, \omega = 0\right) = \frac{1}{1+F_0^s} . \tag{1}$$

$F_0^s = 2\nu^* \Gamma_\omega$ is the standard Landau parameter and Γ_ω is the dynamic ($q = 0$, $\omega \to 0$) effective e-e scattering amplitude between the quasiparticles. Since we

*We recall that the PS is signalled by the occurrence of a diverging compressibility enclosing a region of densities with negative compressibility to be eliminated by a Maxwell construction are interested in the dressing of the e-ph vertex by the e-e interaction, both the wavefunction renormalization z_ω and the vertex Λ in Eq.(1) do not include phonon processes and the dynamic Landau scattering amplitude is due to the e-e interaction only. To explicitly keep memory of this limitation we append a suffix "e" to Γ_ω and to any quantity not involving phononic processes. Thus $F_0^{s(e)} = 2\nu^*\Gamma_\omega^e$.

The relations (1) are exact Ward identities, which must be satisfied irrespective of the details of the e-e interaction and show a drastic difference between the dynamic ($q = 0, \omega \to 0$) and static ($q \to 0, \omega = 0$) limits of the e-ph vertex. Whenever the exchange of a phonon takes place, the vertex corrections must be included leading to a different behaviour of the effective interaction in the two limits. The effective dimensionless e-e interaction mediated by a single-phonon exchange reads

$$\nu^*\Gamma^{ph}_{\text{eff}}(q,\omega) = \nu^*\gamma^2 z_\omega^{e\,2}\Lambda^{e2}(q,\omega)\frac{\omega^2(q)}{\omega^2 - \omega^2(q)} \tag{2}$$

where $\omega(q)$ is the phonon dispersion. Here the presence of z^e_ω indicates that we are considering the effective interaction between quasiparticles and Λ^e expresses the difference of the phonon coupling to the quasiparticles with respect to particles[7].

The effect in $\Gamma^{ph}_{\text{eff}}(q,\omega)$ of the strong $\omega - q$ dependence of the density vertex Λ^e is made apparent in the small q and ω limits, where the relations (1) can be used. Then one obtains

$$\begin{aligned} \nu^*\Gamma^{ph}_{\text{eff}}(q \to 0, \omega \to 0) &= -\gamma^2\nu^*, \qquad \frac{v_F q}{\omega} \to 0 \\ \nu^*\Gamma^{ph}_{\text{eff}}(q \to 0, \omega \to 0) &= -\gamma^2\frac{\nu^*}{\left(1 + F_0^{s(e)}\right)^2} = -\gamma^2\frac{\kappa^{e2}}{\nu^*}; \quad \frac{\omega}{v_F q} \to 0 \end{aligned} \tag{3}$$

where $\kappa^e = \nu^*/\left(1 + F_0^{s(e)}\right)$ is the compressibility of the Fermi liquid in the absence of coupling with the lattice.

The difference between the dynamic and static case can be dramatic in the case of a Fermi liquid with a large mass enhancement $m^*/m \gg 1$, but with a negligible compressibility renormalization ($\kappa^e \simeq \nu_0$). In the case under consideration, $F_0^{s(e)}$ is proportional to the quasiparticle density of states, $\nu^* = (m^*/m)\nu_0 \gg \nu_0$, and one has $F_0^{s(e)} \gg 1$. Eq.(3) then leads to

$$\begin{aligned} \nu^*\Gamma^{ph}_{\text{eff}}(q = 0, \omega \to 0) &= -\gamma^2\left(\frac{m^*}{m}\right)\nu_0 \\ \nu^*\Gamma^{ph}_{\text{eff}}(q \to 0, \omega = 0) &\simeq -\gamma^2\left(\frac{m}{m^*}\right)\nu_0 \end{aligned} \tag{4}$$

so that the effective one-phonon-mediated e-e interaction is large ($\sim m^*/m$) in the dynamic limit and small ($\sim m/m^*$) in the static one.

The strong $\omega - q$ dependence in Eqs.(4) concerns the small-q and small-ω limits. This result relies on quite general (exact) arguments, whereas the case of finite

q's and ω's needs (approximated) analyses of specific models and it will indeed be discussed in the context of the three-band Hubbard model in the next sections. However, the expectation, confirmed by the analysis in Section 3, is roughly that the product $z_w\Lambda$ will be roughly of order one (as in the dynamical limit) all over the region outside the particle-hole continuum, while it strongly deviates from unity (as in the static limit) in the region of the particle-hole continuum, where screening processes take place. Actually, these screening processes can lead, in the large-q limit, to an even larger depression of $z_w\Lambda$ than predicted by Eq.(1).

This strong momentum vs. frequency dependence of the e-ph vertex renders the analysis of the effects of the e-ph coupling particularly delicate, since different physical quantities may involve different $v_F q/\omega$ regimes. In particular the e-ph coupling (and the e-e interaction mediated by phonons) will be depressed by the e-e interaction in all processes involving small energy and large momenta (as in the low-energy lifetime and particularly in transport), specifically in the low-doping regime, where the effects of the strong e-e interaction play a major role[†] This could provide a rational to the irrelevance (often suggested in the literature) of the phonon contributions to the d.c. resistivity in cuprates.

A different behaviour could be obtained in other quantities where large frequencies are more relevant. This is also in agreement with recent calculations[9] performed in a single-band Hubbard model with an on-site "ionic" e-ph coupling, which show, indeed, that the Eliashberg function $\alpha^2F(\omega)$ determining superconductivity, is much less reduced than the analogous quantity determining the transport properties.

The above conclusion on the irrelevance of the e-ph coupling in the static limit is based on a lower order analysis in γ^2. It evidently contrasts with the fact that the stability criterion for positive compressibility, i.e. $F_0^s > -1$, involves the full e-e interaction (including that mediated by phonons) in the dynamical limit, where the e-ph coupling is not depressed by the "pure" e-e vertex corrections. At lowest order in γ^2, F_0^s reads

$$F_0^s = 2\nu^* \left(\Gamma_\omega^e - \gamma^2\right) \tag{5}$$

indicating that a sizable γ^2 can indeed lead to $F_0^s < -1$. It is worth pointing out that $m^*/m \gg 1$ requires a large *bare* repulsion in units of the bare Fermi energy, as in the single- or multi-band Hubbard model near the metal-insulator transition, but this does not imply a large Γ_ω^e.[‡]The lowest-order analysis showing the depression of the e-ph coupling in the low-energy processes maintains its full validity with respect to the inclusion of higher-order terms in γ^2 provided F_0^s in Eq.(5) is positive and still of order m^*/m. On the contrary, near the instability condition $F_0^s = -1$, the phonon contributions to the vertex cannot be neglected and the e-ph interaction is relevant even in the static limit, at least at small q. At large q's the analysis of

[†]Similar results (i.e. that phonons contribute little to the quasiparticle scattering as far as transport properties are concerned) were found in a three-band Hubbard model with intersite "covalent" e-ph coupling[8].

[‡]Notice also that in writing Eq. (5) we are considering an infinite order RPA-like analysis in γ^2 for the compressibility.

Section 3 shows that the e-e interaction mediated by phonons is instead suppressed even near the instability region.

We like to stress that the instability at $F_0^s = -1$ does not imply the softening of the phonon, which stays massive: It rather appears as an overdamping and eventually an instability of the zero sound driven by the e-ph mediated attraction.

The quantitative determination of the needed strength of the e-ph coupling in order to have an instability must rely on the analysis of a specific model. In fact, as it is apparent in its definition, the strength and the sign of F_0^s in Eq.(5) depend on the balance between $\Gamma_\omega^e(q,\omega)$ and γ^2. Moreover, as explicitly seen in the model treated in the following sections, $\Gamma_\omega^e(q,\omega)$ in turn results from a cancellation between the strong bare repulsion and the strong screening processes, which depend on the model one is dealing with.

3. The Three-Band Hubbard Model

The model we shall consider is represented by the following two-dimensional Hamiltonian

$$\begin{aligned} H &= \varepsilon_d^0 \sum_{i\sigma} d_{i\sigma}^\dagger d_{i\sigma} + \varepsilon_p^0 \sum_{i\sigma} (p_{i\sigma x}^\dagger p_{i\sigma x} + p_{i\sigma y}^\dagger p_{i\sigma y}) + \sum_{<ij>\sigma} (t_{ij} d_{i\sigma}^\dagger p_{j\sigma x} + x \to y + \text{h.c.}) \\ &+ \sum_{<ij>\sigma} (t_{ppij} p_{i\sigma x}^\dagger p_{j\sigma y} + \text{h.c.}) + U_d \sum_i n_{di\uparrow} n_{di\downarrow} + \omega_0 \sum_i A_i^\dagger A_i \\ &- \sum_{i,\sigma} \left(A_i + A_i^\dagger\right) \left[G_d \left(n_{di} - \langle n_{di} \rangle\right) + G_p \left(n_{pi} - \langle n_{pi} \rangle\right)\right]. \end{aligned} \tag{6}$$

where ε_d^0 and ε_p^0 are the Cu and O energy levels respectively, $t_{ij} = \pm t_{pd}$ is the Cu-O hybridization, $t_{ppij} = \pm t_{pp}$ is the nearest neighbor O-O hybridization (for the choice of the orbital phases and the related choice of the sign of the hopping constants, see, e.g., Ref.10). U_d is the on site repulsion between holes on copper sites. Starting from a Cu($3d^{10}$)-O($2p^6$) vacuum state, holes on copper $d_{x^2-y^2}$ orbitals or on oxygen p_x or p_y orbitals at site i are created by the $d_i^\dagger$ and $p_i^\dagger$ operators respectively. $n_{di} = \sum_\sigma d_{i\sigma}^\dagger d_{i\sigma}$ is the total density per cell of holes on copper, while $n_{pi} = \sum_{\sigma,\alpha=x,y} p_{i,\sigma,\alpha}^\dagger p_{i,\sigma,\alpha}$ is the total density per cell of holes on oxygen. The boson creation operator $A_i^\dagger$ creates a dispersionless phonon with frequency ω_0 coupled to the local density of copper and oxygen holes by the coupling constants G_d and G_p respectively[§] A similar three-band Hubbard model was considered in Ref.8, where, however, an intersite "covalent" e-ph coupling was considered arising from the ion-position dependence of the hopping integrals.

Since our investigation concerns the interplay between strong interactions and the phonons we take the $U_d \to \infty$ limit. In a standard way we handle the no-double-occupancy constraint on copper sites by means of the slave-boson technique[11,12,13].

[§]Notice that, within this model, the e-ph couplings G_p and G_d do not coincide with the e-ph coupling γ introduced in Section 2. In the particular case when $G_d = G_p = g$ the correspondence is given by the relation $\gamma^2 = g^2/\omega_0$.

Therefore, after performing the usual substitution $d^\dagger_{i\sigma} \rightarrow d^\dagger_{i\sigma} b_i$, $d_{i\sigma} \rightarrow b^\dagger_i d_{i\sigma}$ the constraint becomes $\sum_\sigma d^\dagger_{i\sigma} d_{i\sigma} + b^\dagger_i b_i = 1$. To equip the model with a formal small expansion parameter, we introduce a standard large-N expansion[12], where the spin index σ runs from 1 to N. The constraint is relaxed to assume the form $\sum_\sigma d^\dagger_{i\sigma} d_{i\sigma} + b^\dagger_i b_i = \frac{N}{2}$ and the suitable rescaling of the hopping $t_{pd} \rightarrow t_{pd}/\sqrt{N}$ must, in this model, be joined by the similar rescaling of the hole-phonon coupling $G \rightarrow G/\sqrt{N}$ in order to compensate for the presence of N fermionic degrees of freedom.

At the mean-field ($N = \infty$) level, the model is equivalent to the standard, purely electronic three-band Hubbard model without coupling to the phonons, which has been widely considered in the literature[14,10]. In fact, at mean field level no role is played by the phonons because our electron-lattice coupling depends on the difference between the local and the average density and this difference naturally vanishes in the mean-field approximation. The model displays a T=0 Fermi-liquid behaviour for any finite doping δ, where δ is the deviation from half-filling, when one hole per cell is present in the system. In the Fermi-liquid case the mean-field value of the slave-boson field b_0 multiplicatively renormalizes the hopping thus enhancing the effective mass of the quasiparticles ($b_0/\sqrt{N} \leq \sqrt{1/2}$).

On the other hand, at half-filling the system becomes an insulator if the bare charge-transfer energy difference $\varepsilon^0_p - \varepsilon^0_d - 4t_{pp}$ is larger than a critical value ranging from $3.35t_{pd}$ when $t_{pp} = 0$ to a smaller value $\approx 2.5t_{pd}$ when $t_{pp} = 0.5t_{pd}$[14,10]. In the insulating phase b_0 vanishes leading to an infinite quasiparticle mass ($m^*/m = \infty$) and to a vanishing quasiparticle spectral weight.¶

In order to get new physical effects from the presence of the coupling with the phonons, one needs to consider the fluctuations of the bosonic fields which describe the interactions among quasiparticles.‖

We first report in Fig.1 the interaction (slave-boson) renormalized vertex between phonons and quasiparticles *without phonon processes* included in it as a function of the Matsubara frequency. Two of these quantities can be joined to a bare phonon propagator to give the one-phonon effective scattering amplitude. On the other hand, joining two of the above vertices with a fully renormalized phonon propagator leads to the full phonon-mediated scattering amplitude.

It is clear that the results reported in Fig.1 quantitatively confirm the general qualitative analysis of Section 2. Fig.1 displays the strong dependence of the e-ph vertex on the momentum vs frequency ratio. It is apparent that a rapid increase occurs in the phonon-quasiparticle vertex, when the frequency becomes larger than a screening energy $\omega_{scr}(q)$ ($\propto q$ at small q). This is most evident at low momenta,

¶The infinite quasiparticle mass is an outcome of the large-N analysis. At higher order one expects the narrowing of the bandwidth to be limited by the copper superexchange J[10,15]. In the cuprates this could limit the mass enhancement to $m^*/m \simeq 1.5t/J \simeq 6 - 8$[15] in the small doping limit. A larger mass enhancement could be expected in systems with smaller J/t. A large ratio m^*/m is also obtained in heavy-fermion systems. In the following the limit $m^*/m \rightarrow \infty$ should always be intended with the above provisions.

‖It should be noted that the boson propagators are of order 1/N, thus the quasiparticle scattering amplitudes are of order 1/N.

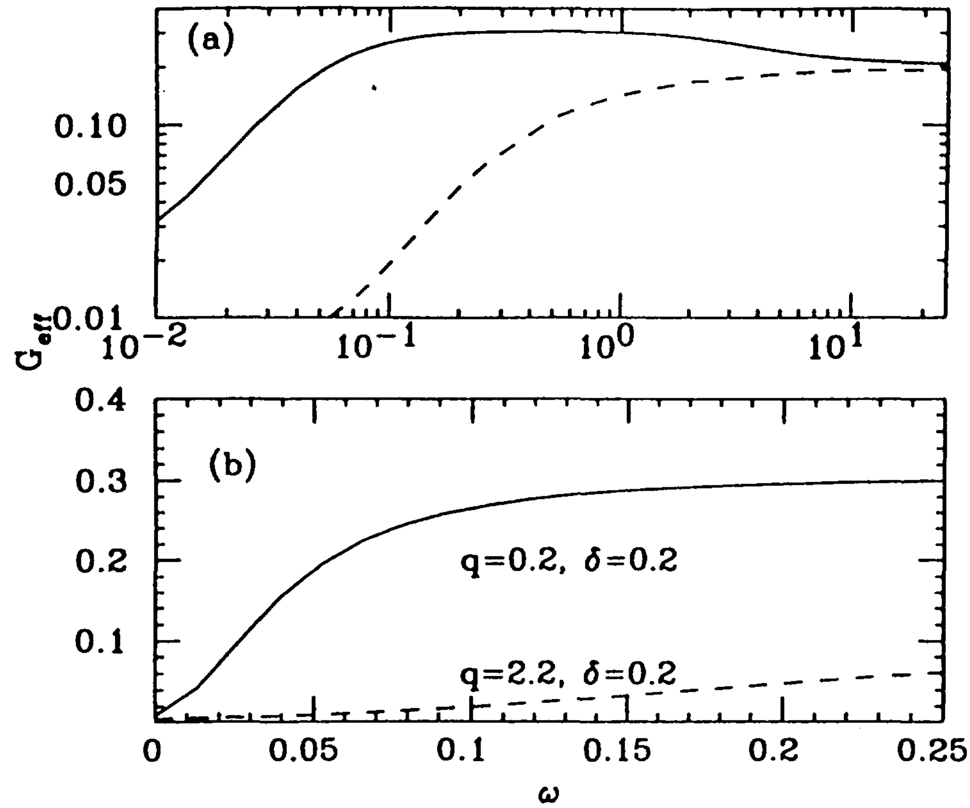

Figure 1: (a) Interaction-renormalized quasiparticle-phonon vertex at fixed low ($q = 0.2$, continuous line) and large momenta ($q = 2.2$, dashed line) as a function of the transferred Matsubara frequency ω. The parameters are $\varepsilon_p^0 - \varepsilon_d^0 = 4.9t_{pd}$, $t_{pp} = 0.2t_{pd}$, $G_d = 0.1t_{pd}$, $G_p = 0.15t_{pd}$, $\omega_0 = 0.02t_{pd}$ and $\delta = 0.225$. The scattered quasiparticles are on the Fermi surface and q is in the y direction, $\mathbf{q} = (0, q)$. (b) Same as (a) in the small frequency region.

where the phonon-quasiparticle vertex G_{eff} increases fast by more than one order of magnitude, from the small value at zero frequency up to a sizable value at large-frequency. The Fermi-liquid analysis of Section 2 would suggest that the screening energy $\omega_{scr}(q)$ would be given by $v_F q$ at small q and by the renormalized bandwidth W at large q. However, our numerical results reveal that the screening energies are much larger and, at least at large q, they are more properly described by the bare bandwidth W_0. Analyzing in detail the contributions coming from the vertex corrections, we also discovered that in the "intermediate-high"-energy region the *interband* screening (specific of the three-band Hubbard model) acts indeed to enhance the vertex at small q's by a factor of about 1.5 with respect to the bare vertex.

The phase diagram in the $(\varepsilon_p^0 - \varepsilon_d^0)/t_{pd}$ vs δ plane resulting from the analysis of the compressibility $\chi(q, \omega = 0)$ is shown in Figs. 2 for a typical parameter set. The instability line indicates where $\chi(q \to 0, \omega = 0)$ diverges and it encloses a region of negative compressibility. The most impressive consequence of the coupling with the phonons is that a large unstable region appears at large doping or at large bare

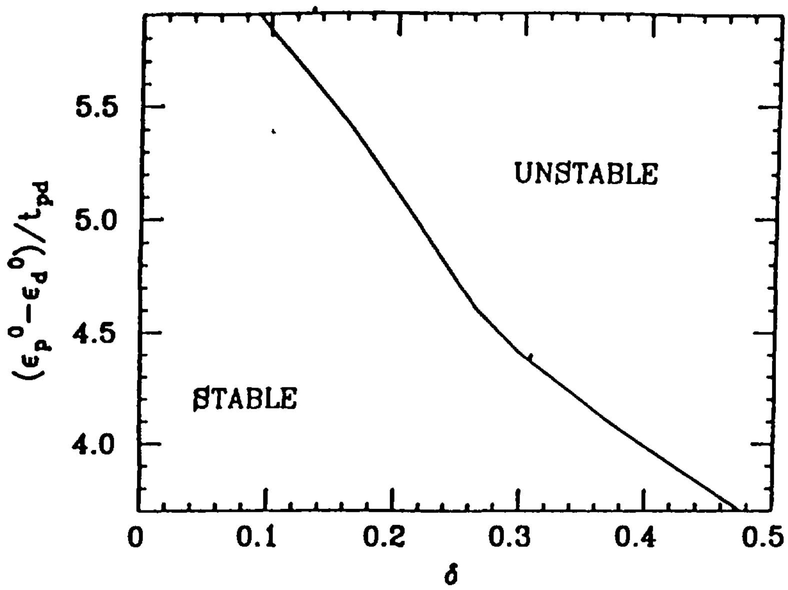

Figure 2: Phase diagram $(\varepsilon_p^0 - \varepsilon_d^0)/t_{pd}$ versus positive doping δ. The parameters are as in Fig.1.

charge-transfer gap. It is worth noting again that the instability is *not* related to any phonon softening. The instability of the phase diagrams in Fig.2 is therefore an electronic instability even if it is driven by the coupling with the lattice. The instability of $\chi(q, \omega = 0)$ first occurs at $q \to 0$ and is therefore a signature of a long-wavelength, static, thermodynamical PS region to be identified by a Maxwell construction.*

This can have relevant physical consequences, once a long-range Coulomb (LRC) force is considered in the model. This latter interaction would stabilize the system in the regions of the phase diagram where $\chi(q, \omega = 0)$ shows divergences at low-q and would prevent the occurrence of PS, but would leave open the possibility of finite-q instabilities. Most probably the introduction of a LRC repulsion would result into the formation of incommensurate CDW or quenched bubbles of different charge densities.† A similar phenomenon was suggested for different models in Refs.16,6 and seems to be confirmed by a static analysis of a pseudospin model in Ref.17. Of course the above argument misses the dynamical aspects of the problem, and should be taken as purely indicative of a possible scenario. One should also take into account that superconductivity can take place near the instability line where $F_0^s = -1$ and act to stabilize the system (i.e. superconductivity could partially or fully preempt the instability region).

Finally we investigated the possibility of Cooper pairing. Superconductivity can indeed appear as a precursor of the PS instability[4,5,6] suggesting that a re-

*Phase separation is also found in a large-N analysis of the single-band infinite-U Hubbard model, with the electron density coupled to the lattice displacement[18].

†In the case of formation of inhomogeneous charge distributions inside the system, different lattice parameters are also naturally expected in the differently doped regions.

δ	0.15	0.20	0.229		δ	0.15	0.20	0.229
λ_{s_1}	-0.58	-0.48	1.2		λ_{d_1}	-0.063	0.017	2.1
λ_{s_2}	-0.55	-0.65	1.5		λ_{d_2}	-0.021	0.012	1.8

Table 1: s_1-, s_2-, d_1- and d_2-wave coupling constants for various doping and for the same parameters as in Fig.1. The instability line is at $\delta_c = 0.23$.

gion of large compressibility is a good candidate in order to find superconductivity. According to the previous experience in other strongly interacting models, we therefore investigated the Fermi-surface average of the particle-particle scattering amplitude using the weights $g_{s_1}(k) = \cos(k_x) + \cos(k_y)$, $g_{s_2}(k) = (\cos(k_x) - \cos(k_y))^2$, $g_{d_1}(k) = \cos(k_x) - \cos(k_y)$ and $g_{d_2}(k) = \sin(k_x)\sin(k_y)$ to project the interaction onto the s-wave and d-wave channels. Whereas the couplings λ_d are found to be generally attractive near (and inside) the unstable region, we find s-wave Cooper instabilities only very close to the instability line. The results are tabulated in Table 1 at various doping concentrations for the set of parameters of Fig.1, for which the critical doping for the occurrence of the instability is $\delta_c = 0.23$. (Notice that $\lambda_l > 0$ means attraction). The closeness to a PS instability appears therefore as a favorable condition in order to obtain high temperature superconductivity from a phonon-mediated attraction similarly to what suggested in the context of purely electronic pairing mechanisms.

However, it should be emphasized that the presence of a s-wave static Cooper instability only in a narrow region, by no means excludes the possibility of having s-wave superconductivity in a much larger area of our phase diagram. Only an appropriate Eliashberg dynamical analysis can allow to draw a conclusion, specially in the light of the considerations on the strong frequency dependence of the vertex corrections discussed in Section 2. Of course the same applies to the attraction in the d-wave channels, which could also be greatly favored by dynamical effects.

The occurrence of superconductivity near the PS region is the consequence of a large attraction at small q. One has, therefore to worry whether the LRC interaction will spoil superconductivity. We could argue that, if the attraction and the LRC force have different energy scales (in the model here considered these energies scales would be ω_0 and the bare bandwidth W_0 respectively, with $\omega_0 \ll W_0$) then a standard Eliashberg treatment would likely reduce the LRC repulsion to a not too large μ^*.

Moreover, as mentioned above, being the LRC forces effective in stabilizing the system al low momenta only, it still remains open the possibility of finite-momenta instabilities. Close to these instabilities a large attraction persists even in the static limit, so that, Cooper pairing could still occur.

4. Conclusions

In the present work we investigated the e-ph interaction in the presence of a strong local repulsion within a general Landau Fermi-liquid framework. Using standard Ward identities, we pointed out the strong dependence of the e-ph vertex on the momentum vs frequency ratio in a strongly interacting system displaying a large effective-mass enhancement, but not too large compressibility. In particular we showed that the dimensionless attractive quasiparticle scattering mediated by a single-phonon exchange coupled to the electronic density is strongly suppressed by vertex corrections due to the e-e repulsion when $v_F q > \omega$. On the contrary the dimensionless effective one-phonon-mediated attraction is strongly enhanced by the effective-mass increase when $v_F q < \omega$. These remarks are obviously relevant in any dynamical analysis of the pair formation in strongly interacting systems like, e.g., high temperature superconducting cuprates, fullerenes or $Ba_{1-x}(K,Pb)_xBiO_3$. In particular, our work calls for a critical reanalysis of the effective potential models in the Eliashberg approach of pairing, when applied to strongly correlated systems. Most of the work in this field is, in fact, carried out by averaging at the outset the momenta on the Fermi surface. This likely leads to overestimate the role of large momenta, and misses the peculiar strong momentum vs frequency dependence revealed in the interacting systems.

We also like to point out that phonon corrections to the e-ph vertex, which are usually neglected according to the Migdal theorem, have recently been considered in Ref.19. This analysis showed that these corrections, which are not negligible if the phonon frequency vs bandwidth ratio is sizable, tend to suppress the e-ph coupling in the large $v_F q/\omega$ region, whereas they tend to enhance it when $v_F q/\omega$ is small![‡‡] These corrections tend therefore to cooperate with the strong interaction effects discussed in the present paper. Specifically the strong interaction effect, particularly near the PS instability, could create the conditions to validate the analysis of Ref.19 in order to increase the superconducting transition temperature via corrections to the Migdal theorem.

A relevant issue related to the work presented here concerns the possible formation of polarons when a strong e-e interaction is present. The treatment of this long standing problem is beyond the scope of our analysis, but it is worth pointing out that the band narrowing occurring at low doping due to strong e-e repulsion could suggest an easier polaron formation.

The high sensitivity of the vertex corrections as a function of frequency and momentum renders the conditions for polaron formation rather ambiguous, since it is not clear which frequency vs momentum regions dominate the e-e screening processes dressing the e-ph coupling g (or γ). A quick inspection into the correlation function $\langle c_i^\dagger c_i (a_i + a_i^\dagger) \rangle$, which represents the correlation between the charge density and the lattice displacement, shows that a rather natural guess is that the relevant frequencies for the establishing of the polaronic regime are those smaller than the phonon frequency ω_0, while the relevant momenta are of the order of k_F. Then, as far as polarons are concerned, the effective quasiparticle-phonon coupling would be

[‡‡]This is in agreement with the analysis of Section 2 when generalized to include phonon corrections to the e-ph vertex.

in the static or dynamic limit according to whether the phonon frequency is larger or smaller than the screening energy $\omega_{scr}(q)$, which, according to our numerical analysis is of the order of the bare bandwidth W_0. In the antiadiabatic regime for the bare bandwidth ($\omega_0 > W_0$) the quasiparticle-phonon interaction would not be screened and the e-e repulsion would favour the polaronic regime by reducing the electronic kinetic energy. The same effect is also present in the adiabatic conditions ($\omega_0 < W_0$), but the polaronic regime will be more strongly disfavoured by the suppression of the e-ph coupling. However, to settle this point would require a further detailed analysis.

A last remark can be made on the possible formation of bipolarons. The expectation is that, even when the antiadiabatic conditions would be realized, the large repulsion would nevertheless be able to stabilize the system against bipolaron formation.

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