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Inelastic electron scattering as an indicator of clustering in wave functions

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While the shell model is the most fundamental of nuclear structure models, states in light nuclei also have been described successfully in terms of clusters. Indeed, Wildemuth and Tang [1] have shown a correspondence between the cluster and shell models, the clusters arising naturally as correlations out of the shell model Hamiltonian. For light nuclei, the cluster model reduces the many-body problem to a few-body one, with interactions occurring between the clusters. These interactions involve particle exchanges, since the nucleons may still be considered somewhat freely moving, with their motion not strictly confined to the clusters themselves. Such is the relation of the cluster model to the shell model. For a realistic shell model then, one may expect some evidence of clustering in the wave functions for those systems in which the cluster model is valid.

A good place to look for this behavior is in the ^{6}Li and ^{7}Li nuclei. Both of these have been described successfully in terms of clusters [2], as $\alpha + d$ in the case of ^{6}Li (or as $\alpha + p + n$ in a three-body description [3]), and $\alpha + t$ for ^{7}Li, although other two-cluster configurations are possible [2]. The simple $0\hbar\omega$ shell model descriptions of these nuclei automatically contain such clustering: the $0s$-shell inert core is the α particle, while the valence nucleons in the $0p$-shell naturally form the other cluster. More recently, large space multi-$\hbar\omega$ shell models have been constructed for these nuclei [4]. Such are required if a shell model approach is to model cluster effects realistically [5].

This clustering behavior in the shell model wave functions can be illustrated by considering the $E2$ transitions to the low-lying excited states in both ^{6}Li, to the $3^+; 0$ (2.186 MeV) state, and ^{7}Li, to the $\frac{1}{2}^-$ (0.478 MeV) and the $\frac{7}{2}^-$ (4.63 MeV) states. The shell model wave functions for both nuclei were constructed in complete $0\hbar\omega$, $(0+2)\hbar\omega$, and $(0+2+4)\hbar\omega$ model spaces using the Cohen and Kurath $(6-16)$2BME (CK) [6], the MK3W [7], and the Zheng interactions [4], respectively. For comparison, the Zheng interaction was also used to construct wave functions in the $0\hbar\omega$ and $(0+2)\hbar\omega$ spaces. All shell model calculations were performed using the code OXBASH [8]. The B($E2$) values, calculated using harmonic oscillator wave functions determined from analyses of elastic electron scattering form factors [9], for those $E2$ transitions are compared in Table 1 to the experimental values [10,11]. The results obtained using the multi-$\hbar\omega$ shell model wave functions are closer in agreement with experiment than the results obtained using the $0\hbar\omega$

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Table 1: B(E2↓) values (in units of e^2 fm^4) for the transitions in ^{6,7}Li as listed.

Nucleus	Transition	$0\hbar\omega$		$(0+2)\hbar\omega$		$(0+2+4)\hbar\omega$	Expt. [8]
		CK	Zheng	MK3W	Zheng		
^{6}Li	$3^+;0 \rightarrow$ g.s.	2.65		4.31		4.07	9.3 ± 2.1
^{7}Li	$\frac{1}{2}^- \rightarrow$ g.s.	3.04	2.51	8.00	6.21	7.23	16.4 ± 1.0
	$\frac{7}{2}^- \rightarrow$ g.s.	1.04	1.30	3.30	2.88	3.32	$3.50, 7.5 \pm 0.8^{\mathrm{a)}}$

a) Ref. [9].

wave functions. Yet in all cases, that level of agreement is not good, with the calculations underpredicting the measured values by at least a factor of two. This indicates that the shell model wave functions do not exhibit clustering behavior, which is expected to manifest itself at small momentum transfer. The exception is the transition to the $\frac{7}{2}^-$ state in ^{7}Li, for which the value obtained from the γ-decay width [10] is in agreement with the value obtained from the MK3W and $(0+2+4)\hbar\omega$ shell model calculations. However, that measured value quoted in the compilation [10] is not referenced, and so one must look to an alternative analysis by which the two measurements may be compared.

We obtained the B($E2$) as a function of momentum transfer from the calculated longitudinal electron scattering form factors [9] using the transformation of Brown, Radhi, and Wildenthal [12]. That transformation removes most of the dependence on momentum transfer from the form factor; the measured B($E2$) value is compared to the $q = 0$ intercept. This is illustrated in Fig. 1, wherein the longitudinal inelastic electron scattering form factor to the $3^+;0$ (2.186 MeV) state in ^{6}Li and the associated B($E2, q$) are displayed in (a) and (b) respectively. The results obtained using the $(0+2+4)\hbar\omega$, $(0+2)\hbar\omega$, and $0\hbar\omega$ wave functions are displayed by the solid, dashed, and dot-dashed lines respectively. They are compared to the data of Bergstrom *et al.* [13] (circles), Yen *et al.* [14] (squares), Bergstrom and Tomusiak [15] (crosses), and Hutcheon and Caplan [16] (triangles). The form factor illustrates the discrepancy in the predictions of the B($E2$) value. For the multi-$\hbar\omega$ models, there is agreement with data above 1 fm^{-1}. However, all models fall below the data below 1 fm^{-1}. The B($E2, q$), displayed in Fig. 1(b), shows that discrepancy more clearly, with the data converging on the measured value (indicated by the diamond data point), and the results of the calculations falling well below that value.

The B($E2, q$) obtained from the longitudinal inelastic electron scattering form factor to the $\frac{7}{2}^-$

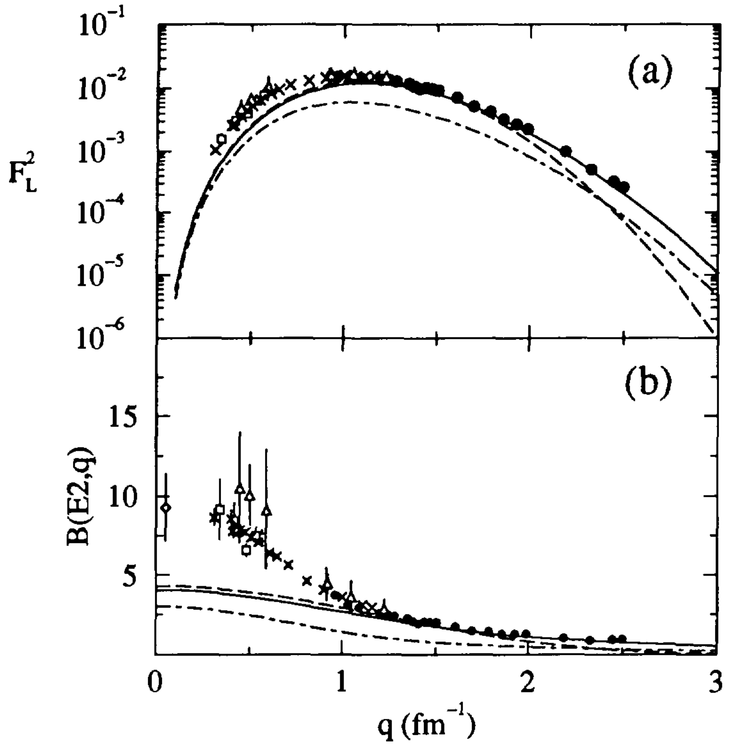

Figure 1: Longitudinal inelastic electron scattering form factor to the $3^+;0$ (2.186 MeV) state in ^{6}Li (a), and the B(E2↓,q) value, in units of e^2 fm^4, as obtained from the form factor (b). The data of Bergstrom *et al.* [13] (circles), Yen *et al.* [14] (squares), Bergstrom and Tomusiak [15] (crosses), and Hutcheon and Caplan [16] (triangles) are compared to the results of the calculations made using the $(0+2+4)\hbar\omega$ wave functions (solid line), the MK3W wave functions (dashed line), and the CK wave functions (dot-dashed line). The B(E2↓) value from the associated γ–decay rate [10] is displayed by the diamond data point in (b).

(4.63 MeV) state in ^{7}Li is displayed in Fig. 2(a). There is some doubt on the measured B(E2↓) for this transition. From the quoted γ–decay rate this is 3.50 e^2 fm^4 [10], however, the source of that measurement is not given in the compilation. The value obtained from an analysis of the longitudinal inelastic electron scattering form factor is 7.5 ± 0.8 e^2 fm^4 [11]. Therein, the B(E2) value for the decay of the $\frac{7}{2}^-$ state is related to that for the $\frac{1}{2}^-$ state, which is well determined. The values obtained from the various shell models are listed in Table 1. Our results obtained from the $(0+2)\hbar\omega$ and $(0+2+4)\hbar\omega$ shell models lie very close to the value obtained from the γ–decay. The data of Lichtenstadt *et al.* [11] (circles), Hutcheon and Caplan [16] (squares), and Bernheim and Bishop [17] (triangles) are compared to our results obtained using the various shell models. A similar discrepancy of our results with data to those observed for the $3^+;0$ state in ^{6}Li [Fig. 1(b)] and the $\frac{1}{2}^-$ state in ^{7}Li, displayed in Fig. 2(b), is now also observed, with the data suggesting a B(E2) value of around 7 or 8 e^2 fm^4. More accurate measurements of the form factor

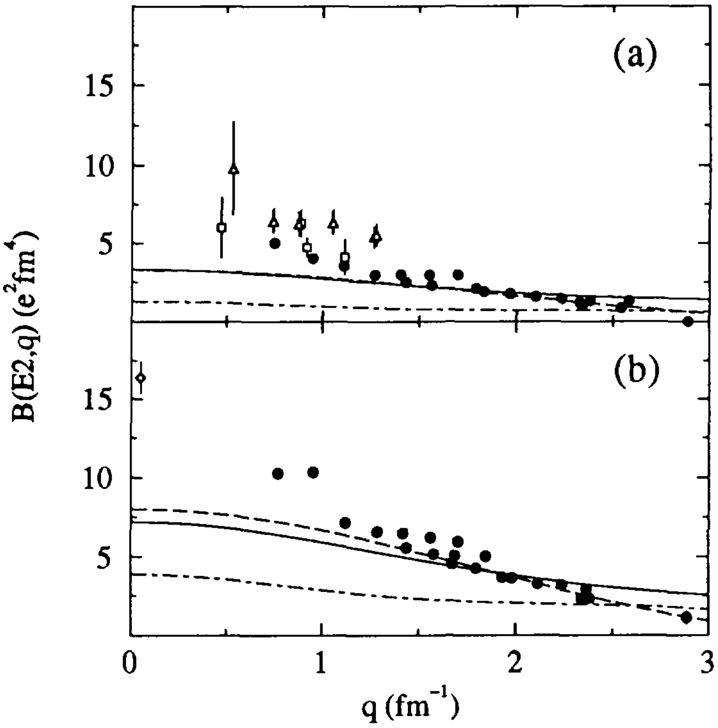

Figure 2: B(E2↓,q) for the $\frac{7}{2}^-$ (4.63 MeV) (a) and the $\frac{1}{2}^-$ (0.478 MeV) (b) states in ^{7}Li. The data of Lichtenstadt *et al.* [11] (circles), Hutcheon and Caplan [16] (squares), and Bernheim and Bishop [17] (triangles) are compared to our results obtained using the various shell models. The measured B(E2↓) value for the $\frac{1}{2}^-$ state in ^{7}Li [10], as determined from the γ-decay rate, is given by the diamond data point.

for the $\frac{7}{2}^-$ state are necessary below $q = 0.5$ fm^{-1} in order to resolve the remaining discrepancy with the quoted γ-decay rate.

From these results, and also of analyses of the electron and proton scattering observables [9], the shell model wave functions, obtained in the $(0 + 2 + 4)\hbar\omega$ model space, do not exhibit the correlations necessary to define the clustering of the wave functions.

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