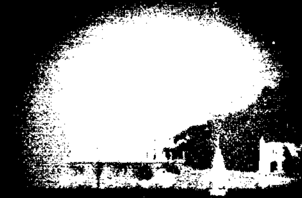
the
abdus salam
international
centre
for theoretical
physics

united nations
educational, scientific
and cultural
organization

international atomic
energy agency



preprint

SPIN-WAVE ANALYSIS OF THE XXZ HEISENBERG MODEL WITH DZYALOSHINSKII-MORIYA INTERACTION

Abdelilah Benyoussef

Abdelrhani Boubekri

and

Hamid Ez-Zahraouy

IC/98/102

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SPIN-WAVE ANALYSIS OF THE XXZ HEISENBERG MODEL WITH DZYALOSHINSKII-MORIYA INTERACTION

Abdelilah Benyoussef
Laboratoire de Magnetisme et de Physique des Hautes Energies,
Departement de Physique, Faculte des Sciences,
B. P. 1014, Rabat, Morocco,

Abdelrhani Boubekri and Hamid Ez-Zahraouy
Laboratoire de Magnetisme et de Physique des Hautes Energies,
Departement de Physique, Faculte des Sciences,
B. P. 1014, Rabat, Morocco
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

The effect of the Dzyaloshinskii-Moriya interaction on the stability of the Neel phase and the energy gap for the XXZ Heisenberg model on a d-dimensional hypercubic lattice is investigated using the linear spin-wave theory. In one-dimension, the disordered phase disappears above a critical value of the easy-axis anisotropy. In two-dimension, the instability of the Neel order occurs below a critical value of the easy-axis anisotropy. We find that for all d-dimensional systems, the energy gap vanishes above a critical value of the DM interaction.

MIRAMARE - TRIESTE

August 1998

I- Introduction

The study of one- and two-dimensional quantum antiferromagnetic Heisenberg models with Dzyaloshinskii-Moriya (DM) interaction [1,2] have received much attention from physicists over the past few years, perhaps due to the advent of high-Temperature superconductivity[3-14]. The DM interaction arises from a mixture of superexchange and spin-orbit coupling. It has been estimated for several cuprate crystal structures as in the compounds $La_{2-x}Ba_xCuO_4$ and $YBa_2Cu_3O_6$ [3-10]. The effect of this interaction on the ground state and excitation spectrum of the one-dimensional Heisenberg Hamiltonian has been studied in the past for both ferromagnetic and antiferromagnetic cases [11-13]. In the ferromagnetic case the spiral structure persists for weak DM interaction and the ferromagnetic state has never been a ground state [12,13], while in the antiferromagnetic case, the effect of the antisymmetric DM interaction was found to produce a resonant hybridization of the singularity at the edge of the continuum and spin-wave excitation [11].

The linear spin-wave theory has been applied to various low dimensional systems, in particular, those with different anisotropies [15-19]. In this paper, we apply this theory to the XXZ Heisenberg model with DM interaction, defined on a d-dimensional hypercubic lattice. Specifically, we shall be interested in one- and two-dimensional cases. In this theory, we always start from a Neel ground state. This means that we consider the case of easy-axis anisotropy and address the stability of the Neel phase. This allows one to find the boundary separating the long-range-order Neel phase from the disordered phase where the energy gap vanishes. In one dimension, our results are in agreement with those derived from the exact solution [12] within the regions of the parameter space where the spin-wave approach is expected to work well (the regions with long-range-order state).

The paper is organized as follows. Section II describes the model and its symmetries on a d-dimensional hypercubic lattice. In section III, we develop the linear spin-wave theory on this system. In section IV, results are given on both one and two dimensional systems. Section V is devoted to the conclusion.

II- The model and its symmetries (hypercubic lattice)

1- Model

Consider a d-dimensional hypercubic lattice, with lattice sites labelled as

$$n = \sum_{\mu=1}^{d} n_\mu \hat{x}_\mu .$$

Here $\{n_\mu\}$ is a set of integers, $(n_\mu = 0, \pm 1, \pm 2, \ldots)$ and $\hat{x}_\mu$ are mutually orthogonal unit vectors

$$\hat{x}_1 = (1,0,0,\ldots,0), \qquad \hat{x}_2 = (0,1,0,\ldots,0), \qquad \hat{x}_d = (0,0,0,\ldots,1).$$

The hypercubic lattice is a bipartite lattice. This means that it can be divided into two sublattices, A and B, such that for a site **n** belonging to the sublattice A all its nearest-neighbouring sites **n+g** belong to the sublattice B, and vice versa. There are Z=2d vectors **g** for a d-dimensional hypercubic lattice:

$$g = \pm \hat{x}_1, \ \pm \hat{x}_2, \ldots, \ \pm \hat{x}_d$$

The Heisenberg model we consider describes interactions between pairs of spins located on neighbouring sites. Assuming that the model is translationally invariant, the XXZ Heisenberg model with Dzyaloshinskii-Moriya (DM) interaction can be written as

$$H = \sum_n \sum_{\mu=1}^{d} \left[J_{xy} \left(S_n^x S_{n+\hat{x}_\mu}^x + S_n^y S_{n+\hat{x}_\mu}^y \right) + J_z S_n^z S_{n+\hat{x}_\mu}^z \right] + J_2 \sum_n \sum_{\mu=1}^{d} \left(S_n^x S_{n+\hat{x}_\mu}^y - S_n^y S_{n+\hat{x}_\mu}^x \right) \qquad (1)$$

The anisotropy parameter is defined as $\Delta = J_z / J_{xy}$.

We shall also need an equivalent form of H:

$$H = \overline{H} + H_{zz} \qquad (2)$$

$$\overline{H} = \sum_n \sum_{\mu=1}^{d} \left[\left(\frac{J_{xy} + i J_2}{2} \right) S_n^+ S_{n+\hat{x}_\mu}^- + \left(\frac{J_{xy} - i J_2}{2} \right) S_n^- S_{n+\hat{x}_\mu}^+ \right] \qquad (3)$$

$$H_{zz} = J_z \sum_n \sum_{\mu=1}^{d} S_n^z S_{n+\hat{x}_\mu}^z . \qquad (4)$$

2- Symmetries

- <u>$J_2 \to -J_2$ *symmetry*</u>

Consider a unitary transformation which represents a global (i.e. the same for all spins S_n) rotation in spin space around the $\hat{x}$-axis by angle π :

$$S_n^x \to S_n^x, \qquad S_n^y \to -S_n^y, \qquad S_n^z \to -S_n^z. \tag{5}$$

Under transformations (5), the Hamiltonian transforms as follows:

$$H(J_z, J_{xy}, J_2) \to H(J_z, J_{xy}, -J_2) \tag{6}$$

which means that the spectrum remains invariant under the sign reversal of J_2.

- <u>$J_{xy} \to -J_{xy}$ *symmetry*</u>

Consider a π-rotation around the $\hat{z}$-axis only for the spins belonging to the sublattice B. Namely,

$$S_n^x \to (-1)^{M_n} S_n^x, \qquad S_n^y \to (-1)^{M_n} S_n^y, \qquad S_n^z \to S_n^z \tag{7}$$

where the site-dependent integer number

$$M_n = \sum_{\mu=1}^{d} n_\mu . \tag{8}$$

Notice that $(-1)^{M_n} = e^{iQ.n}$ where the wave vector $Q = (\pi, \pi, \ldots, \pi)$.

If the site **n** belongs to sublattice A, M_n is even, whereas for **n** belonging to sublattice B, M_n is odd. Then from (8) it follows that the spins of the A-sublattice are not changed at all, while the spins of the B-sublattice are rotated by angle π around the $\hat{z}$-axis:

$$\begin{aligned} S_A^x &\to S_A^x, \qquad S_B^x \to -S_B^x \\ S_A^y &\to S_A^y, \qquad S_B^y \to -S_B^y \\ S_A^z &\to S_A^z, \qquad S_B^z \to S_B^z. \end{aligned} \tag{9}$$

Since, in our Hamiltonian with nearest-neighbour interaction we always have terms with the structure $S_A^\alpha S_B^\beta$, we conclude that, under transformation (9), H_{zz} remains unchanged while $\overline{H}$ effectively changes its sign. This means that

$$H(J_z, J_{xy}, J_2) \to H(J_z, -J_{xy}, -J_2). \tag{10}$$

Combining (6) with (10), we also obtain

$$H(J_z, J_{xy}, J_2) \to H(J_z, -J_{xy}, J_2). \tag{11}$$

Thus we can independently change the signs of J_{xy} and J_2; in both cases the spectrum of the model remains invariant.

- Special nonuniform unitary transformation: $J_{xy} \leftrightarrow J_2$ symmetry

Let us consider now a special, site-dependent, rotation of spins:

$$S_n^+ \to e^{i(\pi/2)M_n} S_n^+, \qquad S_n^- \to e^{-i(\pi/2)M_n} S_n^-, \qquad S_n^z \to S_n^z \tag{12}$$

where M_n is defined in (8). One can understand this transformation as follows. Start from any site **n**, choose one of **d** possible directions and go along the chosen axis. The rotation of the spins along this axis represents a spiral in the xy-plane (of the spin space) with characteristic angle $\pi/2$.

Notice that, under transformation (12)

$$S_n^+ S_{n+\hat{x}_\mu}^- \to S_n^+ S_{n+\hat{x}_\mu}^- e^{i(\pi/2)(M_n - M_{n+\hat{x}_\mu})}. \tag{13}$$

Clearly $M_{n+\hat{x}_\mu} - M_n = 1$.

Therefore

$$S_n^+ S_{n+\hat{x}_\mu}^- \to -i\, S_n^+ S_{n+\hat{x}_\mu}^-, \qquad S_n^- S_{n+\hat{x}_\mu}^+ \to i\, S_n^- S_{n+\hat{x}_\mu}^+. \tag{14}$$

Using (14) in (3), we see that

$$\overline{H}(J_{xy}, J_2) \to \overline{H}(J_2, -J_{xy}) \tag{15}$$

or, for the total Hamiltonian

$$H(J_z, J_{xy}, J_2) \to H(J_z, J_2, -J_{xy}). \tag{16}$$

But due to (6) and (10) we can also write

$$H(J_z, J_{xy}, J_2) \to H(J_z, J_2, J_{xy}). \tag{17}$$

So we have established a symmetry which effectively interchanges the XY and DM parts of the Hamiltonian.

The above symmetries imply that one can always choose a fundamental region in the parameter space (J_{xy}, J_2)

$$0 \le J_2 < J_{xy}, \tag{18}$$

where the properties of the model should be studied for arbitrary values of the anisotropy parameter Δ. This includes the ground state, excitation spectrum, thermodynamics and correlation functions. Using these symmetries, one can always produce a mapping onto any sector of the model starting from (18). Since all transformations are unitary, the spectrum of the model remains unchanged. This means that, for a given value of Δ, the excitation spectrum of the model should not depend on the signs of the coupling constants J_{xy} and J_2 and be symmetric with respect to the interchange $J_2 \leftrightarrow J_{xy}$. However, the above transformations change the wave functions. Therefore the symmetry of the ground state will, generally speaking, depend on the chosen sector of the model; the same applies to the correlation functions.

III- Linear spin-wave theory (hypercubic lattice)

In this section, the antiferromagnetic spin-wave theory introduced by Anderson and Kubo in Ref. [19] is applied to the Hamiltonian (1). Assuming that the classical ground state of the system is a long-range ordered Neel state, we shall study the stability of this state against small quantum fluctuations. Let $\vec{S}_l$ and $\vec{S}_m$ be the spin operators on the sublattices A and B, respectively. The standard Holstein-Primakoff transformation of these operators is defined as follows:

$$S_l^z = S - a_l^+ a_l \qquad\qquad S_m^z = -S + b_m^+ b_m$$
$$S_l^- = \sqrt{2S}\, a_l^+ \sqrt{1 - \frac{a_l^+ a_l}{2S}} \qquad\qquad S_m^- = \sqrt{2S}\sqrt{1 - \frac{b_m^+ b_m}{2S}}\, b_m \tag{19}$$

where a_l, a_l^+, b_m and b_m^+ are the creation and annihilation operators of spin deviations for sublattices A and B, respectively, satisfying the Bose commutation relations,

$$[a_l, a_{l'}^+] = \delta_{l,l'}$$
$$[b_m, b_{m'}^+] = \delta_{m,m'} \tag{20}$$
$$[a_l, b_m^+] = [a_l^+, b_m] = [a_l, b_m] = [a_l^+, b_m^+] = 0.$$

We substitute Eq. (19) into (1) and keep only the lowest order terms denoted by H^{LSWT}. Then

$$
\begin{aligned}
H^{LSWT} &= -N\frac{Z}{2}J_{xy}\,\Delta\,S^2 + J_{xy}\,Z\,S\,\Delta\left\{\sum_l a_l^+ a_l + \sum_m b_m^+ b_m\right\} \\
&+ J_{xy}\,S\sum_l\sum_\delta\left(Y\,a_l b_{l+\delta} + Y^*\,a_l^+ b_{l+\delta}^+\right)
\end{aligned} \tag{21}
$$

where Z denotes the number of adjacent sites, δ denotes the unit vector to adjacent sites, $Y = 1 + i\,\lambda$ and $\lambda = J_2 / J_{xy}$.

We apply the Fourier transformation

$$
\begin{aligned}
a_l &= \sqrt{\frac{2}{N}}\sum_k e^{-ikl}\,a_k \\
b_m &= \sqrt{\frac{2}{N}}\sum_k e^{ikm}\,b_k
\end{aligned} \tag{22}
$$

where k is a vector in a reciprocal lattice of a sublattice. Using Eq. (22) the commutation relations become

$$
\begin{aligned}
&[a_k, a_{k'}^+] = [b_k, b_{k'}^+] = \delta_{k,k'} \\
&[a_k, b_{k'}] = [a_k, b_{k'}^+] = [a_k^+, b_{k'}] = [a_k^+, b_{k'}^+] = 0.
\end{aligned} \tag{23}
$$

Then Eq. (21) is given by

$$
H^{LSWT} = -N\frac{Z}{2}J_{xy}\,\Delta S^2 + J_{xy}\,ZS\sum_k\left(\Delta(a_k^+ a_k + b_k^+ b_k) + \eta\,a_k b_k + \eta^*\,a_k^+ b_k^+\right) \tag{24}
$$

where $\eta = Y\gamma_k$ with $\gamma_k = \frac{1}{Z}\sum_\delta e^{ik\delta}$.

We introduce new Bose operators

$$
a_k = a_{1k}e^{-i\frac{\omega}{2}}, \qquad b_k = b_{1k}e^{-i\frac{\omega}{2}},
$$

where $tg(\omega) = \lambda$. Then $\eta\,a_k b_k = |\eta|\,a_{1k}b_{1k}$ and

$$
H^{LSWT} = -N\frac{Z}{2}J_{xy}\,\Delta\,S^2 + J_{xy}\,ZS\sum_k\left(\Delta(a_{1k}^+ a_{1k} + b_{1k}^+ b_{1k}) + |\eta|(a_{1k}b_{1k} + a_{1k}^+ b_{1k}^+)\right). \tag{25}
$$

Now, one can use the Bogoliubov transformation

$$
\begin{aligned}
a_{1k} &= \cosh(\theta_k)\alpha_k + \sinh(\theta_k)\beta_k^+ \\
b_{1k} &= \sinh(\theta_k)\alpha_k^+ + \cosh(\theta_k)\beta_k.
\end{aligned} \tag{26}
$$

As Eq. (26) is a canonical transformation, the commutation relations (23) are preserved, namely

$$[\alpha_k,\alpha^+_{k'}]=[\beta_k,\beta^+_{k'}]=\delta_{k,k'}$$
$$[\alpha_k,\beta^+_{k'}]=[\alpha^+_k,\beta_{k'}]=[\alpha_k,\beta_{k'}]=[\alpha^+_k,\beta^+_{k'}]=0. \tag{27}$$

Then the Hamiltonian (25) becomes

$$H^{LSWT}=-N\frac{Z}{2}J_{xy}\,\Delta\,S^2+\sum_k\left(W^{LSWT}(k)(\alpha^+_k\alpha_k+\beta^+_k\beta_k)+W^{LSWT}(k)-J_{xy}\,\Delta\,Z\,S\right). \tag{28}$$

Formula (28) should be rewritten as:

$$H^{LSWT}=-N\frac{Z}{2}J_{xy}\,\Delta\,S(S+1)+\sum_k W^{LSWT}(k)\left[(\alpha^+_k\alpha_k+1/2)+(\beta^+_k\beta_k+1/2)\right] \tag{29}$$

where the dispersion relation is given by

$$W^{LSWT}(k)=J_{xy}\,Z\,S\left(\Delta^2-(1+\lambda^2)\gamma^2_k\right)^{\frac{1}{2}}. \tag{30}$$

Therefore α_k, α^+_k, β_k and β^+_k are the creation and annihilation operators of the elementary excitation, and the ground state is their vacuum state. Then the spin deviation averaged over the ground state has the form

$$\Delta S=\left\langle a^+_l a_l\right\rangle=-\frac{1}{2}+\frac{1}{2}\int_{-\pi}^{\pi}\frac{d^dk}{(2\pi)^d}\frac{J_{xy}\,\Delta\,Z\,S}{W^{LSWT}(k)}. \tag{31}$$

The ground state energy per spin is

$$\frac{E^{LSWT}_{gs}}{N}=-J_{xy}\frac{Z}{2}\Delta\,S(S+1)+\frac{1}{2}\int_{-\pi}^{\pi}\frac{d^dk}{(2\pi)^d}W^{LSWT}(k). \tag{32}$$

The energy gap has the form

$$E_g=W^{LSWT}(\pi,\ldots,\pi)=J_{xy}\,Z\,S\left(\Delta^2-\lambda^2-1\right)^{\frac{1}{2}}. \tag{33}$$

Notice that these formulas (31)-(33) will be treated numerically.

IV- Results and discussion

The spin-wave theory results for quantum spin-1/2 antiferromagnetic anisotropic Heisenberg systems with DM interaction are given in one- and two-dimensional cases.

A- One-dimensional case

The sublattice magnetization behaviour as a function of DM interaction, $\lambda=J_z/J_{xy}$, (Fig. 1) and the phase diagram as a function of the easy-axis anisotropy, $\Delta\geq 1$, and DM interaction (Fig. 2) show the existence of two different phases namely, ordered and disordered phases. For $1 \le \Delta < \Delta_c = 1.43$, the disordered phase appears in the region $\lambda_{c_1} \le \lambda \le \lambda_{c_2}$, while ordered Neel phase occurs in the region $0 \le \lambda < \lambda_{c_1}$ and $\lambda > \lambda_{c_2}$. The critical values λ_{c_1} and λ_{c_2} of the DM interaction are functions of the anisotropy Δ. For $\Delta \ge \Delta_c$, the disordered phase disappears (Fig. 2). Sublattice magnetization vanishes continuously at $\lambda = \lambda_{c_1}$ and $\lambda = \lambda_{c_2}$, where transitions between ordered and disordered phases occur. This means that the transitions are of second-order type (Fig. 1). Moreover, for $J_2 = 0$, we recover the usual spin-wave sublattice magnetization for anisotropic chain which is in qualitative agreement with the results obtained by Baxter [20].

The effect of the DM interaction on the energy gap is displayed in Fig. 3. However, the energy gap of the anisotropic chain opens for $\Delta > 1$ and $0 \le \lambda < \lambda_c$, and decreases with increasing value of λ; while for $\lambda > \lambda_c$ the energy gap vanishes. For $J_2 = 0$, we recover the usual spin-wave energy gap for anisotropic chain which is in qualitative agreement with the results obtained by Parkinson and Bonner [21].

In conclusion the DM interaction favours the closing of the energy gap. This result is in excellent agreement with the exact result obtained by Alcaraz and Wreszinski [12].

B- Two-dimensional case

The sublattice magnetization and phase diagram as a function of the easy-axis anisotropy, $\Delta \ge 1$, and DM interaction $\lambda = J_2 / J_{xy}$, which are displayed in Figs. 4, 5, respectively, show the instability of the Neel phase in the region $\lambda'_{c_1} \le \lambda < \lambda'_{c_2}$ for all anisotropy $1 \le \Delta \le \Delta_c = 1.42$. λ'_{c_1} and λ'_{c_2} are functions of anisotropy Δ. Indeed, the Neel order occurs for $0 \le \lambda \le \lambda'_{c_1}$ and $\lambda \ge \lambda'_{c_2}$; but for $\Delta \ge \Delta_c$, the Neel order appears for any value of λ (Fig. 5). For $J_2 = 0$ (Fig. 4), we recover the usual spin-wave sublattice magnetization for anisotropic square lattice [17].

The energy gap of anisotropic square lattice opens for $\Delta \ge 1$ and $0 \le \lambda \le \lambda_c$, and decreases with increasing the DM interaction λ; while for $\lambda > \lambda_c$ the energy gap vanishes (Fig. 3). For $J_2 = 0$, we recover the usual spin-wave energy gap for anisotropic two-dimensional system [17] which is in excellent agreement with the known isotropic results [22,23].

The ground state energy given in Eq. (32) decreases with increasing DM interaction or decreasing easy-axis anisotropy for $\lambda \leq 1$, while for $\lambda \geq 1$ it increases with increasing λ or the anisotropy Δ.

V- Conclusion

We have studied the XXZ Heisenberg model with DM interaction and its symmetries on a d-dimensional hypercubic lattice. Furthermore, we have studied the influence of DM interaction on the magnetic order and energy gap for one and two dimensional quantum spin-1/2 antiferromagnetic anisotropic Heisenberg systems. In the one-dimensional case, the disordered phase exists below a critical anisotropy. In the two-dimensional case, the instability of the Neel phase occurs below a critical value of the anisotropy. However, above a critical DM interaction the energy gap vanishes for all d-dimensional systems. The known exact result in one-dimensional system of Alcaraz and Wreszinski [12] is also obtained.

Acknowledgments

A. Bo. and H. Ez. wish to thank the International Atomic Energy Agency (IAEA) and United Nations Educational Scientific and Cultural Organization (UNESCO) for the hospitality at the Abdus Salam International Centre for Theoretical Physics (AS-ICTP), Trieste Italy. One of the authors (H. Ez.) thanks the Arab Foundation for Scholarship Association. One of the authors (A. Bo.) acknowledges helpful discussions with Professor A. Nersesyan and his careful reading of the manuscript.

References

[1] I. Dzyaloshinskii, J. Phys. Chem. Solids **4**, 241 (1958).

[2] T. Moriya, Phys. Rev. **120,** 91 (1960).

[3] J. D. Axe, A. H. Moudden, D. Hohlwem, D. E. Cox, K. M. Mohanty, A. R. Moodenbaugh and Y. Xu, Phys. Rev. Lett. **66**, 2751 (1989).

[4] T. Thio, C. Y. Chen, B. S. Freer, D. R. Gabbe, H. P. Jenssen, M. A. Kastner, P. J. Picone, N. W. Preyer and R. J. Birgeneau, Phys. Rev. **B41**, 231 (1990).

[5] Y. Maeno, N. Kakehi, A. Odagawa and T. Fujita, Physica **B165**, 1689 (1990).

[6] D. Coffey, K. S. Bedell and S. A. Trugman, Phys. Rev. **B42**, 6509 (1990).

[7] D. Coffey, T. M. Rice and F. C. Zhang, Phys. Rev. **B44**, 10112 (1991).

[8] L.Shekhtman,O.Entin-Wohlman and A.Aharony, Phys.Rev.Lett.**69**, 836 (1992).

[9] W. Koshibae, Y. Ohta and S. Maekawa, Phys. Rev. **B47**, 3391 (1993).

[10] N. E. Bonesteel, Phys. Rev. **B47**, 11302 (1993).

[11] Q. Xia and P. S. Riseborough, J. Appl. phys. **67**, 5478 (1990).

[12] F. C. Alcaraz and W. F. Wreszinski, J. Stat. Phys. **58**, 45 (1990).

[13] I. Bose and U. Bhaumik, J. Phys. condens. Matter **6**, 10617 (1994).

[14] K. Hanzawa, J. Phys. Soc. Jpn. **63**, 264 (1994).

[15] A. Benyoussef, A. Boubekri and H. Ez-Zahraouy, Phys.Lett.**A226**, 117 (1997).

[16] A. Benyoussef, A. Boubekri and H. Ez-Zahraouy, Phys.Lett.**A238**, 398 (1998).

[17] A. Benyoussef, A. Boubekri and H. Ez-Zahraouy, to appear in Physica scripta.

[18] A. Benyoussef, A. Boubekri and H. Ez-Zahraouy, to appear in Journal of Mag. Mag. Mat.

[19] P. W. Anderson: Phys.Rev. **86** (1952) 694; R. Kubo: Phys.Rev. **87** (1952) 568.

[20] R. J. Baxter, J. Stat. Phys. **9**, 145 (1973).

[21] J. B. Parkinson and J. C. Bonner, Phys. Rev. **B32** 4703 (1985).

[22] N. Trivedi and D. M. Ceperley, Phys. Rev. **B41**, 4552 (1990).

[23] A. Parola, S. Sorella and Q. F. Zhong, Phys. Rev. Lett. **71**, 4393 (1993).

Figure captions

Fig. 1: The dependence of the sublattice magnetization on the DM interaction $\lambda = J_2 / J_{xy}$ for S=1/2 and d=1. The number accompanying each curve denotes the value of the easy-axis anisotropy, $\Delta = J_z / J_{xy}$.

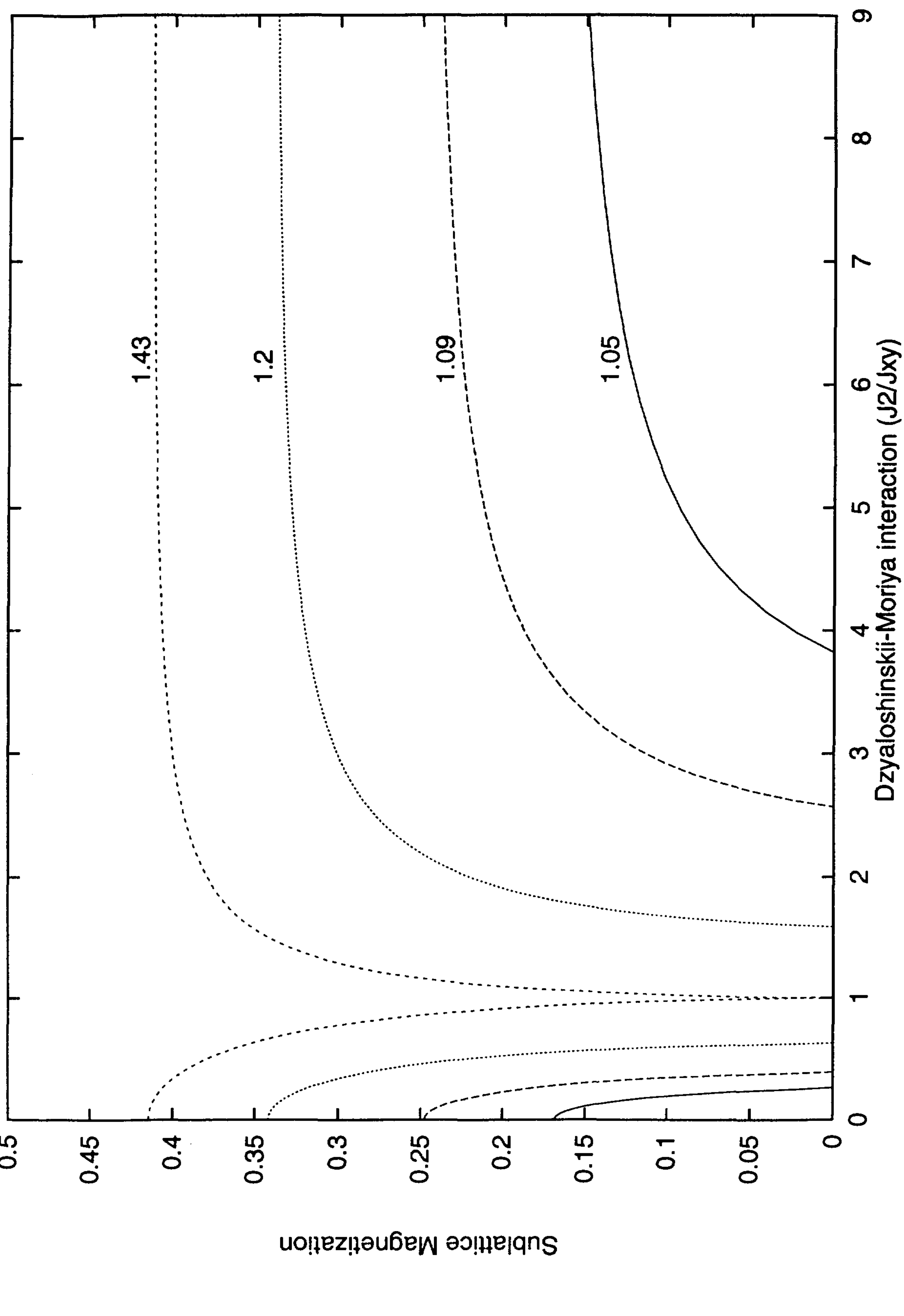
Fig. 2: Phase diagram in ($\Delta = J_z / J_{xy}, \lambda = J_2 / J_{xy}$) plane, for S=1/2 and d=1.

Fig. 3: Phase diagram showing the gap and gapless phases as a function of the anisotropy parameter, $\Delta = J_z / J_{xy}$, and DM interaction, $\lambda = J_2 / J_{xy}$, for all d-dimensional systems.

Fig. 4: The dependence of the sublattice magnetization on the DM interaction $\lambda = J_2 / J_{xy}$ for S=1/2 and d=2. The number accompanying each curve denotes the value of the easy-axis anisotropy, $\Delta = J_z / J_{xy}$.

Fig. 5: Phase diagram in ($\Delta = J_z / J_{xy}, \lambda = J_2 / J_{xy}$) plane, for S=1/2 and d=2.

Fig. 1



Sublattice Magnetization
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Dzyaloshinskii-Moriya interaction (J2/Jxy)
0
1
2
3
4
5
6
7
8
9
1.43
1.2
1.09
1.05

Fig. 2

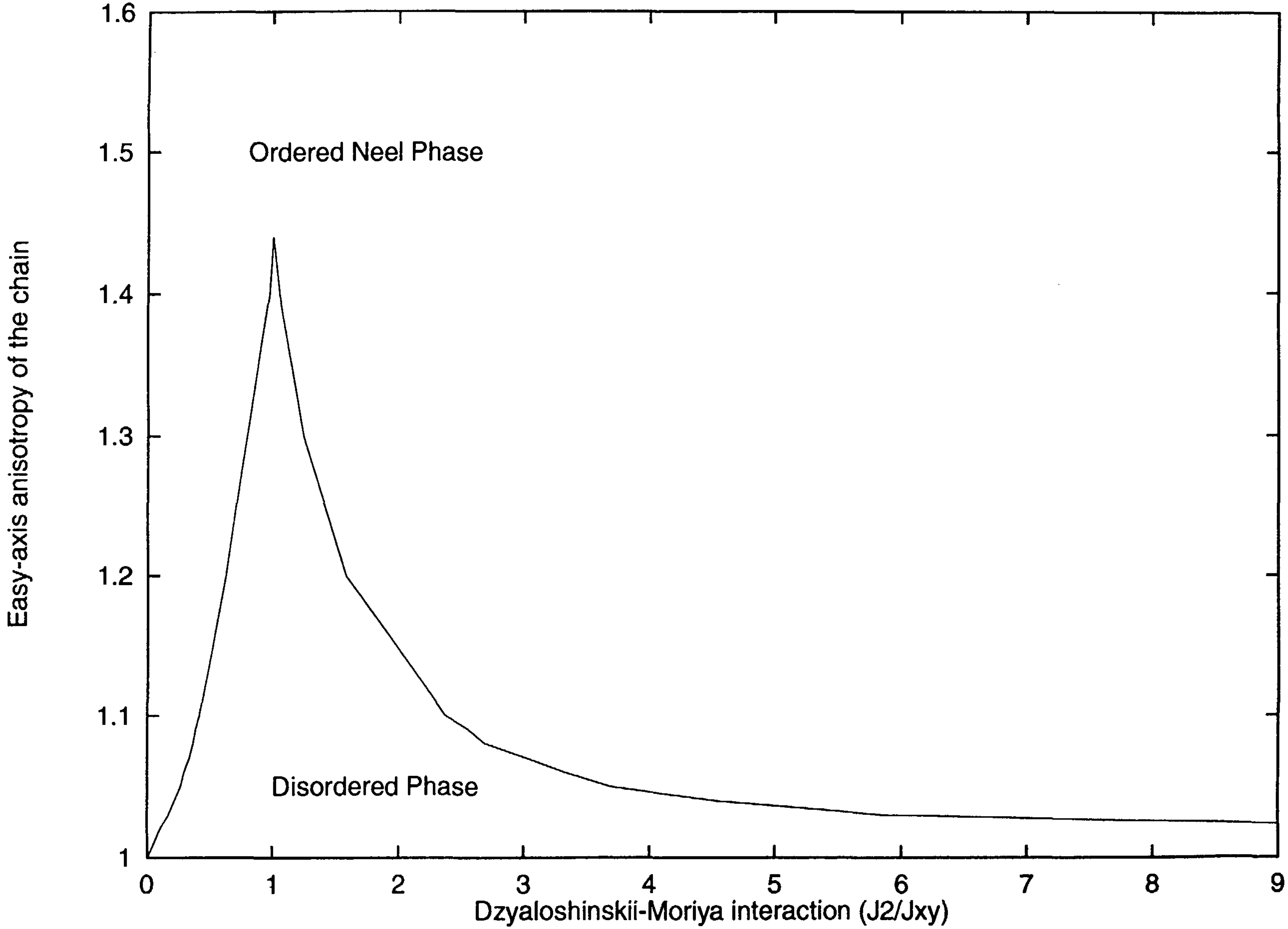

Ordered Neel Phase
Disordered Phase
Easy-axis anisotropy of the chain
Dzyaloshinskii-Moriya interaction (J2/Jxy)
1.6
1.5
1.4
1.3
1.2
1.1
1
0
1
2
3
4
5
6
7
8
9

Fig.3

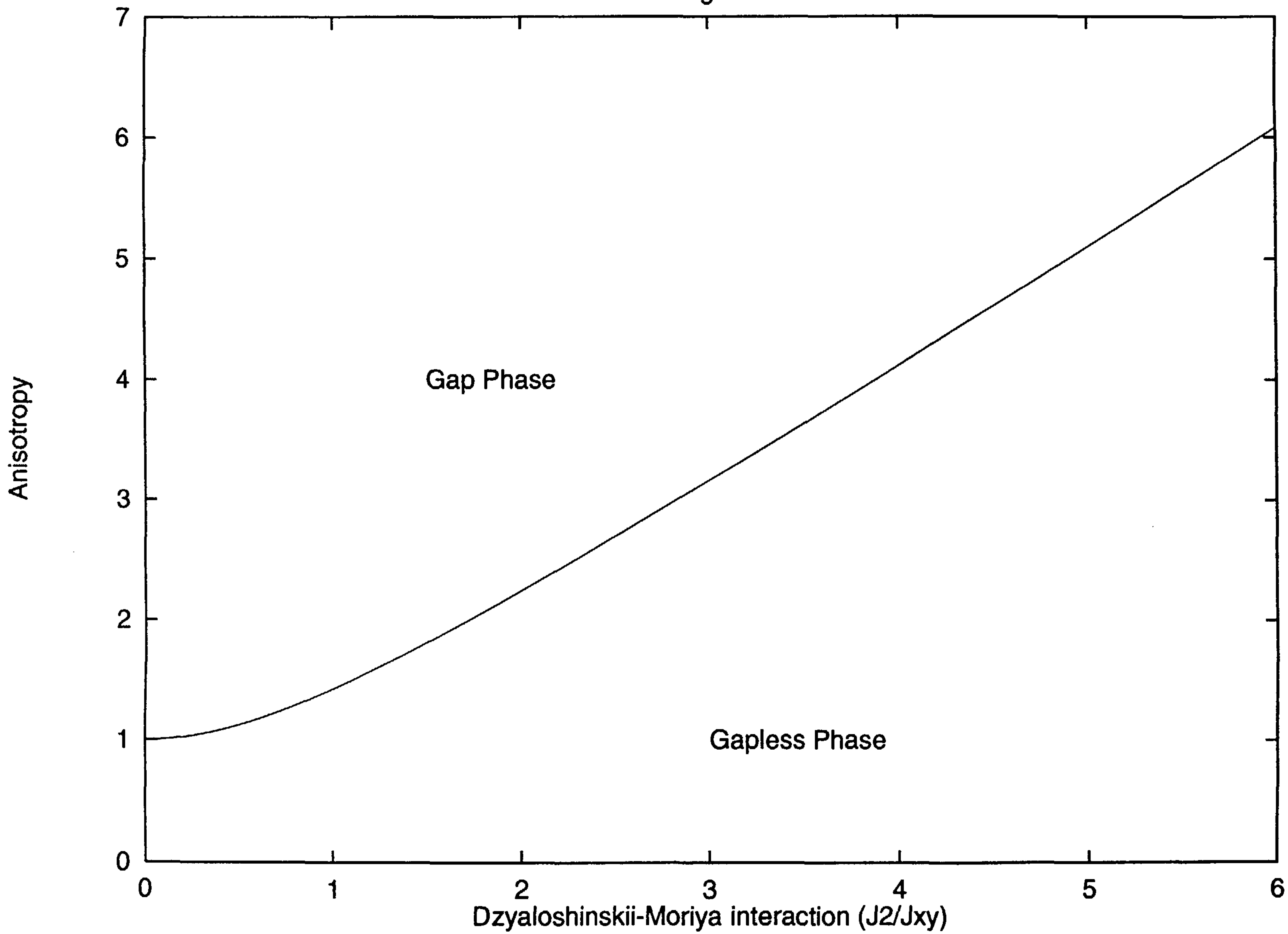
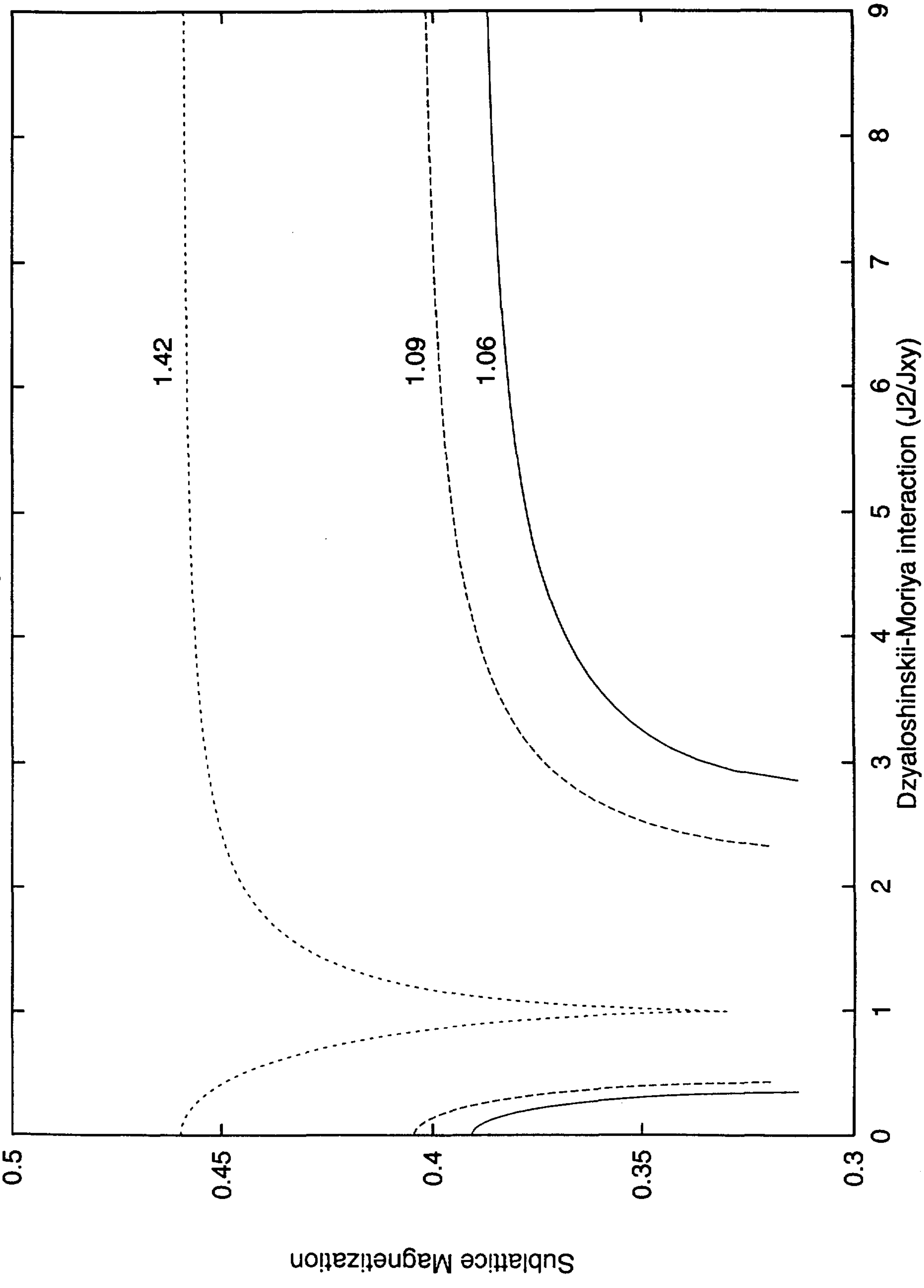
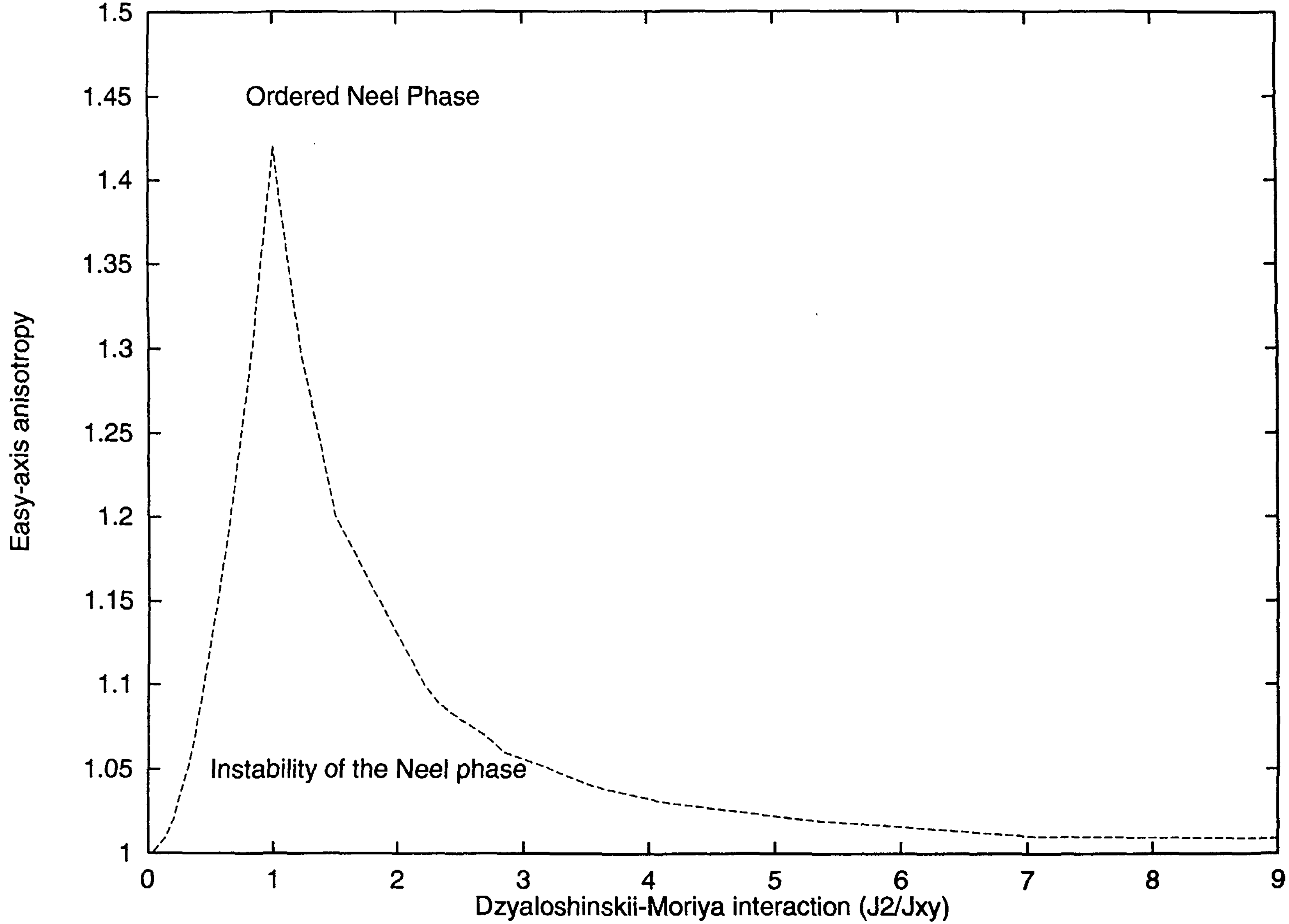

Anisotropy
Dzyaloshinskii-Moriya interaction (J2/Jxy)
Gap Phase
Gapless Phase
0
1
2
3
4
5
6
7

Fig. 4



Sublattice Magnetization
0.5
0.45
0.4
0.35
0.3
0
1
2
3
4
5
6
7
8
9
Dzyaloshinskii-Moriya interaction (J2/Jxy)
1.42
1.09
1.06

Fig. 5



Ordered Neel Phase
Instability of the Neel phase
Easy-axis anisotropy
Dzyaloshinskii-Moriya interaction (J2/Jxy)
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
1
2
3
4
5
6
7
8
9