Applications of fully microscopic theories of electron and proton scattering from nuclei

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Electron form factors and proton scattering data to discrete states in light mass nuclei have been analyzed using fully microscopic theories of the scatterings. Large basis shell model calculations have been used to determine the unique set of one body density matrix elements (OBDME) for electron and proton scattering in both elastic and inelastic channels. When meson exchange current (MEC) corrections are taken into account with the electron form factors and when the proton scattering calculations include all exchange effects with realistic two nucleon ($NN$) interactions in the nuclear medium, the electron scattering form factors and the cross sections and analyzing powers from proton scattering (in the energy range 65 to 200 MeV) to discrete states of nuclei can be very well predicted. With credible large basis shell model descriptions of the structure, no a posteriori adjustments to the results of calculations with this approach need be made to reproduce most current data.

1. Introduction

Successful analyses of electron scattering form factors and of proton (elastic and inelastic) scattering cross sections and analyzing powers, are complementary tests of the details of the structure of nuclei. Such is predicated upon scattering in both cases being dominated by processes that can be described on a one body (bound nucleon) level and for which only the OBDME from nuclear structure are required. Many scatterings from light nuclei and to low excited discrete states seem very appropriate for this treatment [1-3]. We show herein the results of analyses of electron and proton scattering from $^{12}$C and from $^{6}$Li. Our analyses of scattering from $^{12}$C justify the confidence we have in our ability to predict the results of proton scattering when electron scattering form factors are used as the first criterion of the quality of the structure model chosen for the nucleus. Our interest with Lithium stems mainly from the belief that it should exhibit clustering effects, and we seek to assess how well large basis shell model prescriptions can portray that clustering.

Studies of the elastic (and inelastic) data from the scattering of electrons and of protons from $^{12}$C are particularly useful in a quest to identify appropriate descriptions of discrete states of light mass nuclei as well as for the 'best' effective $NN$ and nucleon-nucleus ($NA$) interactions at intermediate energies. First there are many states below 20 MeV in excitation which can be resolved easily by scattering of protons with incident energies up to 800 MeV. Second, that spectrum includes both negative and positive parity states of natural and unnatural spin–parity and of isospin 0 and 1. Further there is high quality scattering data available, especially of cross sections and analyzing powers with proton scattering, against which the chosen models of structure can be tested. Finally, the target is light enough that reasonable large basis models of its structure can be made for use in our microscopic model analyses of scattering.

If, as we shall assume, the dominant scattering processes for electron and proton scattering from nuclei are one body (single bound nucleon) in character, there are three basic ingredients that must be specified to calculate transition amplitudes. They are the nuclear structure properties of the OBDME, the bound nucleon wave functions, and the specifics of the reaction mechanism.

With electron scattering, the reactions are promoted by the (known) electromagnetic interaction. Yet there is still some uncertainty with regard to the appropriate form of the nuclear currents that are required in analyses of form factors. In particular, both the current and charge densities contain in principle one-, two-, up to $A$-body components, corresponding to the exchange of charged bosons responsible for the nuclear interaction. Analyses of data usually have involved a restriction to just the one–body terms, although some recent analyses of intermediate energy photonuclear reaction data for which magnetic interactions are important have found it necessary to include in the current operator, two-body components from MEC. Specific MEC have also been used for magnetic electron scattering and as such require two body density matrix elements from structure. A further difficulty arises as all practical nuclear shell model calculations involve truncated basis spaces and, in most cases, interactions specified only by matrix elements. Such influence the specification of operators to be used to obtain form factors. That operator form must be both utilitarian and capable of accurate reproduction of data. In the case of the electric multipoles, the difficulties discussed above may be partially circumvented by the use of Siegert's theorem, with which one can replace the current density by the charge density in such a way that the effects of omitted currents are incorporated implicitly [4].

With proton elastic scattering the optical potentials must be generated. Microscopically that requires an effective $NN$ interaction, the elastic scattering OBDME (equivalently the shell occupancies of nucleons in the target) and the single particle bound state wave functions. The resulting $NA$ optical potential is complex, energy dependent and nonlocal; the latter being due in large part to antisymmetrizing the $(A+1)$ particle scattering state. Those optical potentials then are used to provide the distorted waves in distorted wave approximation.

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analyses of inelastic proton scattering. In our approach, the same effective $NN$ interaction is chosen as the transition operator that promotes each transition, and the structure information of both single nucleon bound states and the many nucleon target wave functions (from which the inelastic scattering OBDME are obtained) are defined from the shell model calculation that gave the ground state properties.

To date, analyses of electron scattering data has been foremost in the overall process by which details of proposed nuclear structure are tested because of the greater certainty there has been as to specifics of the reaction operators compared to those required in analyses of proton scattering. Such need no longer be the case, although we continue to use analyses of electron form factors to place constraints upon the character of the nucleon bound state wave functions.

2. Electron scattering form factors

A brief development of the electron scattering form factors is given herein to identify those elements of nuclear structure that enter analyses and to specify how the one body form of the transverse electric operators take into account MEC effects. Electron scattering form factors involving angular momentum transfer $J$, may be expressed as

$$|F_J(q)|^2 = \frac{1}{2J_1 + 1} \left( \frac{4\pi}{2^2} \right) |\langle \Psi_{J_1} || T_J^a(q) || \Psi_{J_1} \rangle|^2,$$

where $a$ indicates either longitudinal, transverse electric, or transverse magnetic form factors. Assuming one body forms for the operators, the reduced matrix elements may be expressed in the form

$$\langle \Psi_{J_1} || T_J^a(q) || \Psi_{J_1} \rangle = \frac{1}{2^2} \sum_{J_2, \alpha} \left[ S_{J_1J_2}^{(\alpha)(a)} M_{J_1J_2}^{(\alpha)(a)} \right],$$

where, with $\alpha$ being the nucleon type label, $S_{J_1J_2}^{(\alpha)}$ is the matrix of transition densities (OBDME), and $M_{J_1J_2}^{(\alpha)}$ are the single particle matrix elements. The OBDME and the single particle matrix element are specified by

$$S_{J_1J_2}^{(\alpha)} = \left( \Psi_{J_1} \left[ a_{J_2}^{(\alpha)} \times a_{J_1}^{(\alpha)} \right]^J \right) \Psi_{J_1} \text{ and } M_{J_1J_2}^{(\alpha)} = \left( \phi_{J_2}^{(\alpha)} || T_J^a(q) || \phi_{J_1}^{(\alpha)} \right).$$

In the calculations we have made, the single particle wave functions $\{\phi_{J_1}^{(\alpha)}\}$ are either harmonic oscillator (HO) or Woods-Saxon (WS) potential functions. The forms for the single particle transition operators are well known [5] and we consider only how MEC effects can be included. With magnetic transitions the particle currents are involved explicitly in the specification of the excitation operator, for which MEC effects also must be evaluated explicitly. On the other hand, MEC effects in the electric transition operators can be subsumed to a redefinition of the one body operators.

In the notation of Friar and Haxton [3], the bare one–body electric operators are defined by

$$T_{JM}^{(1)}(q) = \frac{f_{SN} q_J q_M}{2M} \sum_{i=1}^{A} \left\{ \Delta_{JM}(k)[1 + \tau_{5}(k)] + \Sigma_{JM}(k) \frac{1}{2} \mu^* + \mu + \tau_{5}(k) \right\},$$

where

$$\Delta_{JM}(k) = \left[ -\sqrt{\frac{J}{J+1}} M_{J+1}^{JM}(q\bar{r}_k) + \sqrt{\frac{J+1}{2J+1}} M_{J-1}^{JM}(q\bar{r}_k) \right] \cdot \frac{1}{q} \nabla(k),$$

and

$$\Sigma_{JM}(k) = M_{J}^{JM}(q\bar{r}_k) \cdot \sigma(k).$$

with the functions $M_{J}^{JM}(q\bar{r}_k)$ defined by

$$M_{J}^{JM}(q\bar{r}_k) = \langle j_{L}(qr) Y_{JM}^{M} \rangle_{r}.$$
Friar and Fallieros [6] have derived a form for the transverse electric operator in which current conservation is invoked for arbitrary wavelength. However, using that form (designated as $T_{et}^{(1)}$) in an analysis of the transverse $E2$ form factor for the $2^+_0$ (4.44 MeV) state in $^{12}$C, Friar and Haxton [3] found that the calculated form factor did not have the correct high $q$ dependence. This they attributed to an unphysical singularity in the Siegert–like part of the current density operator which contributed significantly because the nuclear wave functions did not give current conservation. With wave functions that give current conservation, any contribution from that singularity is canceled by a corresponding term from the magnetic moment current density [3]. Indeed, by construction, all operator forms give identical matrix elements if a consistent system of wave functions and corresponding conserved charge-current operators are used. However, in practical nuclear structure calculations the free-nucleon one-body current and restricted space basis states do not form such a consistent system; effective two-body currents would have to be generated to account for the omission of meson-exchanges (corresponding to the actual nuclear interaction) and for the omission of basis states outside the assumed space.

3. The effective interaction and nucleon scattering from nuclei

Nonrelativistic many body theories of the NA optical potentials are framed around the $NN t$ matrices which, in momentum space and for channels $\{JST\}$, are solutions of the Lippmann–Schwinger equation,

$$t_{LL'}^{(JST)}(p', p; k) = V_{LL'}^{(JST)}(p', p) + \frac{2}{\pi} \sum_{i} \int_{0}^{\infty} V_{Li}^{(JST)}(p', q) \frac{1}{q^2 - \frac{1}{4} k^2 - i\epsilon} t_{Li}^{(JST)}(q; p; k) q^2 dq ,$$

with $k$ being the relative on-shell momentum. But when the struck nucleon is embedded in a nuclear medium, it is more appropriate to use medium modified $NN g$ matrices in optical model calculations. The $g$ matrices we have used are solutions of the Brueckner–Bethe–Goldstone (BBG) equation for infinite nuclear matter ($k_f$ being the Fermi momentum),

$$g_{LL'}^{(JST)}(p', p; K, k_f) = V_{LL'}^{(JST)}(p', p) + \frac{2}{\pi} \sum_{i} \int_{0}^{\infty} V_{Li}^{(JST)}(p', q) [\mathcal{H}] g_{Li}^{(JST)}(q; p; K, k_f) q^2 dq ,$$

where

$$\mathcal{H}(q, k, K, k_f) = \frac{\tilde{Q}(q, K; k_f)}{\tilde{E}(q, K; k_f) - \tilde{E}(k, K; k_f) - i\epsilon} .$$

Therein $\tilde{Q}(q, K; k_f)$ is an angle averaged Pauli operator with an average center of mass (CM) momentum, $K$, and the energies in the propagators include auxiliary potentials, $U$, defined by

$$\tilde{E}(q, K; k_f) = (q^2 + K^2) + \frac{m}{k_f^2} [U(|q + K|) + U(|q - K|)] .$$

Details of the calculations have been given previously [7]. In a free $NN$ collision the struck nucleon initially has zero momentum, but when it is embedded in (local) nuclear matter that struck nucleon can have a range of momentum values and the relative and CM momenta are not the same. An average must be taken. Finally we have used the Paris interaction [8] as the input $NN$ potential.

To study nucleon elastic and inelastic scattering, we have used the program DWBA91 of Raynal [9] and so require an effective interaction in coordinate space. To parametrize the momentum space $t$- and $g$ matrices into an effective interaction in coordinate space, first one must choose the set of input data for which a fit is to be optimized. We consider the complex, half-off-shell $g$ matrices that are solutions of the BBG equations in 15
channels \(\{J^*, L, L'/\}\) where \(J\) is the total angular momentum of the \(NN\) system, \(S\) the spin, \(T\) the isospin, \(L\) and \(L'\) the orbital angular momenta. As the struck nucleon is embedded in nuclear matter, a selected (average) value for the \(NN\) energy, based upon the incident energy of interest with scattering, is used to specify the on-shell momentum. The coordinate space form of the local effective transition operator required to specify not only the optical potential for elastic scattering but also the transition operator in the DWA calculations of inelastic scattering is

\[
g^{(i)ST}_{\text{eff}}(r; E; k_f) = \sum_{j=1}^{n_i} S_j^{(i)}(E; k_f) \frac{e^{-s_j(r)}}{r}; \tag{15}
\]

the index \(i\) specifying the central, tensor and spin-orbit components. Therein \(S_j^{(i)}(E)\) are complex, energy and density dependent strengths and \(j\) represents the set of ranges chosen. In principle, the number of strengths and ranges \((n_i)\) chosen can be as large as one likes, though for all operators \(n_i = 4\) seems sufficient to reproduce accurately the half-off-shell \(g\) matrices for laboratory energies between 65 and 200 MeV [10]. Note that with \(k_f = 0\), we obtain the \(NN\) \(t\) matrices.

4. The \(NA\) optical potential and the DWA for inelastic scattering

As noted previously, in our microscopic approach, three quantities are required to specify the \(NA\) optical potential. They are the OBDME of the \(A\)-body spectroscopy, the single particle bound state wave functions, and the effective \(NN\) interaction. The OBDME (single nucleon shell occupancies in most cases of elastic scattering) for many light mass nuclei have been obtained from small \((0\omega)\) and large \((0 + 2)\omega\) and \((0 + 2 + 4)\omega\) basis shell model calculations. The effective interaction required is basis of the form we have discussed above and it is folded with the nuclear structure functions allowing explicitly for the antisymmetrization of the projectile with each and every target nucleon. Further, a kinematic correction to the effective \(NN\) interaction is required to describe a given \(NA\) system (see Eq. (19) in Ref. [11]). It remains then only to determine the specific radial variation of the Fermi momentum which is defined by \((k_f(r))^2 = 3\pi^2 \rho(r)/2\) where \(\rho(r)\) is the density profile of the target. The result of this folding is a complex, energy dependent, nonlocal optical potential that has central and spin–orbit attributes. By this full folding process we include modifications due to the presence of the nuclear medium. We have obtained such optical potentials also by limiting the effective interaction to be that defined by mapping the free \(NN\) \(t\) matrices. Note that this corresponds to the choice \(k_f(r) = 0\) for all radii. Results obtained with the density dependent interaction will be identified by the label ‘DD’ while those found by eliminating medium effects will be identified as ‘free’.

The forms of the optical potentials follow from identification of the net effective interaction between the projectile and each and every nucleon in the target as

\[
V_{\text{eff}}(|r|; E, k_f) = \sum_{LST} (2L + 1) V_L^{ST}(r_1, r_2; E, k_f) P_L(\cos \theta) P^S P^T, \tag{16}
\]

where \(|r| = |r_1 - r_2|\) and \(V_L^{ST}(r_1, r_2; E, k_f)\) are two body spin \((S)\) and isospin \((T)\) multipoles. Those multipoles are related to the effective interaction by a series of Fourier and Bessel transformations [12], viz.

\[
V_L^{ST}(r_1, r_2; E, k_f) = \frac{2}{\pi} \int W^{ST}(q; E, k_f) j_L(q r_1) j_L(q r_2) q \, dq, \tag{17}
\]

where

\[
W^{ST}(q; E, k_f) = \sum_i \int \exp(iq \cdot r) g^{(i)ST}_{\text{eff}}(|r|; E, k_f(r)) \, dr. \tag{18}
\]

Antisymmetrizing the \(NA\) wave function leads to a nonlocal optical potential which usually takes the form

\[
U(r_1, r_2; E) = \delta(r_1 - r_2) \sum_n \zeta_n \int \varphi^*_n(s) \, v^D(r_1, E; \rho(k_f(s))) \, \varphi_n(s) \, ds \\
+ \sum_n \zeta_n \varphi^*_n(r_1) \, v^{EX}(r_2, E; \rho(k_f(r_2))) \, \varphi_n(r_2) \\
\Rightarrow U_D(r_1; E) + U_{EX}(r_1, r_2; E). \tag{19}
\]

Here \(v^D\) and \(v^{EX}\) are appropriate combinations [12] of the \(ST\) channel elements of the effective interaction of Eq. (16), \(\varphi_j(r)\) are the single (bound) nucleon wave functions and, \(\zeta_n\) are the shell occupancies in the target. We use these microscopically defined optical potentials to give the distorted waves for the DWA analyses we make.
of inelastic proton scattering. With $A_{01}$ being a two nucleon antisymmetrization operator, the DWA transition amplitudes can be written as

$$T_{j_1 j_2}^{M_{j_1} M_{j_2}}(\theta) = \left\langle \chi^{(+)}(k_0) \right| \left( \Psi_{J_1 M_1} (1 \ldots A) \right| A_{01} \left\{ \left| \chi^{(+)}(k_0) \right| \left| \Psi_{J_2 M_2} (1 \ldots A) \right\} \right\rangle . \quad (20)$$

wherein the distorted wave functions are denoted by $\chi^{(+)}(k_0)$ for an (incoming/outgoing) proton with spin projection $\mu$, wave vector $k$ and coordinate set 'q'. Since all pairwise interactions between the projectile and every target nucleon is taken to be the same, it is convenient to make a cofactor expansion of each A-body nuclear state, viz. to use

$$|\Psi_{JM}(1 \ldots A)\rangle = \frac{1}{\sqrt{A}} \sum_{j, m, \alpha} \left| \varphi_j^{(a)}(1) \right\rangle a_{jm} |\Psi_{JM}(1 \ldots A)\rangle . \quad (21)$$

Therewith all dependences upon the coordinate '1' selected to be the active entry in the 'g_{eff}' can be isolated so that the transition amplitudes expand to the form

$$T_{j_1 j_2}^{M_{j_1} M_{j_2}}(\theta) = \sum_{j_1, j_2, m_1, m_2, \alpha} \left\langle \Psi_{J_1 M_1} \left| a_{j_1 m_1}^d a_{j_2 m_2} \right| \Psi_{J_2 M_2} \right\rangle_{\alpha}$$

$$\times \left\langle \chi^{(+)}(k_0) \right| \left| \varphi_j^{(a)}(1) \right\rangle g_{eff}(0, 1) A_{01} \left\{ \left| \chi^{(+)}(k_0) \right| \left| \varphi_j^{(a)}(1) \right\} \right\rangle . \quad (22)$$

From this it is evident that besides the distorted waves and effective $NN$ interaction, we require the bound state wave functions of target nucleons and the transition OBDMER since

$$\left\langle \Psi_{J_1 M_1} \left| a_{j_1 m_1}^d a_{j_2 m_2} \right| \Psi_{J_2 M_2} \right\rangle = (-)^{(j_1 - m_1)}(j_1 j_2 m_1 - m_2) [2I + 1]^{-1} \langle J_1 IM_1 | J_2 M_2 \rangle S_{j_1 j_2 I}^{(a)} . \quad (23)$$

5. A study of $^{12}$C — predictions of the models

The spectroscopy of $^{12}$C has been determined using the program OXBASH [13] and the MK3W interaction. The $0\hbar \omega$ calculation forms only a positive parity spectrum in poor agreement with data. But that is not the case with the spectrum from the large basis study. While the positive parity states of $^{12}$C were calculated in a complete $(0 + 2)\hbar \omega$ space and with this interaction, the negative parity states were calculated in a restricted $(1 + 3)\hbar \omega$ space. In both calculations the same single particle basis of $0s$ up to and including the $0f^{1p}$ shell was used. The restriction from a full $(1 + 3)\hbar \omega$ study is that we have not included the $0g^{1d^{2}}s$ shell. With exceptions, notably the $3^{-}_{1} ; 0$ state at 9.64 MeV and the superdeformed $0^{+}_{1}$ ; 0 state at 7.65 MeV, that calculated spectrum is in agreement with observation. All known spin—parity assignments are matched by ones from our large basis shell model calculations within 700 keV [1].

![Fig. 1. The elastic (electron) longitudinal form factor compared with the $0\hbar \omega$ (dashed curve) and the $(0 + 2)\hbar \omega$ (solid curve) results.](image)

With regard to the ground state of $^{12}$C, the large basis calculation has not varied the ground state occupancies much from those found with the $0\hbar \omega$ model. It is not a surprise then that, with either set, we find a good fit to the longitudinal form factor from elastic scattering of electrons when proper single particle (bound state) wave functions are used. For both HO and WS potential bound states, the fits to the experimental data are shown in Fig. 1. They are very good over the range of momenta transfer shown (to 3.5 fm$^{-1}$). With the same OBDMER and single particle bound state wave functions, we folded the effective interactions to specify the complex nonlocal $p^{+}-^{12}$C optical potentials. Using them in one pass for solutions of the Schrödinger equations gave the results for the elastic scattering of 200 MeV protons that are presented in Fig. 2. The solid curves given therein were obtained by using WS wave functions, and those shown by the dashed curves were found by using HO ones. The results in the left panels were obtained using our effective interactions but with the restriction that they are only those mapping the free $NN$ t matrices. The cross section is predicted well, but the full medium modified effective interaction gives a very much better prediction of the measured data. In fact the WS bound state result is in quite exceptional agreement with the data to 60° scattering. However it is with predictions of
the analyzing power that there is most improvement when our medium modified effective $NN$ interactions are used. Clearly with the 'free' interaction, the results reflect only the gross structures in the measured data. Such mismatches have been found by others and sometimes have been attributed to the neglect of relativistic effects. However most other calculations do not allow satisfactorily for the exchange amplitudes, and at best those amplitudes are approximated with an 'equivalent local' representation. It is evident from our results that the medium modifications to the effective $NN$ interaction are essential to give as good agreement with the data, especially the analyzing power. We also believe that the nonlocality due to antisymmetrization should be treated as exactly as possible, especially as we have analyzed the 200 MeV proton elastic scattering data from many other nuclei (22 in fact [14]) as well as a few at energies of 135, 160, and 180 MeV, with equal success. A compendium of these results exists [15] which includes all of our 65 MeV proton scattering results.

The calculated longitudinal electron scattering form factors for the transition to the $2^+; 0$ (4.44 MeV) state in $^{12}$C are compared with the data in Fig. 3. The results of calculations with the $0\hbar\omega$ and the $(0 + 2)\hbar\omega$ models OBDME are shown by the solid and the dashed curves respectively. Both calculations were made by using WS single particle wave functions. The $0\hbar\omega$ model calculation underestimates the data by a factor of two. But clearly by using the $(0 + 2)\hbar\omega$ wave functions we get far better agreement with the data. The additional transition strength obtained by inclusion of $2\hbar\omega$ components in the wave functions provide the necessary boost in magnitude to give that agreement. Such enhancement has been previously designated as 'core polarization'. The improvement in the description of the longitudinal form factor is also reflected in the transverse $E2$ form factor. The data from electron scattering to the $2^+$ state are compared with the results of our diverse calculations in Fig. 4. The $0\hbar\omega$ and the $(0+2)\hbar\omega$ shell model results are displayed in sections (a) and (b) respectively.

In both segments, the solid, dotted and dashed curves depict the results of calculations made using the $T^{el}$, $T^{nr}$ and $T^{nr'}$ operators respectively. Clearly there is significant improvement in fits to the data by changing from the standard $T^{el}$ operator to $T^{nr'}$ so confirming the result of Friar and Haxton [3]. The effect is particularly significant for low $q$, where meson exchange currents are expected to be negligible, indicating that the $T^{nr'}$ operator corrects for the lack of cross shell nucleonic currents with the $0\hbar\omega$-shell model. With the bigger basis calculation (b) the effect of replacing $T^{el}$ by $T^{nr'}$ is not as dramatic, presumably because some of the cross shell currents are now included explicitly. Rather the correction is essentially just a scale shift. The shift is not sufficient to fit the measured data, a further enhancement of 1.6 in the result using $T^{nr'}$ would be required to bring about agreement over the whole range of momentum transfer. The results using $T^{nr'}$ indicate the same anomalous behavior at high momentum transfer as found in other calculations [3]. But more data in the range $q \sim 2 - 3$ fm$^{-1}$ are needed.

Fig. 2. Differential cross section and analyzing power from the elastic scattering of 200 MeV polarized protons from $^{12}$C.

Fig. 3. The longitudinal form factor from the excitation of the $2^+; 0$ (4.44 MeV) state in $^{12}$C.

Fig. 4. The transverse electric form factor from the excitation of the $2^+; 0$ (4.44 MeV) state in $^{12}$C.
The results of our DWA calculations for the cross section and analyzing power from 200 MeV protons exciting the $2f_0^2; 0$ (4.44 MeV) state in $^{12}$C are compared with the data in Fig. 5. With both the free and density dependent (DD) effective interactions, the larger basis structure calculations increase the predicted magnitudes above those given with the $0\hbar\omega$ wave functions into quite good agreement with the data. The results reflect the effects observed when those wave functions were used in the calculations of the electron form factors. In this case though the dashed and solid curves depict the $0\hbar\omega$ and $(0 + 2)\hbar\omega$ model results respectively. Clearly the density dependent effective interaction leads to cross sections in best agreement with the data reproducing the shoulder effect in the $20 - 40^\circ$ region in particular. The analyzing power results are not as good fits to the measured values. They do reflect the general shape of the data however, the density dependent results more so than the free ones in the scattering angles to $50^\circ$. We stress that no core polarization corrections have been applied to the calculated cross sections.

The isovector $E2$ form factor from inelastic electron scattering to the 16.11 MeV $2^+$ state in $^{12}$C is of interest as past studies suggest that $0\hbar\omega$ wave functions give a good specification of the transition. We find that the larger basis structure calculation modulates the $0\hbar\omega$-shell components but partially compensates changes that brings with numerous small additional elements. Unlike the case of the 4.44 MeV transition, now only the $0p \leftrightarrow (0f1p)$ elements are non-trivial. The net effect of core polarization effects in this isovector excitation, as measured by the variation in the form factors when the $0\hbar\omega$- and $(0 + 2)\hbar\omega$ shell model OBDME are used, are not as dramatic as in the isoscalar excitations. Furthermore they are destructive in nature as is evident from the results that are displayed in Fig. 6. The dashed and solid curves depict the calculations made using the $0\hbar\omega$ and the $(0 + 2)\hbar\omega$ spectroscopic information respectively. The $T^{ef}$ form of the operators have been used.

The $2^+_1; 1$ (16.11 MeV) excitation results are shown in Fig. 7. The diminution of transition strength expected with larger basis calculations of such transitions occurs. Both the free and density dependent interactions, with $(0 + 2)\hbar\omega$ spectroscopy, give results in good agreement with data. The reduction in cross section predictions with the larger basis spectroscopy matches that noted with the transverse form factor from electron scattering to this state. In this case, all features of the transition operator are important in the calculations, and again we stress that no core polarization corrections have been used to seek better agreement with the data.

6. A study of $^6$Li – evidence of clustering

A good place to look for cluster behavior in nuclei is in the $^6$Li and $^7$Li nuclei. Both of these have been...
described successfully in terms of clusters; as $\alpha + d$ or as $\alpha + p + n$ in a three-body description of $^6\text{Li}$, and $\alpha + t$ or two-cluster configurations for $^7\text{Li}$. The simple $0\hbar\omega$ shell model descriptions of these nuclei automatically contain such clustering as the $0s$-shell is an inert core (the $\alpha$ particle) and the extra, valence, nucleons occupy the $0p$-shell. More recently, large space multi-$\hbar\omega$ shell models have been constructed for these nuclei [16]. Such are required if a shell model approach is to model cluster effects realistically. We consider shell model wave functions within the $0\hbar\omega$, $(0+2)\hbar\omega$, the $(0+2+4)\hbar\omega$, and $(0+2+4+6)\hbar\omega$ model spaces for $^6\text{Li}$. However, within the largest of these, because of the dimension of the space, only a calculation of the ground state properties has been made. The choice of model space dictates the choice of interaction (for details see Ref. [17]). All calculations of the wave functions and of the OBDME, which specify the structure changes in the scattering events to be studied, were carried out using the code OXBASH [13].

A first assessment of the cluster-like behavior of the shell model wave functions for $^6\text{Li}$ is given by examining the ground state properties predicted by each model. In particular, we examined the root-mean-square (r.m.s.) charge radius, and the magnetic and quadrupole moment predictions. For $^6\text{Li}$ (and for $^7\text{Li}$), the r.m.s. radius is predicted adequately by all model calculations and is insensitive to the addition of excitations to the $0\hbar\omega$ model space. The effect of increasing the size of the model space is more noticeable in the magnetic and quadrupole moments. There is a dramatic decrease in the calculated value of the magnetic moment in adding $2\hbar\omega$ excitations to the model space, from $0.869\mu$ to $0.848\mu$, compared with the experimental value of $0.822\mu$. Proceeding from the $(0+2)\hbar\omega$ space to the $(0+2+4+6)\hbar\omega$ there is a steady decrease in the value of the magnetic moment, but that change is more gradual. In the case of the quadrupole moment, the effect of increasing the model space is most dramatic. There is little change to the result of $-0.264\text{ e fm}^2$, obtained using the $0\hbar\omega$ space wave function, when $2\hbar\omega$ excitations are allowed. However, adding $4\hbar\omega$ excitations gives a value of $-0.012\text{ e fm}^2$, a correction of more than an order of magnitude. In comparison to the experimental value of $-0.083\text{ e fm}^2$, the calculation actually has overcompensated. Clearly large variations in the prediction of the (small) quadrupole moment may be produced with small perturbations added to the shell model functions. Similar results are obtained from calculations of the structure of $^7\text{Li}$ [17], although in that case no structure model could reproduce the very large experimental value of $-4.06\text{ e fm}^2$. The elastic electron scattering form factors for $^6\text{Li}$ are as shown in Fig. 8. The results obtained using the wave functions of the $(0+2+4)\hbar\omega$, $(0+2)\hbar\omega$, and $0\hbar\omega$ model spaces are given by the solid, dashed, and dot-dashed lines, respectively. In Fig. 8(a), the (elastic) longitudinal form factor data are compared to our results obtained by using the various shell model structures of the nucleus, and with WS single nucleon bound state wave functions. All are good fits to the data (to $3\text{ fm}^{-1}$) and those fits are dominated by the $C0$ contribution. As shown in Fig. 8(b), the $C2$ contribution is some orders of magnitude smaller than that of $C0$. The transverse $M1$ elastic electron scattering form factor for $^6\text{Li}$ is compared with the results of our calculations in Fig. 8(c). Only with the $(0+2+4)\hbar\omega$ spectroscopy can we predict the shape of the form factor at higher momentum transfers adequately.

The results of our calculations of the elastic scattering for 200 MeV protons on $^6\text{Li}$ are compared with the data in Fig. 9. The cross sections are shown in the top segments; the analyzing powers are given in the bottom ones. The results for the $(0+2)\hbar\omega$ and $(0+2+4)\hbar\omega$ models are given in the left and the right hand panels respectively. For the single particle bound states, we chose either HO (dashed curves) or WS (solid curves) wave functions as determined by the fits to the electron scattering (longitudinal) form factor. In this scattering, non-zero multipoles are possible and their effects were calculated in the DWA using DWBA91 [9]. Recoil corrections to the cross section and analyzing power have
also been included. When such is done and the OBDME from either the \((0+2)\hbar\omega\) or the \((0+2+4)\hbar\omega\) models is used, we find very good agreement with not only the cross section but also the analyzing power. Together with the very good agreement these structures give with elastic electron scattering data, this suggests that the ground state wave functions obtained in the multi-\(\hbar\omega\) spaces have converged.

The longitudinal inelastic electron scattering form factor to the \(3^+; 0\) (2.185 MeV) state is shown in panel (a) of Fig. 10. This form factor is dominated by the \(C2\) component. The results found using our big bases OBDME, reproduce the magnitude of the measured form factor for \(q \geq 1\) fm\(^{-1}\). Both contain strength from transitions outside of the \(0p\)-shell which enhance the \(C2\) contributions above those given by the \(0\hbar\omega\) model. The \((0+2+4)\hbar\omega\) model structure is most favored as there is almost exact agreement with the data in that region of momentum transfer. However, the quoted \(B(E2)\) value for the associated \(\gamma\)-decay of this \(3^+; 0\) state is \(9.3 \pm 2.1\) e\(^2\) fm\(^4\), and the values obtained by all of our calculations are significantly smaller. All results require substantial renormalization to reproduce the measured value. That is confirmed by predictions of the electron scattering form factor at low momentum transfer. Below 1 fm\(^{-1}\) all of the calculated results are less than observation. Yet that degree of renormalization is not as suggested by the results of the calculations of the form factor at higher momentum transfer. While this suggests that the internal (nucleon) dynamics of the nucleus are well described by the inclusion higher \(\hbar\omega\) excitations in the model space, such cannot account for the asymmetries of the structure. At large radii, which most influence scattering at low momentum transfer, the clustering of the wave function is not reproduced by the shell model in which up to \(4\hbar\omega\) excitations are included. This deviation of all the calculated results away from the data is illustrated further in Fig. 10(b) which displays the \(B(E2|q)\) value as a function of momentum transfer for the \(3^+; 0\) (2.186 MeV) state in \(^6\)Li, and as determined from the measured and predicted longitudinal inelastic form factors. This is achieved by removing from the form factor most of the dependence on the momentum transfer, according to the transformation given by Brown, Radhi, and Wildenthal [18]. The \(B(E2)\) value as related to the associated \(\gamma\)-decay is given by the \(q = 0\) intercept.

The cross sections and analyzing powers obtained from the various shell models for the inelastic scattering of 200 MeV protons to the \(3^+; 0\) (2.186 MeV) state in \(^6\)Li are compared to the data in Fig. 11. HO single particle wave functions were used to find the results shown in the left hand panels while those of WS form were used to obtain the results displayed in the right hand ones. The cross sections displayed are the sum of all possible angular momentum transfers which may contribute. Consistent with the analysis of the inelastic electron scattering form factor, the \(I = 2\) component is the most dominant. It is evident from Fig. 11 that the result found by using the simple \(0\hbar\omega\) model is deficient. Larger space structures give cross sections on the order of magnitude greater and are now in quite good agreement with data apart from the region around 20° which is still too weak. This problem at low momentum transfer is consistent with the analyses of the inelastic electron scattering data to this state. The analyzing power varies in a similar way and with both large basis model calculations we reproduce the data well. We note that as the model space is increased, the \(I = 2\) scattering amplitudes are the most enhanced to give improved fits to the data. More details and the results of analyses of other excitation data (as well as those for \(^7\)Li) are to be found in a recent publication [17].
7. Conclusions

Microscopic theories of electron and of proton scattering from nuclei have been used as complementary tests of details of nuclear structure. Both scattering theories are predicated upon the scattering operators being one body (struck nucleon) in form so that the analyses probe the OBDME of the nuclear spectroscopy to be assessed and the character of the single nucleon bound state wave functions. Also as our aim is to predict scattering results so that we can identify any shortcomings of the structure adopted, the bases specifying those nuclear structure models must be large enough to give realistic spectral properties of the nuclei. For the two nuclei considered herein, $^{12}\text{C}$ and $^6\text{Li}$, the former is very well described by a shell model using a complete $(0 + 2)\hbar\omega$ space. For the cluster type nucleus, $^6\text{Li}$, we find that even $(0 + 2 + 4)\hbar\omega$ space calculations do not quite match well all ground state and low excited state properties. But they are quite good nevertheless.

With realistic sets of OBDME, one must then specify the interaction operators by which the different scattering events are promoted. The electromagnetic interaction is known but for transverse form factor analyses, MEC effects need be considered. For the electric transverse case, the continuity equation leads to equivalent one body operator forms of the transition operators that incorporates the effects of (two nucleon density matrix element) MEC. For proton scattering from nuclei, we need the effective interaction between the projectile and each and every nucleon within the target. For projectile energies in the range 65 to 200 MeV, such have been established by mapping the half-off-shell $NN g$ matrices. The effective interactions are complex, and energy and density dependent. Full folding of those effective interactions then defined the proton-nucleus optical potential. That potential is complex, energy dependent and nonlocal.

Electron form factors and proton scattering (cross sections and analyzing powers) from $^{12}\text{C}$ were reported. With the large basis model OBDME and Woods-Saxon bound state wave functions, our predictions of the elastic, $2^+;0$ (4.44 MeV), and the $2^+;1$ (16.11 MeV) transitions all aged very well with data. No a posteriori adjustments of any kind were required to achieve those results. Electron and proton scattering then were used to test our structure models for $^6\text{Li}$. While the predicted r.m.s. radii for $^6\text{Li}$ (and $^7\text{Li}$) are insensitive to the size of the model space and yet reproduce the measured values, the magnetic dipole and quadrupole moments show dramatic convergence towards the experimental values as the size of the model space is increased. There is still some degree of renormalization necessary even in the results obtained using the $(0+2+4+6)\hbar\omega$ wave functions, while in the case of the quadrupole moments, the results obtained are still far from the experimental values. The same is indicated especially in the analyses of the electron scattering form factors and of the proton scattering observables. In the analyses of the scattering exciting the $3^+;0$ state in $^6\text{Li}$, no result of our calculations could reproduce the data at low momentum transfer. This is associated with the underestimation in the B(E2) value of each transition of about a factor of 2. Yet the high momentum transfer data for those scatterings are well reproduced when using the multi-$\hbar\omega$ wave functions. This indicates that the internal nucleon dynamics are well described, and that the asymptotics at large radius, where clustering is expected to appear, as yet are not well reproduced. But there is remarkable agreement between experiment and theory in the transverse electron scattering form factors, which also is indicative that our large space model spectroscopies give reasonable descriptions of the internal nucleon dynamics.

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References