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Thermodynamics of pion gas using states predicted from k-deformed Poincaré Algebra

Claudette E. Cordeiro, Antonio Delfino and Jishnu Dey

Av. Litorânea s/n - BoaViagem
24210-340 - Niterói - RJ -Brazil
Phone 55-21-719-6735 Fax Ext.: 2032

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Claudette E. Cordeiro,
Antonio Delfino
and
Jishnu Dey *
Instituto de Física, Universidade Federal Fluminense,
24210, Niterói, R. J., Brasil

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Abstract : κ-deformed Poincaré algebra, which preserves rotational and translational symmetries, can successfully predict the angular and radial excited states of the pion. At high temperature, T, these states can be excited in the pion gas, in addition to the usual momentum excitation. We exploit this to look at pion free energy finding it increases linearly with T. The energy per particle and the entropy show evidence of a smooth phase transition after T = 0.2 GeV.

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Laws of nature are required to be invariant with respect to Lorentz rotations as well as space-time translations, so we have to deal with the Poincaré invariance. Therefore, the ordinary Poincaré group is appropriate to treat the classical and the quantum special relativity. In general, deformed groups bring new symmetry structures into physics [1]. By deforming a group we allow the extension of this deformation to the physical theory related with the original group. It is by itself interesting to see how a determined theory or equation behaves under a new symmetry structure generated by a group deformation. For the Poincaré group, the deformation structure is not unique. A particular form due to [2], (hereafter LNR) considered in this paper shows that the three dimensional rotation and the translation subgroups are not deformed and the algebra of Lorentz boosts is modified, both for bosons and fermions. The relevant q-deformation parameter, is called κ, in this case, and when this goes to infinity we recover the undeformed algebra [2]. This κ-deformed version has two Casimir invariants C_1 and C_2 [2]. The κ-deformed Dirac equation has recently been found [3], and it factorizes the second Casimir invariant, quite unlike the usual case which factorizes the first one. Applications of LNR algebra have been done recently in order to see which would be the impact of change in the standard theories governed by the ordinary quantum special relativity. The following problems have been studied.

a) The definition of mass with different non-relativistic limits [4],

b) the non-additivity of masses and its relation to the interesting dark matter puzzle [5],

c) the classical electrodynamics problem of finding the acceleration of charged particle in a 1-dimensional homogenous electric field [6],

d) gauging the deformed Dirac equation, applying it to the quantum relativistic hydrogen atom and solving the Dirac-Coulomb problem [7].

e) calculating the Landau deformed levels [8],

f) explanation of the flattening of the experimental hadron spectrum, [11], [10],

g) application of the new mass-energy relation of κ-deformed algebra to the model of Nambu and Jona-Lasinio, *now with a natural cut-off* $1/\epsilon$ *provided by the theory* [11].

From one of these studies, namely the case (d), it turns out that for negligible deformation, the normal Dirac equation is recovered. So they expand in the deformation parameter and obtained the remarkable result that the first order effect vanishes identically. This means clearly that there is no change in the energy spectrum in the first order of perturbation theory. This is not, however, what happens in the deformed Landau levels [8] , which are expected to shift already in the same first order perturbation theory. As we can see, people are getting interested to see how a determined theory or equation behaves under a new symmetry structure generated by a group deformation. The flattening of the experimental hadron spectrum which is explained by the deformed algebra, in the case (f), lead to interesting smooth phase transitions at finite T. The most important case is that of pion gas, since boson gases generated in collisions will decay into a pion gas. In this paper we report our results and show that the results are not inconsistent with standard QCD expectations.

In the notation of Biedenharn, Mueller and Tarlini [7], (we will use this here) who use ϵ as the inverse of κ, to avoid confusion with the Dirac quantum number. This ϵ parameter is claimed to work as a length scale which must be less than 1 fm. Although the authors have stated that this self-consistent estimate of the Poincaré length is both speculative and need not be very accurate, it poses the question whether this may then affect particle and nuclear physics phenomenology, since the relevant length scale there is indeed the fermi.

To describe the κ-deformed algebra we start with the commutation relations [2] between the components of energy-momentum P_ν (ν = 0,1,2,3), angular momentum M_i and Lorentz boosts L_i:

$$\begin{gathered}[P_i, P_j] = 0, \quad [P_i, P_0] = 0, \quad [M_i, P_j] = i\varepsilon_{ijk}P_k, \\ [M_i, P_0] = 0, \quad [L_i, P_0] = iP_i, \quad [L_i, P_j] = i\epsilon^{-1}\delta_{ij}sinh(\epsilon P_0), \\ [M_i, M_j] = i\varepsilon_{ijk}M_k, \quad [M_i, L_j] = i\varepsilon_{ijk}L_k, \\ [L_i, L_j] = -i\varepsilon_{ijk}\left(M_k cosh(\epsilon P_0) - \frac{\epsilon}{4}P_k P_l M_l\right), \end{gathered} \tag{1}$$

where i, j, k = 1, 2, 3. Following is the first Casimir operator, C_1, which commutes with all the generators :

$$C_1 = \left[\frac{2}{\epsilon}sinh\left(\frac{\epsilon P_0}{2}\right)\right]^2 - P_i P_i. \tag{2}$$

In the rest frame ($P_i = 0$) :

$$C_1 = \left[\frac{2}{\epsilon}sinh\left(\frac{m\epsilon}{2}\right)\right]^2 \tag{3}$$

The mass of the particle is $\sqrt{C_1}$, but for small ϵ like ours, is practically equal to m. In the deformed Dirac equation, however, the Casimir operator itself is the mass ([7] see comment after their equation 2.8). For a general frame $(E, \mathbf{P})$ $(E \equiv P_0)$

$$\left[\frac{2}{\epsilon}sinh\left(\frac{\epsilon E}{2}\right)\right]^2 = \mathbf{P}^2 + \left[\frac{2}{\epsilon}sinh\left(\frac{m\epsilon}{2}\right)\right]^2. \tag{4}$$

This gives

$$E = \frac{2}{\epsilon}sinh^{-1}\left[\left(\frac{\epsilon}{2}\right)^2 p^2 + sinh^2\left(\frac{m\epsilon}{2}\right)\right]^{1/2}. \tag{5}$$

The idea of [9] was to put the squared momentum to be equal to J/α' in the spirit of Regge phenomenology. This, however, does not exhaust all the possible excitations of the hadron. For the pion for example, we have the radial excited states at 1.3 and 1.77 GeV. This can also be tackled in the κ-deformed scheme by assuming that the radial excitations are also linear. The only justification we know of this comes from the classic work of t'Hooft, [12] where he showed that in large N limit [1] one exactly gets *this* behaviour for the excitations for a lower dimensional model. This has recently been used also by Iachello et al.[14], who however could not fit the pion or the upsilon, since the radial excitations do not obey an exact linear law. It was found recently by Dey et al.[10] that the κ-deformed scheme provides the necessary non-linearity. The final expression for the states of a hadron with ground state m is given by

$$E = \frac{2}{\epsilon} sinh^{-1}\left[\left(\frac{\epsilon}{2}\right)^2 \left(\frac{J}{\alpha'} + nb\right) + sinh^2\left(\frac{m\epsilon}{2}\right)\right]^{1/2}. \tag{6}$$

where b, estimated by t' Hooft [12] is $b = 4\pi^2\alpha_s m_q m_q'$. α_s is the strong coupling constant which we take to be 0.55 and the best pion fit is obtained for b = 1.93 implying a reasonable mass as expected by him, $m_q = m_q' = 298$ MeV. This of course is a constituent mass obtained by him after summing all planar gluon diagrams in a Bethe-Salpeter approach and the number is close to his expectation. We should clarify, however, that in a lower dimensional model t' Hooft could not use the angular momentum J-excitation, since there is no J in such models. However we use his idea in the spirit of an ansatz in the real 3+1 dimensional model and so can justifiably use both the J and the n excitation.

In Table 2 we give the radial excited states of the pion. In Table 1 and Table 2, the fits are better than a percent. There are two parameters α' and b, and we fit five states. The $\pi(1.3)$ state has an unconfirmed J = 2 partner at 2.1 GeV. We predict it at 1.998 GeV.

The expressions for free energy, energy and number are now easy to calculate. We do not have to worry about the lack of experimental information about pion excitations beyond 2 GeV, but use the above equation to compute them. If we label the solutions of the above equation as E_i (the label i is a generic symbol for J and n), then

$$E_k = [E_i^2 + k^2]^{1/2} \tag{7}$$

and the Bose-function can be written as

$$f(E_k) = \frac{1}{e^{E_k/T} - 1} \tag{8}$$

[1] we note that there is renewed interest in large N models[13]

The free energy density is then

$$F(T) = -\frac{T}{2\pi^2}\int_0^\infty k^2 \sum_{n,J} g_{n,J}\, ln \mid 1 - e^{-E_k/T} \mid dk \tag{9}$$

and the energy and the number densities are

$$E(T) = \frac{1}{2\pi^2}\int_0^\infty k^2 \sum_{n,J} g_{n,J}\, f(E_k)\, E_k\, dk \tag{10}$$

and

$$N(T) = \frac{1}{2\pi^2}\int_0^\infty k^2 \sum_{n,J} g_{n,J}\, f(E_k)\, dk \tag{11}$$

The degeneracy factor $g_{n,J}$ is $3\,(2J+1)$, where the factor 3 comes from the pion isospin.

In fig. 1 we plot the free energy density as a function of temperature T, and find that it increases sharply after T = 0.3 GeV. In fig. 2 however, the number density is also found to increase in a similar fashion so that the free energy per particle (fig. 3) is very nearly linear in T. In fig. 1 and 2, we also plot the results using a truncated set, consisting of the experimental states alone. This is the result found in [15]. For this case the increase is not sharp, showing clearly the need for the additional states. The energy per particle, (fig. 4) has three regions, one upto 0.2 GeV, the next one from T = 0.2 to 0.3 GeV (or 0.35) and a third region from the last value onwards. The result with the truncated set has a more or less contant slope of about πT and this region was found in [15] where there is a comparison with lattice results. The first and the third regions have much smaller slope. If the second region had infinite slope, one would say that is a discontinuity in the energy per particle and there is a first order phase transition. As such the curve shows a change in the average particle energy which is smooth, suggesting the phase transition is of higher order. The entropy in fig. 5 also supports this interpretation.

If one takes the J = 1,2 angular momentum excitations, b(1.235) and $\pi_2(1.67)$, "cousins of the π" and extrapolate to find the ground state in conventional Regge theory, one finds a mass of about 0.57 GeV for the pion $\pi(.138)$. As far as we know this was first pointed out by Johnson and Nohl [16] and was attributed to short range interaction, which differentiates between the π and the ρ family, for example. This is acceptable as such [14], but if one is assuming this one cannot do thermodynamics of the pion. We find a good fit to the pion and the ρ meson families *simultaneously with the same set of parameters* and this implies that one can get an effective fit to the hadrons, including the effect of the short range forces. The advantage is obvious, since we can enrich the description of the pion gas : instead of assuming it is just the ground state pion with lot of momentum excitation, we can include excitation of the discrete states ***as well as the momentum excitations in the right way.*** The importance of the pion gas in estimating contributions of hadrons to heavy ion and other applications of hadronic thermodynamics need not be freshly emphasized, it is well known.

We have found indications of a smooth phase transition using a purely hadronic model of the pion gas. In [15] it was shown that such a model may coexist with a model involving quarks in a big bag. If one may indulge into somewhat more exotic speculation : it may be that the densely populated hadronic states available from κ-deformed Poincaré algebra at high J and n, may indeed come from string-like models. An indication that this may be so is already evident in the fact that one can fit the baryons into the scheme as well as the mesons [9], [10], indicating a kind of effective supersymmetry in the hadronic scale that has been discussed a lot in recent years [17].

The energy of the thermal pion is found to change by about $\Delta E = 3$ GeV when T changes from about 0.2 GeV to about 0.4 GeV in fig. 4. The radius of the thermal pion in this range is not known but it is likely to be large. Let us assume it to be constant over this range and assume that it is 1 fm. Then we can get the change in the energy density $\Delta\rho_E = \Delta E/V$ where V is the volume of the thermal pion. In the simplest QCD-motivated model, - the MIT bag - the change in energy density between the confined and the quark-gluon phase is estimated to be 4B. Assuming that $\Delta\rho_E$ somehow matches the QCD-motivated model, we get a bag pressure of $(193\ MeV)^4$. This is the current standard accepted value and this indicates that the two models are perhaps compatible. We find this exciting in a time when people speculate about "bigger symmetry groups, perhaps giving a unification of $\hbar$ and α' - parameters that control quantum mechanical and stringy corrections." [18].

To summarize κ-deformed Poincaré algebra the gives us a way of fitting the experimental pion states almost exactly, therefore one can use it to extrapolate the available experimental data and use it to predict the thermodynamics of the pion gas. The distinctive feature of the deformed algebra is that it shows a flattening of the states with increasing angular momentum, J, or the radial excitation quantum number, n. It is seen that this predicts a smooth higher order phase transition for the pion gas. Thus we have a found a purely hadronic model which provides a smooth phase transition at high temperature and which could in principle, co-exist with QCD description of the pion gas.

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Table 1. Angular momentum excited states of the pion.

J	κ-deformed state	experiment
0	.138	π(.138)
1	1.2233	b(1.235)
2	1.6545	π_2(1.67)

Table 2. Radial excited states of the pion.

Radial quantum number n	κ-deformed state	experiment
0	.138	π(.138)
1	1.3157	pi(1.3)
2	1.7707	π(1.77)

Fig. 1. Free energy density (GeV^4) as a function of temperature (GeV). The solid line refers to our result which includes the predicted states, while the dashed curve refers to a truncated set provided by the known experimental states.

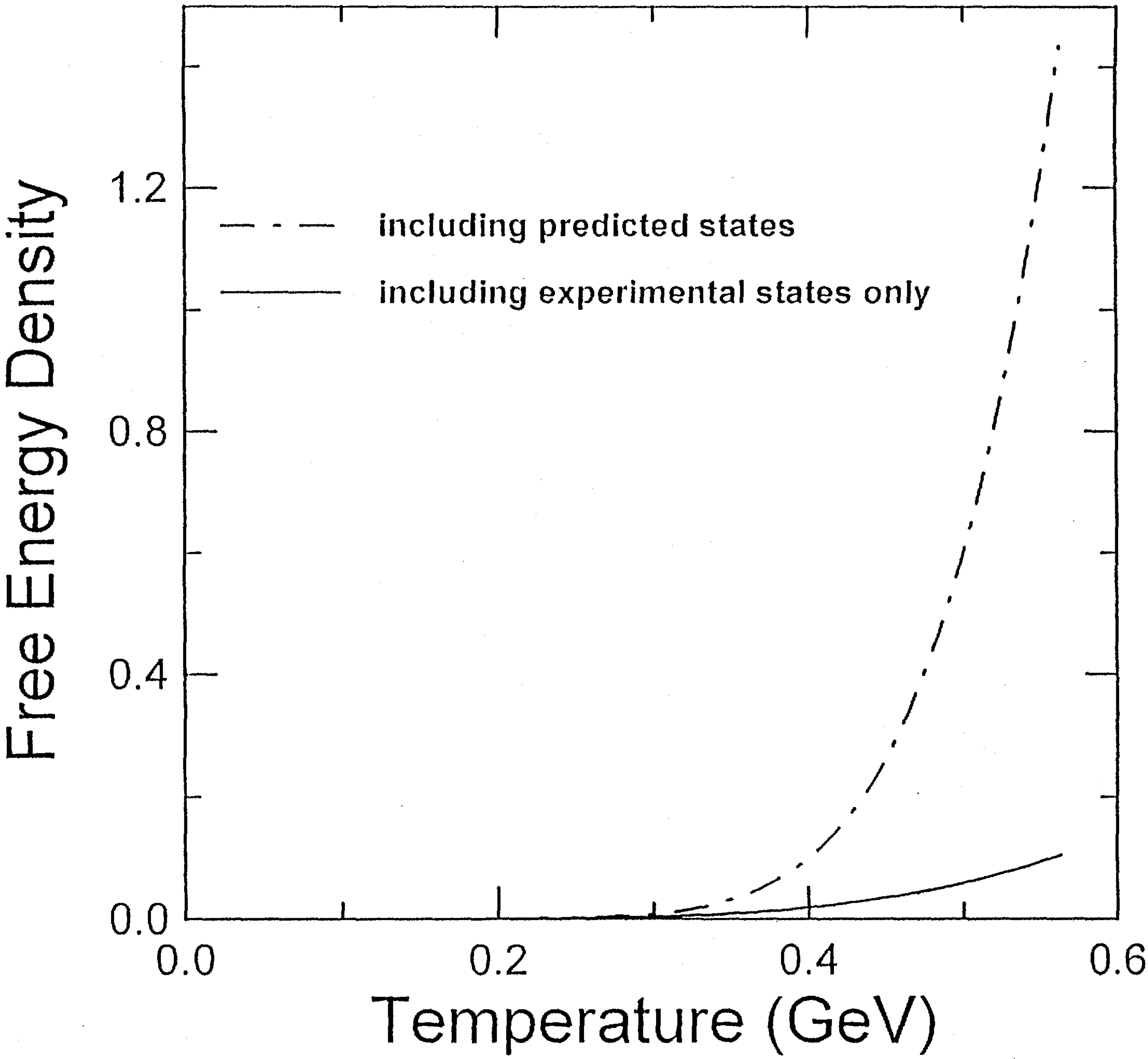
Fig. 2. Number density (GeV^3) as a function of temperature. For explanation of the solid and dashed curves see caption of Fig. 1.

Fig. 3. Free energy per particle (GeV) as a function of temperature.

Fig. 4. Energy density per particle (GeV) as a function of temperature. For explanation of the solid and dashed curves see caption of Fig. 1.

Fig. 5. Entropy density (GeV^3) as a function of temperature. For explanation of the solid and dashed curves see caption of Fig. 1.

Fig. 1



Free Energy Density
1.2
0.8
0.4
0.0
0.0
0.2
0.4
0.6
Temperature (GeV)
including predicted states
including experimental states only

Fig. 2

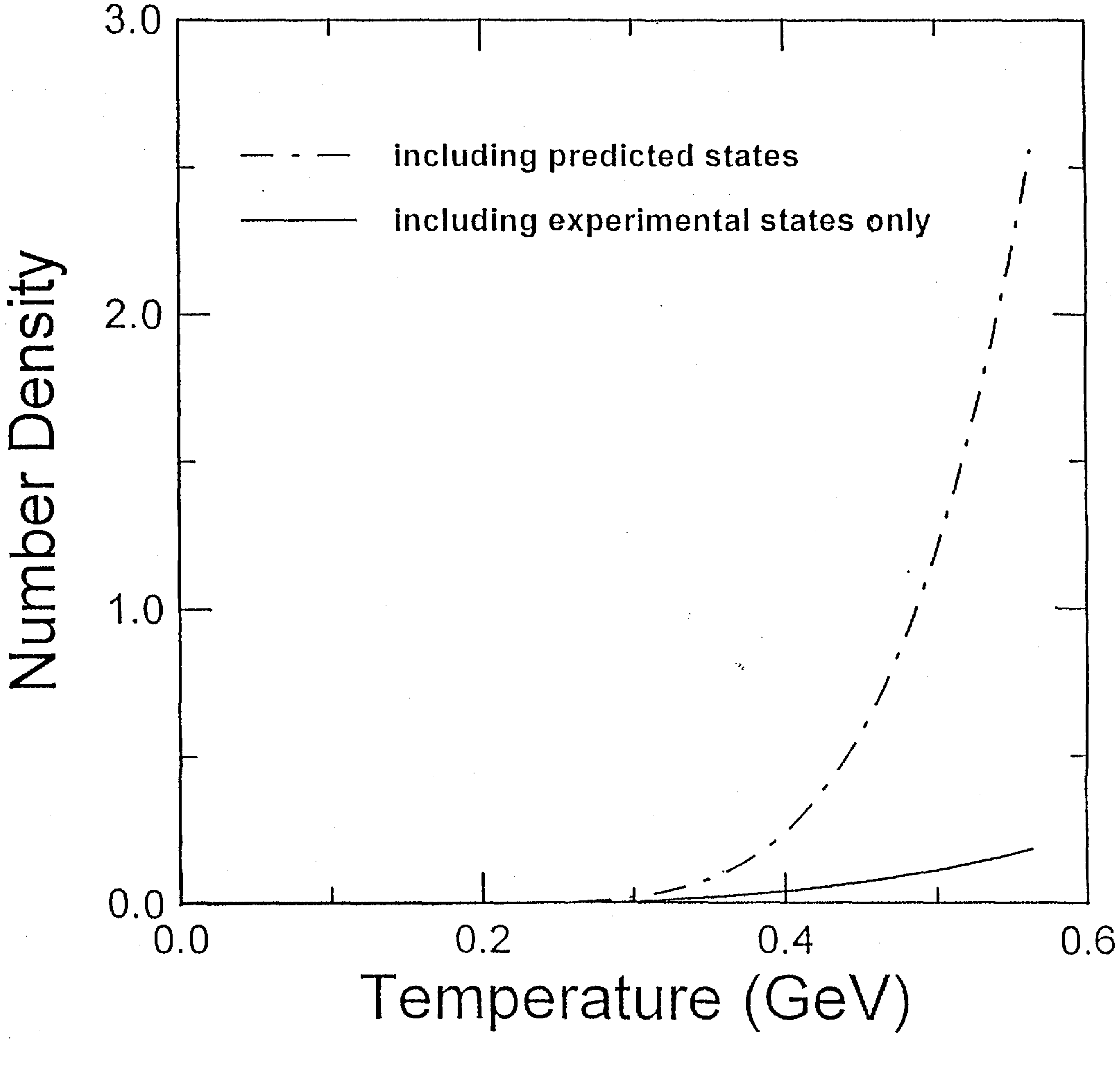

3.0
2.0
1.0
0.0
0.0
0.2
0.4
0.6
Number Density
Temperature (GeV)
including predicted states
including experimental states only

Fig. 3

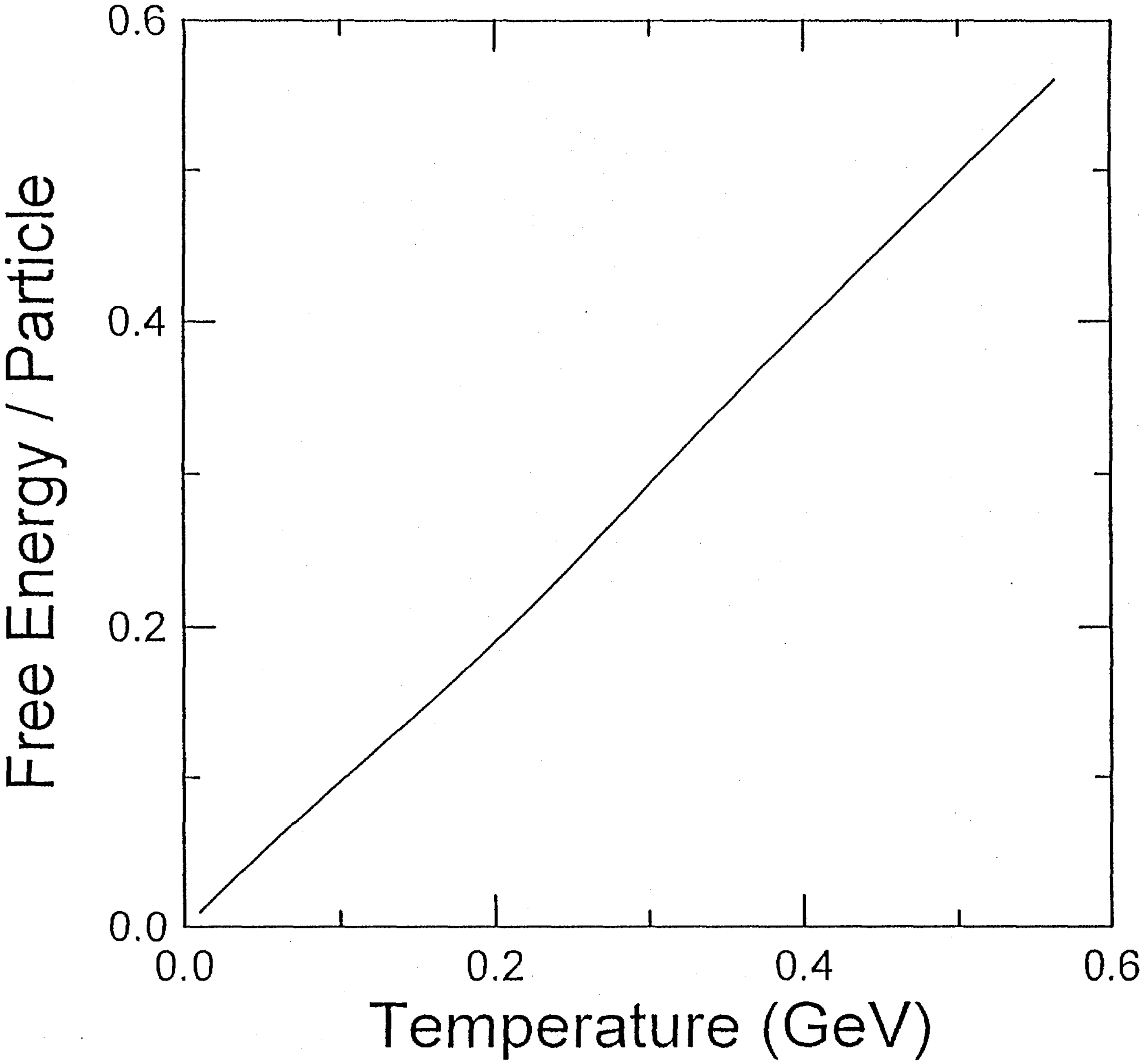


Fig. 4

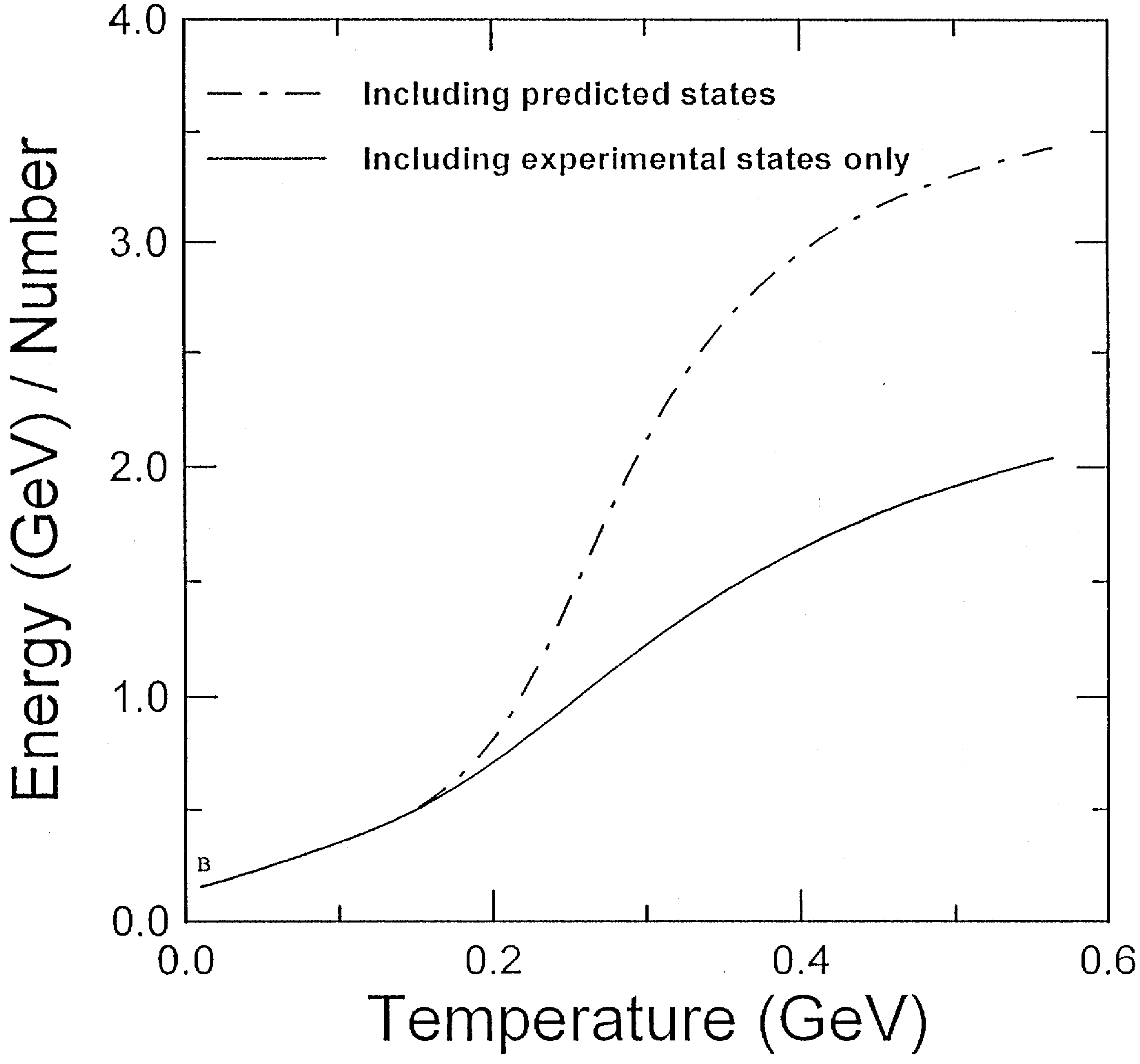
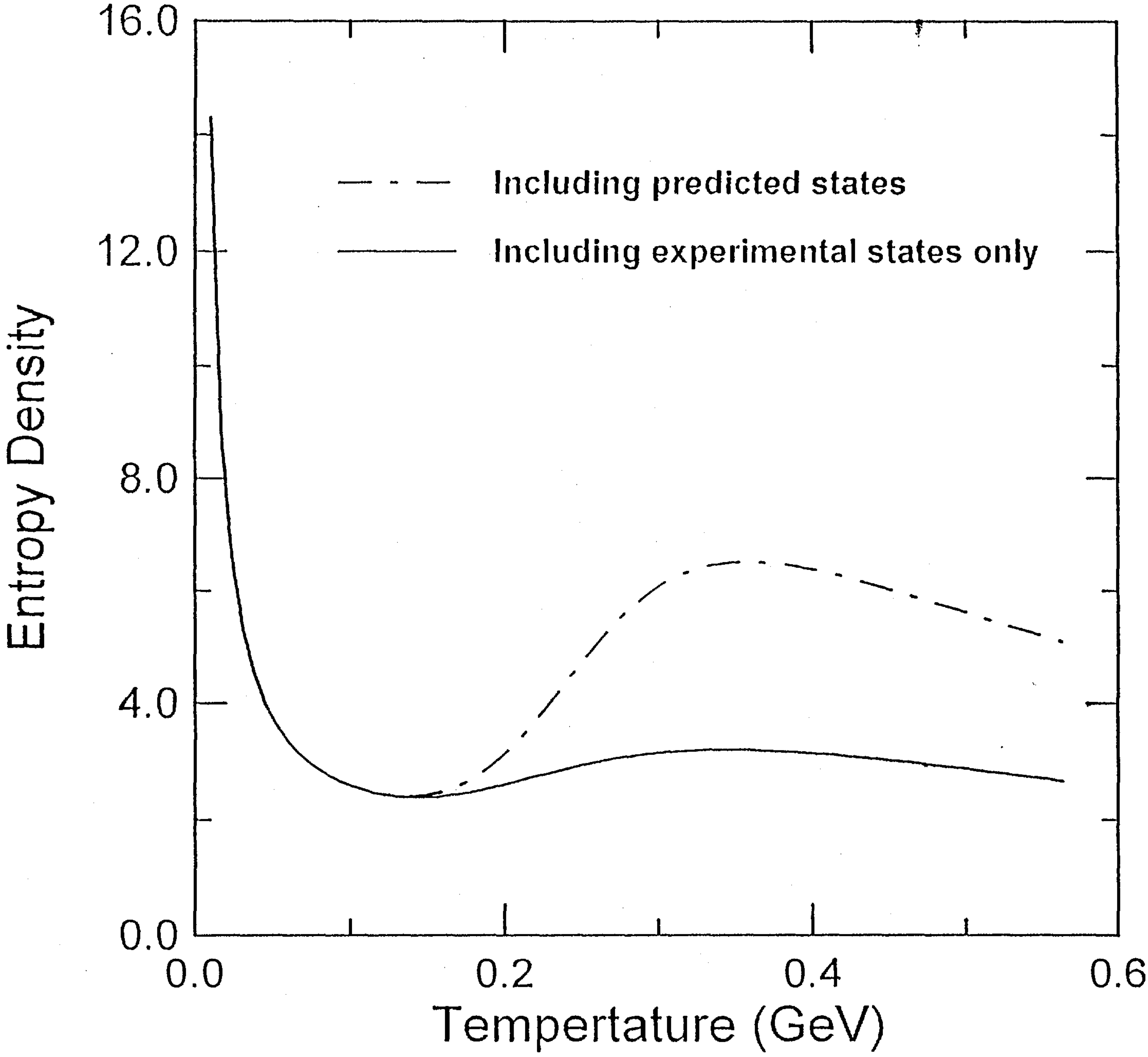

Including predicted states
Including experimental states only
Energy (GeV) / Number
Temperature (GeV)
4.0
3.0
2.0
1.0
0.0
0.0
0.2
0.4
0.6
B

Fig. 5



Including predicted states
Including experimental states only
16.0
12.0
8.0
4.0
0.0
Entropy Density
0.0
0.2
0.4
0.6
Tempertature (GeV)