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Electrostatic images for underwater anisotropic conductive half spaces

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Abstract – A static image principle makes it possible to derive analytical solutions to some basic geometries for DC fields. The underwater environment is especially difficult both from the theoretical and practical point of view. However, there are increasing demands that also the underwater geological formations should be studied in detail. The traditional image of a point source lies at the mirror point of the original. When anisotropic media is involved, however, the image location can change and the image source may be a continuos, sector-like distribution. In this paper some theoretical considerations are carried out in the case where the lower half space can have a very general anisotropy in terms of electrical conductivity, while the upper half space is assumed isotropic. The reflection potential field is calculated for different values of electrical conductivity.

The case of two conducting half spaces is relevant when we study deep-seated underwater bottom and geological layers beneath it using such electrical systems in which we lower transmitters and receivers in deep water near or within bottom. What is meant by 'deep-seated bottom' is of course relative and it is dependent on the scale of interest. In our model this means that the boundary between water and air can be omitted.

This paper shows an effort to determine the electric potential due to static (or low-frequency) current source in the vicinity of anisotropically conducting half space. The duality principle gives us the possibility to utilise recent results derived for electrostatic permittivity problems. In [1], the well-known image theory for a point charge in front of an isotropic half space, is generalised to an anisotropic half space, where the permittivity tensor characterising the anisotropy is symmetric and positive-definite, i.e., it has three orthogonal eigenvectors and the corresponding eigenvalues are positive. The derivation of the theory resembles Heaviside operational calculus [2] involving pseudodifferential operator manipulations. Although the derivation of the image requires considerable amount of algebraic labour the final result, the image function itself, is compact and readily usable in eg. electric potential calculations.

Consider two conducting half spaces with a plane boundary at $z = 0$, as in the figure (1.a). A point current source $i_0(\mathbf{r}) = I_0\delta(z - z_0)$ is located in an isotropic conducting half space at $z = z_0$, above an anisotropic half space ($z < 0$). Using the image principle, we replace the lower half conductivity with the same, isotropic one, as the upper half is, and compensate the change with an image source. For general anisotropy the image consists of a point source and a surface current distribution over an angular sector, and in [1] it was given in the form

$$i^r(\mathbf{r}) = -I_0\frac{\alpha - 1}{\alpha + 1}\delta(\mathbf{r} + \mathbf{u}_z z_0) + I_0\frac{2\alpha\beta^2(-(z + z_0))}{\pi\sqrt{\beta^2(z + z_0)^2 - x^2}} \times$$

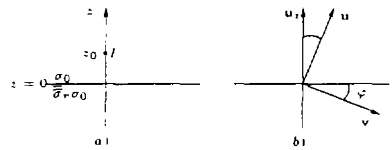

Figure 1: a) geometry of the problem: b) tilted anisotropic medium: it can viewed as rotated $\mathbf{u}_x$, $\mathbf{u}_y$, $\mathbf{u}_z$ axial medium

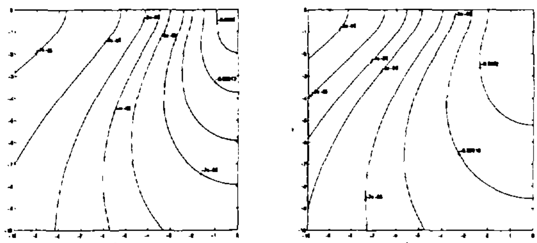

Figure 2: left: reflected potential at $z = 0$ caused by the sector-like image. $\sigma_0 = 3.2$ [S/m], $\sigma_u = 0.1$, $s_v = 0.02$, $s_w = 0.01$, $\varphi = 10°$, due to symmetry the contours can be mirrored with respect to the x and y axes to get the equipotentials for x, y=-10..+10; right: as in left, $\varphi = 65°$

$$\frac{\alpha^2[\beta^2(z+z_0)^2 - x^2] - x^2}{(\alpha^2[\beta^2(z+z_0)^2 - x^2] + x^2)^2}\delta(y)\Theta[\beta^2(z+z_0)^2 - x^2]\Theta[-(z+z_0)] \tag{1}$$

Here, $\alpha^2(1+\beta^2)$ and α^2 are the eigenvalues of a two-dimensional, symmetric and positive-definite dyadic $\overline{\overline{A}}$ which is related to the conductivity dyadic

$$\overline{\overline{A}} = -\det\overline{\overline{\sigma}}_r(\mathbf{u}_z \times \overline{\overline{\sigma}}_r^{-1} \times \mathbf{u}_z) = \alpha^2(1+\beta^2)\mathbf{u}_x\mathbf{u}_x + \alpha^2\mathbf{u}_y\mathbf{u}_y \tag{2}$$

In general, the eigenvectors of σ_r are distinct from those of $\overline{\overline{A}}$. The step functions Θ confine the sector to the lower half space, the apex is at $z = -z_0$.

For a tilted anisotropy (figure (1.b)) we get

$$\alpha = \sqrt{\sigma_w(\sigma_v \sin^2\varphi + \sigma_u \cos^2\varphi)}, \quad \beta = \sqrt{\frac{\sigma_u\sigma_v - \sigma_w\sigma_v \sin^2\varphi - \sigma_w\sigma_u \cos^2\varphi}{\sigma_w\sigma_v \sin^2\varphi + \sigma_w\sigma_u \cos^2\varphi}} \tag{3}$$

and the reflected electric potential in the upper half space is calculated from the integral $\int i^r(\mathbf{r})dV'/(4\pi s_0|\mathbf{r} - \mathbf{r}'|)$ over the image charge, figure 2.

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References

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[2] O. Heaviside. Electromagnetic Theory. Ernest Benn. Ltd, London, 1925. 2nd reprint. Vols. I, II, III.