THE ORDER PARAMETERS OF A SPIN-1 ISING FILM IN A TRANSVERSE FIELD

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Abstract

Using the effective field theory with a probability distribution technique that accounts for the self-spin correlation functions, the layer longitudinal magnetizations and quadrupolar moments of a spin−1 Ising film and their averages are examined. These quantities as functions of the temperature, the ratio of the surface exchange interactions to the bulk ones, the strength of the transverse field and the film thickness are calculated numerically and some interesting results are obtained.

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1 Introduction

The development of the molecular-beam-epitaxy technique and its application to the growth of thin metallic films has simulated renewed interest in both experimental and theoretical thin-film magnetism [1, 2]. Magnetic films can be studied as models of the magnetic size effect and quasi-two-dimensional systems [3]. Part of this theoretical activity has been devoted to the study of the magnetic and phase transition properties of semi-infinite Ising systems. The surface magnetism of these systems is very interesting [4-12]. When the ratio of the surface exchange interactions to the bulk ones $R = J/J_s$ is larger than a critical value R_c, the system becomes ordered on the surface before it is ordered in the bulk, and the critical temperature of the surface T_c^S/J is higher than that of the bulk T_c^B/J. For a film composed of a number L of atomic layers parallel to the surfaces, one cannot differentiate between the critical temperatures of the surface and the internal layers, and the film possesses a unified critical temperature T_c/J [13-18]. For a film in which there is a great number of atomic layers, its magnetism is similar to that of a semi-infinite system [18], in the sense that there exists a critical value R_c of the parameter R such that when $R \leq R_c$, the film critical temperature is smaller than that of the bulk system and approaches the last one for large values of the film thickness L and when $R \geq R_c$ the film critical temperature is larger both than the surface and the bulk critical temperatures of the corresponding semi-infinite system and approaches the surface critical temperature for large values of the film thickness [18].

The critical properties of ordinary, amorphous and transverse Ising films were calculated and discussed [13-14, 16-18]. The magnetic properties (the magnetizations and the susceptibilities,...) have also been investigated [10-18]. All the studies mentioned above are concerned with spin Ising systems of magnitude $\frac{1}{2}$. In addition, there is not a lot of studies of the critical properties and magnetic properties of the transverse Ising film with a higher spin. To our knowledge, Benyoussef et al. [19] have studied the semi-infinite spin-1 Ising model using the mean field approximation and Jia-Lin et al. [20], within the frame of the effective field theory with the differential operator technique, have investigated the critical behavior of a transverse Ising ferromagnetic thin film with spin-1 but unfortunately in their calculations the derivation of the expression for the longitudinal quadrupolar moment is incorrect and consequently this yields incorrect phase diagrams.

Our aim in this paper is to study the layer longitudinal magnetizations and quadrupolar moments of a spin-1 Ising film in a uniform transverse field within the framework of the effective field theory with a probability distribution technique [21, 22]. This technique is believed to give more exact results than those of the standard mean-field approximation. In section 2, we outline the formalism and derive the equations that determine the layer magnetizations and quadrupolar moments, and their averages. The longitudinal magnetizations and quadrupolar moments curves of the film as functions of the temperature, the ratio of the surface exchange interactions to the bulk ones, the strength of the transverse field and the film thickness are discussed in section 3 and comparison has been made with the results of the longitudinal magnetizations obtained by Wang et al. [15] for the spin-$\frac{1}{2}$ case. The last section 4 is devoted to a brief conclusion.

2 Formalism

We consider a spin-1 Ising film of L layers on a simple cubic lattice with free surfaces parallel to the (001) plane, in a uniform transverse field Ω. The Hamiltonian of the system is given by

$$H = - \sum_{(i,j)} J_{ij} S_{iz} S_{jz} - \Omega \sum_i S_{ix} \tag{1}$$

where S_{iz} and S_{ix} denote the z and x components of a quantum spin $\vec{S}_i$ of magnitude $S = 1$ at site i, $J_{i,j}$ is the strength of the exchange interaction between the spins at nearest-neighbor sites i and j, which is equal to J_s if both spins i and j are on the surface layers and to J otherwise. The statistical properties of the system are studied using an effective field theory that employs the probability distribution technique, which is fully described in [21] for a diluted spin Ising system of magnitude S and in [22] for the diluted spin-$\frac{1}{2}$ Ising system. This method accounts for the single-site correlations and it has been applied successfully in the study of various physical problems concerning the critical properties of Ising spin systems with magnitude $S \geq \frac{1}{2}$ [23-30] and in particular it has been applied to the study of the spin-S Ising model in a transverse model [23]. In the case of a spin-1 Ising model in a transverse field, following Elkouraychi et al. [26], we find in the current situation for a fixed configuration of neighboring spins of the site i that the longitudinal magnetization and quadrupolar moment of any spin at site i are given by

$$
\begin{aligned}
m_{iz} &= < S_{iz} > \\
&= < \frac{(\sum_j J_{ij} S_{jz})}{[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]^{\frac{1}{2}}} \frac{2 sinh(\beta[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]^{\frac{1}{2}})}{1 + 2cosh(\beta[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]^{\frac{1}{2}})} > \\
&= < f_{1z}[(\sum_j J_{ij} S_{jz}), \Omega] >
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
q_{iz} &= < (S_{iz})^2 > \\
&= < \frac{1}{[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]} \frac{\Omega^2 + [2(\sum_j J_{ij} S_{jz})^2 + \Omega^2]cosh(\beta[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]^{\frac{1}{2}})}{1 + 2cosh(\beta[(\sum_j J_{ij} S_{jz})^2 + \Omega^2]^{\frac{1}{2}})} > \\
&= < f_{2z}[(\sum_j J_{ij} S_{jz}), \Omega] >
\end{aligned}
\tag{3}
$$

where m_{iz} and q_{iz} are respectively the longitudinal magnetization and quadrupolar order parameter of the ith site, $\beta = 1/k_B T$, $< ... >$ indicates the usual canonical ensemble thermal average for a given configuration and the sum is over all the nearest neighbors of the site i, In a mean field approximation, one would simply replace these spin operators by their thermal values m_z (the longitudinal magnetizations). However, it is at this point that a substantial improvement to the theory is made by noting that the spin operators have a finite set of base states, so that the averages over the functions f_{1z} and f_{2z} can be expressed as an average over a finite polynomial of spin operators belonging to the neighboring spins. This procedure can be effected by the combinatorial method and correctly accounts for the single site kinematic relations. Up to this point the theory is exact, but the right-hand sides of Eqs. (2)-(3) will contain multiple spin correlation functions. To perform thermal averaging on the right-hand side of Eqs. (2)-(3), one now follows the general approach described in refs. [21, 22]. Thus, with the use of the integral

3

representation method of the δ Dirac's distribution, Eqs. (2)-(3) can be written in the form

$$m_{iz} = \int d\omega f_{1z}(\omega, \Omega) \frac{1}{2\pi} \int dt \exp(i\omega t) \Pi_j < \exp(-it J_{ij} S_{jz}) > \tag{4}$$

$$q_{iz} = \int d\omega f_{2z}(\omega, \Omega) \frac{1}{2\pi} \int dt \exp(i\omega t) \Pi_j < \exp(-it J_{ij} S_{jz}) > \tag{5}$$

where

$$f_{1z}(y, \Omega) = \frac{y}{[y^2 + \Omega^2]^{\frac{1}{2}}} \frac{2 sinh(\beta[y^2 + \Omega^2]^{\frac{1}{2}})}{1 + 2cosh(\beta[y^2 + \Omega^2]^{\frac{1}{2}})} \tag{6}$$

$$f_{2z}(y, \Omega) = \frac{1}{[y^2 + \Omega^2]} \frac{\Omega^2 + (2y^2 + \Omega^2)cosh(\beta[y^2 + \Omega^2]^{\frac{1}{2}})}{1 + 2cosh(\beta[y^2 + \Omega^2]^{\frac{1}{2}})} \tag{7}$$

In the derivation of the equations (4) and (5), the commonly used approximation has been made according to which the multi-spin correlation functions are decoupled into products of the spin averages.

To make progress, the simplest approximation of neglecting the correlations between different sites will be made. This is achieved by introducing the probability distribution of the spin variables S_{iz} which is given in [21] by

$$P(S_{iz}) = \frac{1}{2}[(q_{iz} - m_{iz})\delta(S_{iz} + 1) + 2(1 - q_{iz})\delta(S_{iz}) + (q_{iz} + m_{iz})\delta(S_{iz} - 1)] \tag{8}$$

with

$$m_{iz} = < S_{iz} > \tag{9}$$

and

$$q_{iz} = < S_{iz}^2 > \tag{10}$$

Allowing for the site magnetizations and quadrupolar moments to take different values in each atomic layer parallel to the surface of the film, and labeling them in accordance with the layer number in which they are situated, the application of Eqs. (2), (4) and (8) yields the following set of equations for the layer longitudinal magnetizations

$$m_{1z} = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} 2^{\mu+\mu_1} C_\mu^N C_\nu^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} (1 - 2q_{1z})^\mu (q_{1z} - m_{1z})^\nu$$

$$(q_{1z} + m_{1z})^{N-\mu-\nu} (1 - 2q_{2z})^{\mu_1} (q_{2z} - m_{2z})^{\nu_1} (q_{2z} + m_{2z})^{N_0-\mu_1-\nu_1}$$

$$f_{1z}(J[R(N - \mu - 2\nu) + (N_0 - \mu_1 - 2\nu_1)], \Omega) \tag{11}$$

...

$$m_{nz} = 2^{-N-2N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} \sum_{\mu_2=0}^{N_0} \sum_{\nu_2=0}^{N_0-\mu_2} 2^{\mu+\mu_1+\mu_2} C_\mu^N C_\nu^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} C_{\mu_2}^{N_0} C_{\nu_2}^{N_0-\mu_2}$$

$$(1 - 2q_{nz})^\mu (q_{nz} - m_{nz})^\nu (q_{nz} + m_{nz})^{N-\mu-\nu} (1 - 2q_{n-1,z})^{\mu_1} (q_{n-1,z} - m_{n-1,z})^{\nu_1}$$

$$(q_{n-1,z} + m_{n-1,z})^{N_0-\mu_1-\nu_1} (1 - 2q_{n+1,z})^{\mu_2} (q_{n+1,z} - m_{n+1,z})^{\nu_2} (q_{n+1,z} + m_{n+1,z})^{N_0-\mu_2-\nu_2}$$

$$f_{1z}(J[R(N - \mu - 2\nu) + (2N_0 - \mu_1 - \mu_2 - 2\nu_1 - 2\nu_2)], \Omega) \quad ;$$

$$for \ n = 2, 3, ..., L - 1 \tag{12}$$

...

4

$$m_{Lz} = 2^{-N-N_0} \sum_{\mu=0}^{N} \sum_{\nu=0}^{N-\mu} \sum_{\mu_1=0}^{N_0} \sum_{\nu_1=0}^{N_0-\mu_1} 2^{\mu+\mu_1} C_\mu^N C_\nu^{N-\mu} C_{\mu_1}^{N_0} C_{\nu_1}^{N_0-\mu_1} (1-2q_{Lz})^\mu (q_{Lz}-m_{Lz})^\nu$$

$$(q_{Lz}+m_{Lz})^{N-\mu-\nu} (1-2q_{L-1,z})^{\mu_1} (q_{L-1,z}-m_{L-1,z})^{\nu_1} (q_{L-1,z}+m_{L-1,z})^{N_0-\mu_1-\nu_1}$$

$$f_{1z}(J[R(N-\mu-2\nu)+(N_0-\mu_1-2\nu_1)], \Omega) \tag{13}$$

where N and N_0 are the numbers of nearest neighbors in the plane and between adjacent planes respectively ($N = 4$ and $N_0 = 1$ in the case of a simple cubic lattice which is considered here) and C_k^l are the binomial coefficients, $C_k^l = \frac{l!}{k!(l-k)!}$.

The equations of the longitudinal quadrupolar moments are obtained by substituting the function f_{1z} by f_{2z} in the expressions of the layer longitudinal magnetizations. This yields

$$q_{nz} = m_{nz}[f_{1\alpha}(y, \Omega) \rightarrow f_{2z}(y, \Omega)] \tag{14}$$

We have thus obtained the self consistent equations (11)-(14) for the layer longitudinal magnetizations m_{nz} and quadrupolar moments q_{nz}, that can be solved directly by numerical iteration. No further algebraic manipulation is necessary.

The corresponding layer transverse magnetizations and quadrupolar moments m_{nx} and q_{nx} can be obtained from the longitudinal ones by interchanging $(\sum_j J_{ij}S_{jz})$ and Ω. This yields

$$m_{nx} = m_{nz} \left\{ f_{1z}[(\sum_j J_{ij}S_{jz}), \Omega] \rightarrow f_{1z}[\Omega, (\sum_j J_{ij}S_{jz})] \right\} \tag{15}$$

$$q_{nx} = q_{nz} \left\{ f_{2z}[(\sum_j J_{ij}S_{jz}), \Omega] \rightarrow f_{2z}[\Omega, (\sum_j J_{ij}S_{jz})] \right\} \tag{16}$$

The longitudinal and transverse magnetizations and quadrupolar moments of the film are defined as the averages of the layer ones and are determined respectively by

$$\tilde{m}_\alpha = \frac{1}{L} \sum_{n=1}^{L} m_{n\alpha} \tag{17}$$

$$\tilde{q}_\alpha = \frac{1}{L} \sum_{n=1}^{L} q_{n\alpha} \tag{18}$$

Because we are interested in the study of the longitudinal ordering in the transverse spin-1 Ising film, we limit our studies to the longitudinal order parameters (the longitudinal magnetizations and quadrupolar moments) and for simplicity, in the numerical calculation, we should note the symmetry of the film and thus the amount of calculation can be decreased greatly. After selecting some values of R, Ω/J and L, one can obtain the layer magnetizations and quadrupolar moments from Eqs. (11)-(14) and their averages from Eqs. (17)-(18).

3 Order parameter curves

In this section, we are interested in the study of the layer longitudinal magnetizations and quadrupolar moments of a spin-1 Ising film in a transverse field. These quantities depend on the temperature, the ratio of the surface exchange interactions to the bulk ones $R = J_s/J$,

the strength of the transverse field and the film thickness L. As far as we know, apart the study by Zhong et al. [20] of the layer longitudinal magnetizations of the transverse spin-$\frac{1}{2}$ Ising film, the order parameters of the transverse spin-1 Ising film have not been studied in the past. In addition the transverse field dependence of the order parameters of the transverse Ising systems was considered only rarely. Therefore it is very interesting to investigate the temperature and transverse field dependences of the order parameters (the longitudinal magnetization and quadrupolar moment) of the transverse spin-1 Ising film. Because of the symmetry of the film, we give only L/2 layer order parameters as functions of temperature and transverse field in the corresponding figures. We will show only some typical results and we take the bulk exchange coupling J as the unit of the energy in our calculations.

3.1 Temperature dependences of the order parameters

After selecting a value of the transverse field, a value of the ratio of the surface exchange interactions to the bulk ones and a value of the film thickness, one can obtain the layer longitudinal magnetizations and quadrupolar moments from equations (11)-(14) as function of temperature, and then their averages from equations (17)-(18).

First we consider the variation of the film longitudinal magnetization $\bar{m}_z$ and the bulk magnetization m_z^{Bulk} of the system with the temperature. They are shown in figures 1a and 1b for two values of the ratio of the surface exchange interactions to the bulk ones $R = 0.1$ and 2 respectively, for various film thickness $L = 4$, 8 and 12 and for two values of the strength of the transverse field $\Omega /J = 0$ and 2. The dotted curves correspond to $\Omega /J = 0$ and the solid curves to $\Omega /J = 2$. It is clear from these figures that the magnetizations decrease with the increase of temperature and transverse field as expected. The bulk longitudinal magnetization vanishes at the bulk critical temperature T_c^B/J which depends on Ω/J only and the film longitudinal magnetization vanishes at the film critical temperature T_c/J which depends on R, Ω/J and L. For $\Omega /J = 0$ and $T /J = 0$, the bulk longitudinal magnetization and the longitudinal magnetizations of the film with $L = 4$, 8 and 12 are 1 (saturation magnetization $m_{z0} = 1$). For $R = 0.1$, $\Omega /J = 2$ and $T /J = 0$, the film longitudinal magnetization decreases and, the thinner the film, the smaller the longitudinal magnetization. The bulk critical temperatures for $\Omega/J = 0$ and 2 are respectively 3.5187 an 3.3229. The film critical temperatures for $\Omega /J = 0$ and $L = 4$, 8 and 12 are 2.9392, 3.3879, 3.4627 respectively, and those for $\Omega /J = 2$ and $L = 4$, 8 and 12 are 2.7009, 3.1840, 3.2635. The film critical temperature decreases with the increase of L and it is always smaller than that of the bulk system. For $R = 2$, $\Omega /J = 0$ and $T /J = 0$, the longitudinal magnetizations of the film with $L = 4$, 8 and 12 are 1 (saturation magnetization $m_{z0} = 1$) but, for $\Omega /J = 2$ and $T /J = 0$, the film longitudinal magnetization decreases and, the thinner the film, the bigger the longitudinal magnetization. The critical temperatures for $\Omega /J = 0$ and $L = 4$, 8 and 12 are 4.6263, 4.5664, 4.5657 respectively, and those for $\Omega /J = 2$ and $L = 4$, 8 and 12 are 4.4799, 4.4180, 4.4173. The film critical temperature decreases with the increase of L and it is always larger than the bulk critical temperature.

In figures 2a and 2b, we show the variation of bulk longitudinal and the film longitudinal

quadrupolar moments for the same values of the parameters as in figures 1a and 1b respectively. These quantities vary in the same way as the magnetizations, initially falling with the increase of the temperature at low temperatures, passing through a cusp at the transition temperature, before going to their asymptotic disordered state value $\frac{2}{3}$. Here the case of $R = 0.1$ is slightly more sensitive to the transverse field strength.

The bulk and the layer longitudinal magnetizations are exhibited in figures 3a and 3b when the film thickness L is equal to 8, for $R = 0.1$, $R = 2$ and for $\Omega/J = 0$ (Fig. 3a) and for $\Omega/J = 2$ (Fig. 3b). From these figures we see that the first layer or what is often called the surface layer has in the case where $R = 0.1$ ($R = 2$) the smallest (largest) longitudinal magnetization and all the layer magnetizations are smaller (larger) than the bulk one. We observe also the magnetization of the first layer is sensitive to the strength of the transverse field.

In figures 4a and 4b, we show the variation of bulk longitudinal and the film longitudinal quadrupolar moments for the same values for the parameters as in figures 3a and 3b respectively. These quantities vary in the same way as the magnetizations, initially falling with the increase of the temperature at low temperatures, passing through a cusp at the transition temperature, before going to their asymptotic disordered state value $\frac{2}{3}$. We observe also the first layer quadrupolar moment is more sensitive to the strength of the transverse field.

In order to clarify the effect of the strength of the transverse field on the layer order parameters, we have calculated many curves of the variation of the order parameters with temperature for several values of the parameters R, Ω/J, and L. All results are qualitatively the same and the main important result is that it should exist a critical value R_c of the parameter R depending on the value of Ω/J such that when $R > R_c$ the longitudinal magnetization and quadrupolar moment of the first layer are always greater than those of the internal layers which also are greater than those of the bulk system and the opposite situation occurs when $R < R_c$. We remark also from the above figures that in all case the layer longitudinal magnetizations vanish at the same transition temperature which is the film critical temperature which is smaller (larger) than the bulk critical temperature for $R = 0.1$ ($R = 2$). A critical value R_c at which the film and the bulk systems order simultaneously should exist. According to our results the critical value R_c of the parameter R depends on Ω/J and their values for $\Omega/J = 0$ and 2 are greater that 0.1 and less than 2. Here we are not interested to evaluate these values.

3.2   Transverse field dependence of the order parameters

Solving equations (11)-(14) numerically, we can also obtain the layer longitudinal and transverse magnetizations and quadrupolar moments as functions of the transverse field. The variation of the order parameters on the strength of the transverse field were rarely considered in the past [20]. In this subsection, we assume the film to be composed of $L = 8$ atomic layers parallel to the surfaces as an example to understand the transverse field dependences of the order parameters. The dependence of the bulk and the layer longitudinal magnetizations on the strength of the transverse field are exhibited for $R = 0.1, 2$ and for $T/J = 0$ in Fig. 5a and for $R = 0.1, 2$ and $T/J = 2$ in Fig. 5b. The bulk critical transverse field Ω_c^B/J and the film critical transverse

fields Ω_c/J at which the bulk and the film critical temperatures go respectively to zero are $\Omega_c^B/J = 5.2588$ and Ω_c/J equal to 5.0625 for $R = 0.1$ and 6.8039 for $R = 2$. These figures show that the magnetization of the surface layer is obviously different from those of the other layers. Figure 5b shows that, as the temperature is increased, the magnetization of the surface layer for $R = 0.1$ ($R = 2$) decreases (increases) rapidly. The variation of the bulk and the layer quadrupolar moments as functions of the strength of the transverse field are displayed for $R = 0.1, 2$ and for $T/J = 0$ in Fig. 6a and for $R = 0.1, 2$ and for $T/J = 2$ in Fig. 6b Comparing figure 6a with figure 6b, we see that increasing temperature more obviously decreases the quadrupolar moments of the bulk system and of the film. Comparing the results obtained from figures 2a to 6b, we see that for $R = 0.1$ in all cases the longitudinal magnetization of the surface layer is smaller than of the second layer which is smaller than of the third layer which is smaller than of the fourth layer which is smaller than of the bulk system and it is the same for the longitudinal quadrupolar moment. We have the contrary situation for $R = 2$. The results are in agreement with those of [18] and in disagreement with those of [15] in their study of the layer longitudinal magnetizations of the transverse spin-$\frac{1}{2}$ Ising film.

4 Conclusion

In conclusion, using the effective-field theory with a probability distribution technique that accounts for the self-spin correlation functions, we have studied the longitudinal magnetizations and quadrupolar moments of the spin-1 Ising film with a simple cubic structure in a transverse field. For a few sets selected parameters we have calculated the averages and layer order parameters as functions of the temperature, the strength of the transverse field, the ratio R of the surface exchange interactions to the bulk ones and the film thickness and sown that there exists a critical value R_c depending on Ω/J such that when $R > R_c$ we have $m_{1z} > m_{2z} > ... > m_z^{Bulk}; q_{1z} > q_{2z} > ... > q_z^{Bulk}$ and when $R < R_c$ we have $m_{1z} < m_{2z} < ... < m_z^{Bulk}; q_{1z} < q_{2z} < ... < q_z^{Bulk}$. The film possesses one well defined critical temperature which is smaller (larger) than the bulk one for $R < R_c$ ($R > R_c$) and as for the bulk system the film longitudinal magnetization and quadrupolar moment vanishes and passes through a cusp at the film critical temperature respectively and they decrease with the increase of T/J or Ω/J. Other works that concern the evaluation of the phase diagrams such the determination of R_c as function of Ω/J for the spin-1 Ising film in a transverse field are in progress and will be published.

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Figure Captions

Figure 1: The film longitudinal magnetization as a function of the temperature.
a) $R = 0.1$,
b) $R = 2$.
The dotted and solid curves correspond respectively to $\Omega/J = 0$ and 2.

Figure 2: The film longitudinal quadrupolar moment as a function of the temperature.
a) $R = 0.1$,
b) $R = 2$.
The dotted and solid curves correspond respectively to $\Omega/J = 0$ and 2.

Figure 3: Layer longitudinal magnetizations as a function of the temperature for a film with $L = 8$ layers.
a) $\Omega/J = 0$,
b) $\Omega/J = 2$.
The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

Figure 4: Layer longitudinal quadrupolar moments as a function of the temperature for a film with $L = 8$ layers.
a) $\Omega/J = 0$,
b) $\Omega/J = 2$.
The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

Figure 5: Layer longitudinal magnetizations as a function of the strength of the transverse field for a film with $L = 8$ layers.
a) $T/J = 0$,
b) $T/J = 2$.
The solid and dotted curves correspond respectively to $R = 0.1$ and 2.

Figure 6: Layer longitudinal quadrupolar moments as a function of the strength of the transverse field for a film with $L = 8$ layers.
a) $T/J = 0$,
b) $T/J = 2$.
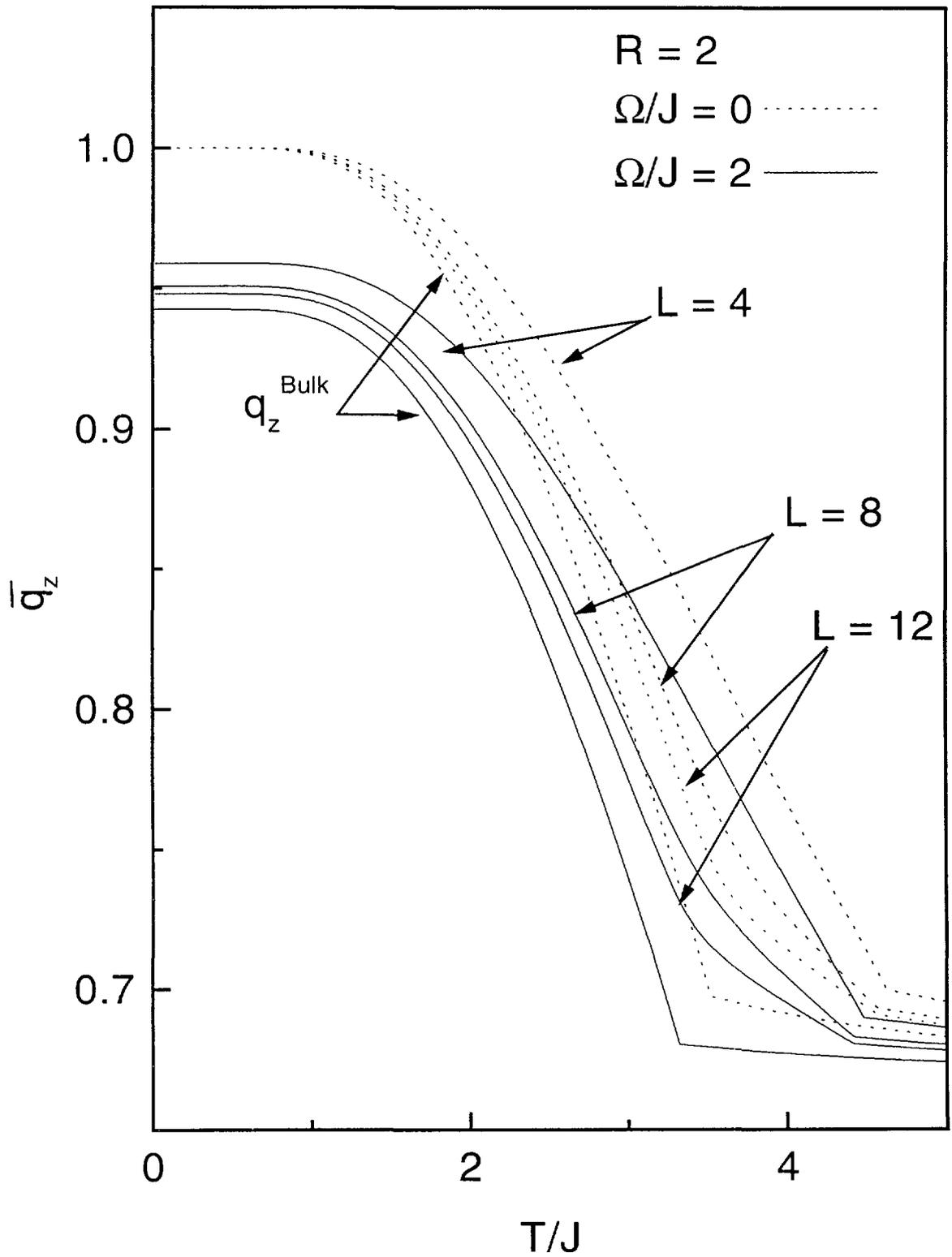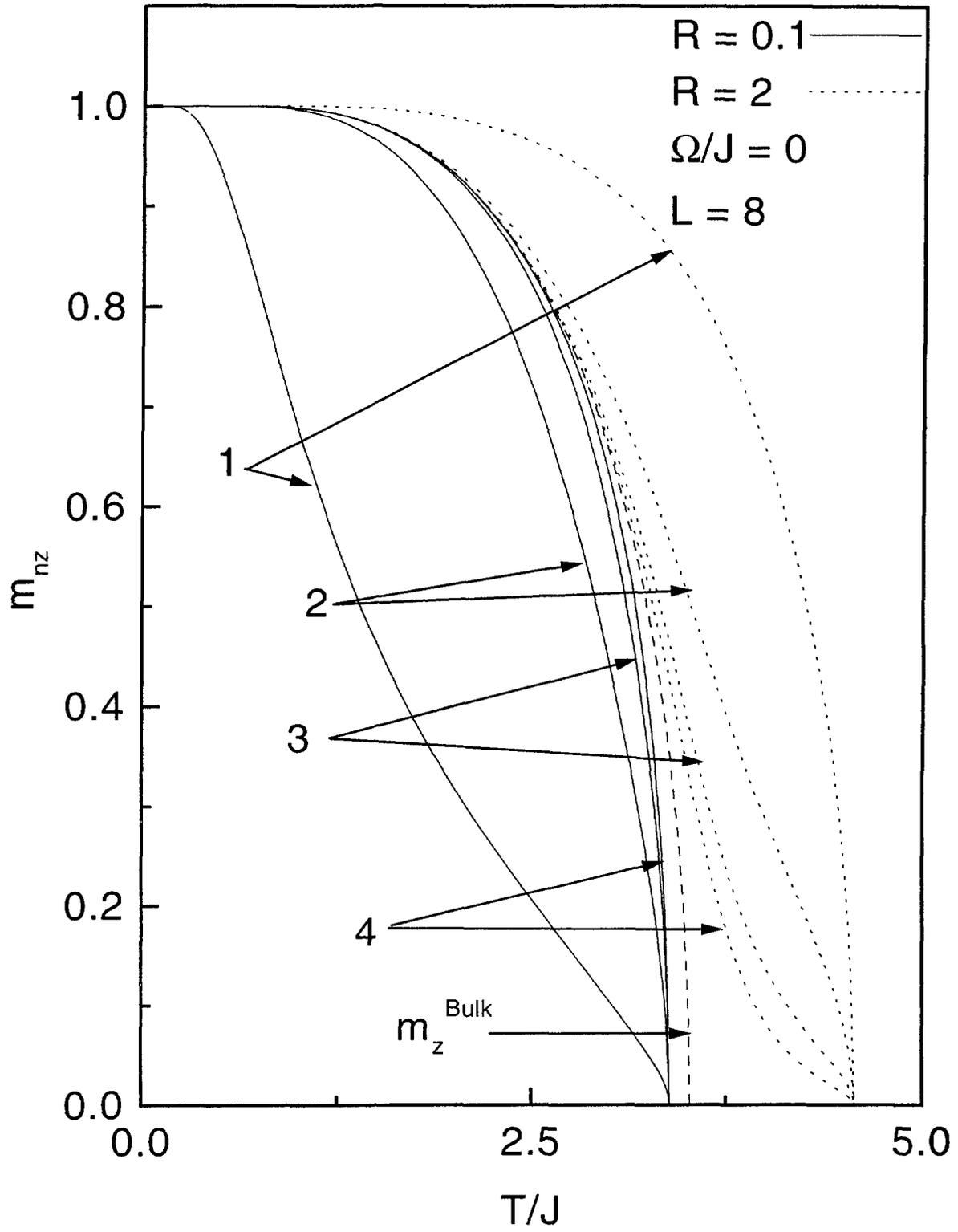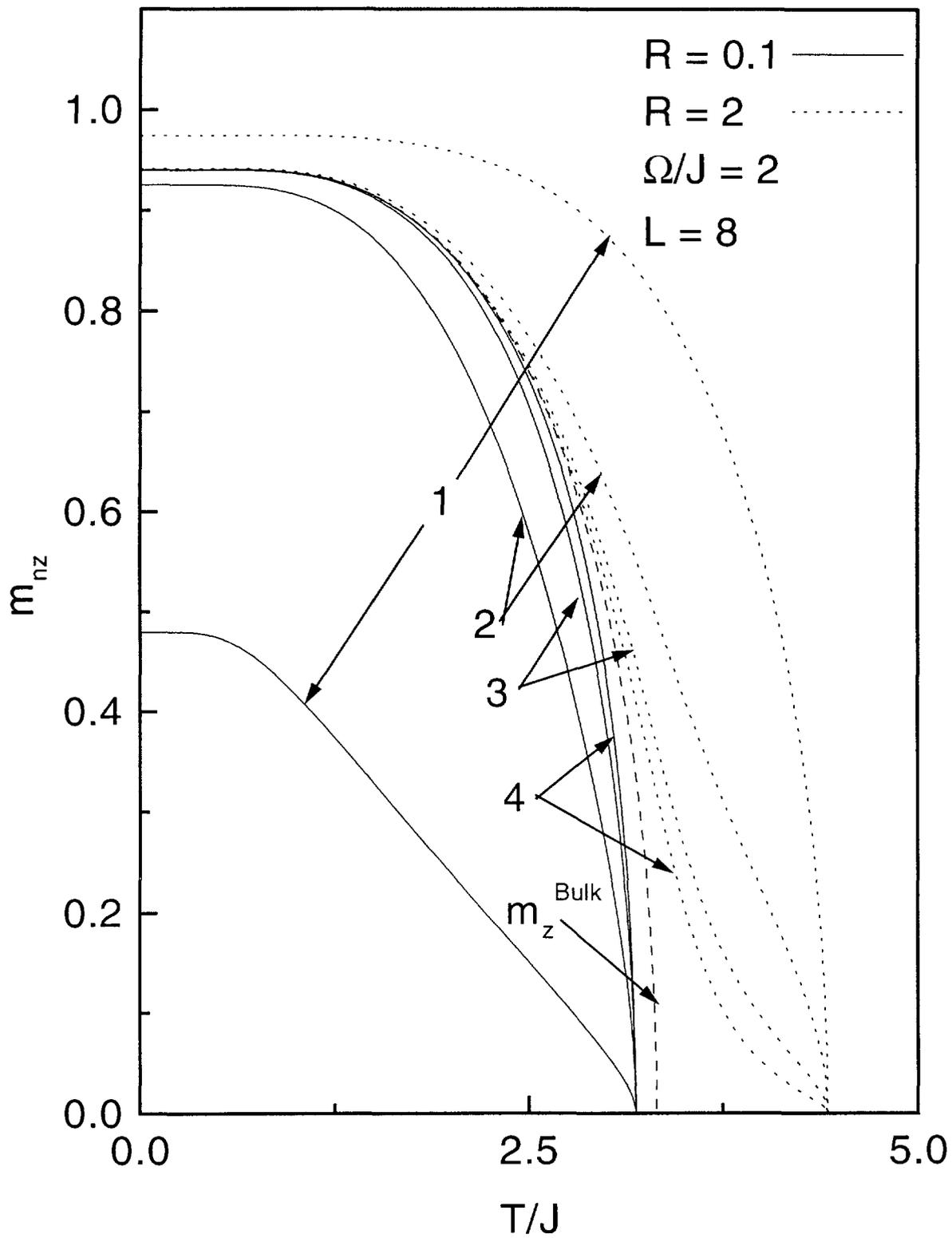The solid and dotted curves correspond respectively to $R = 0.1$ and 2.
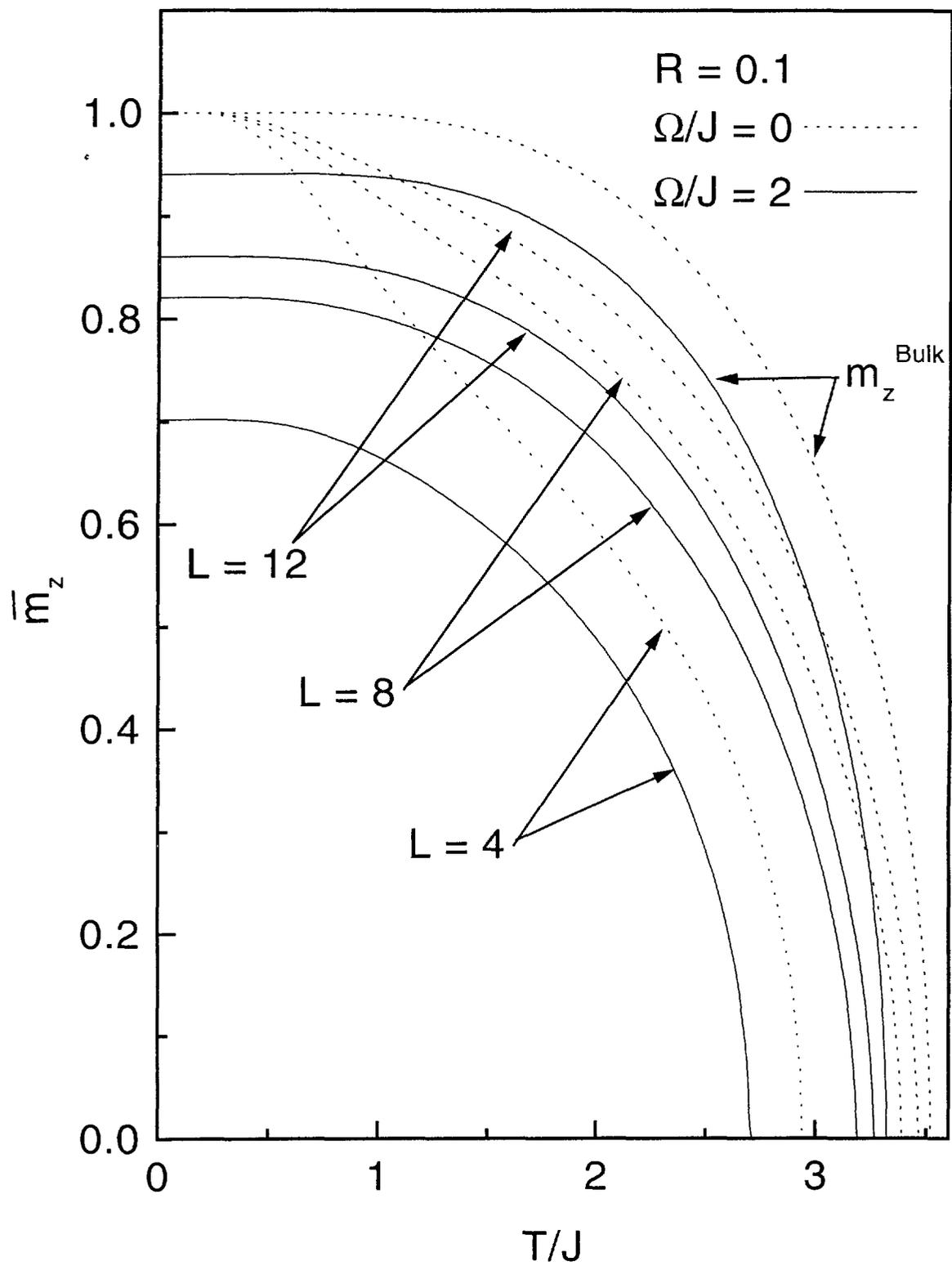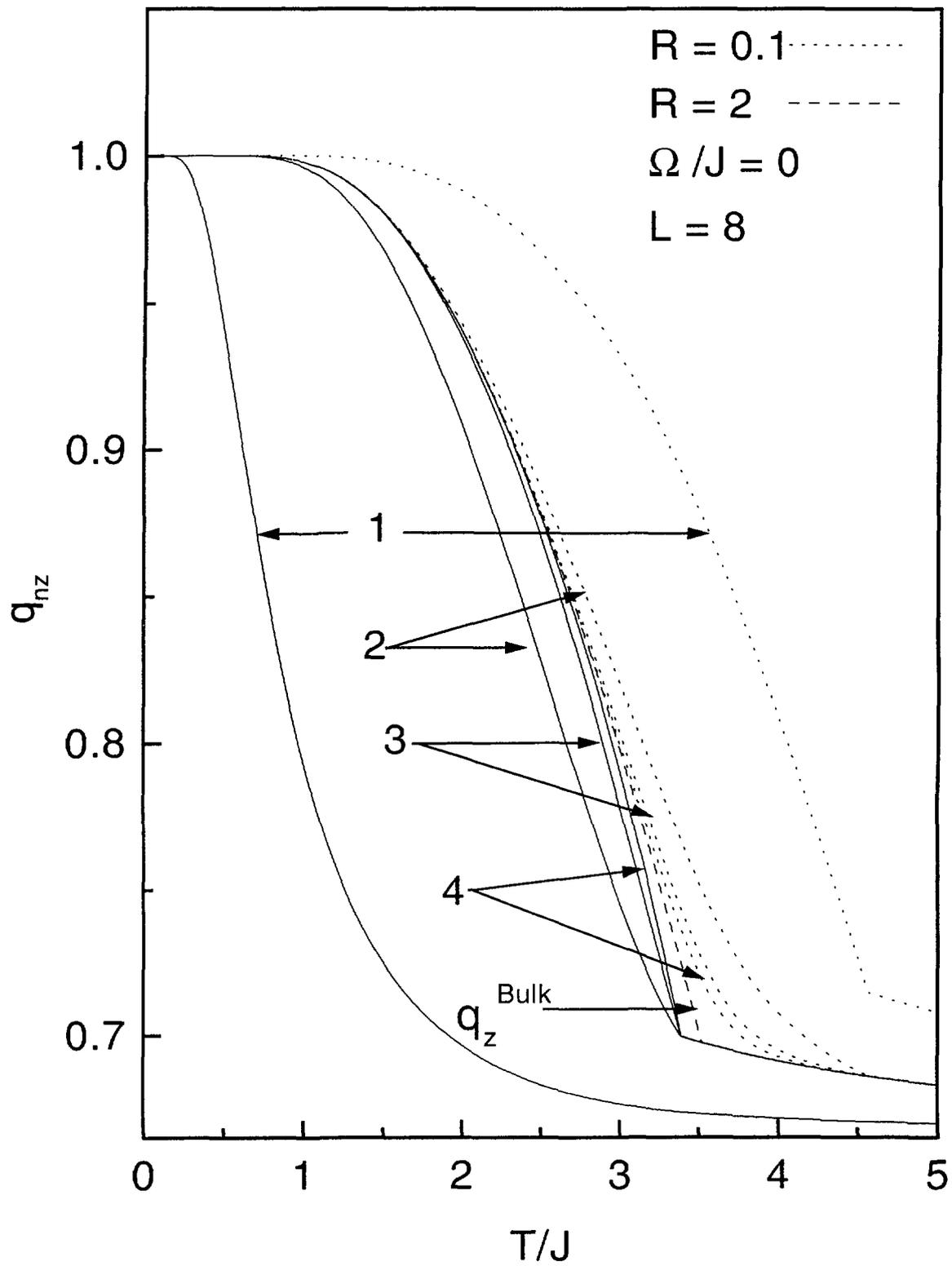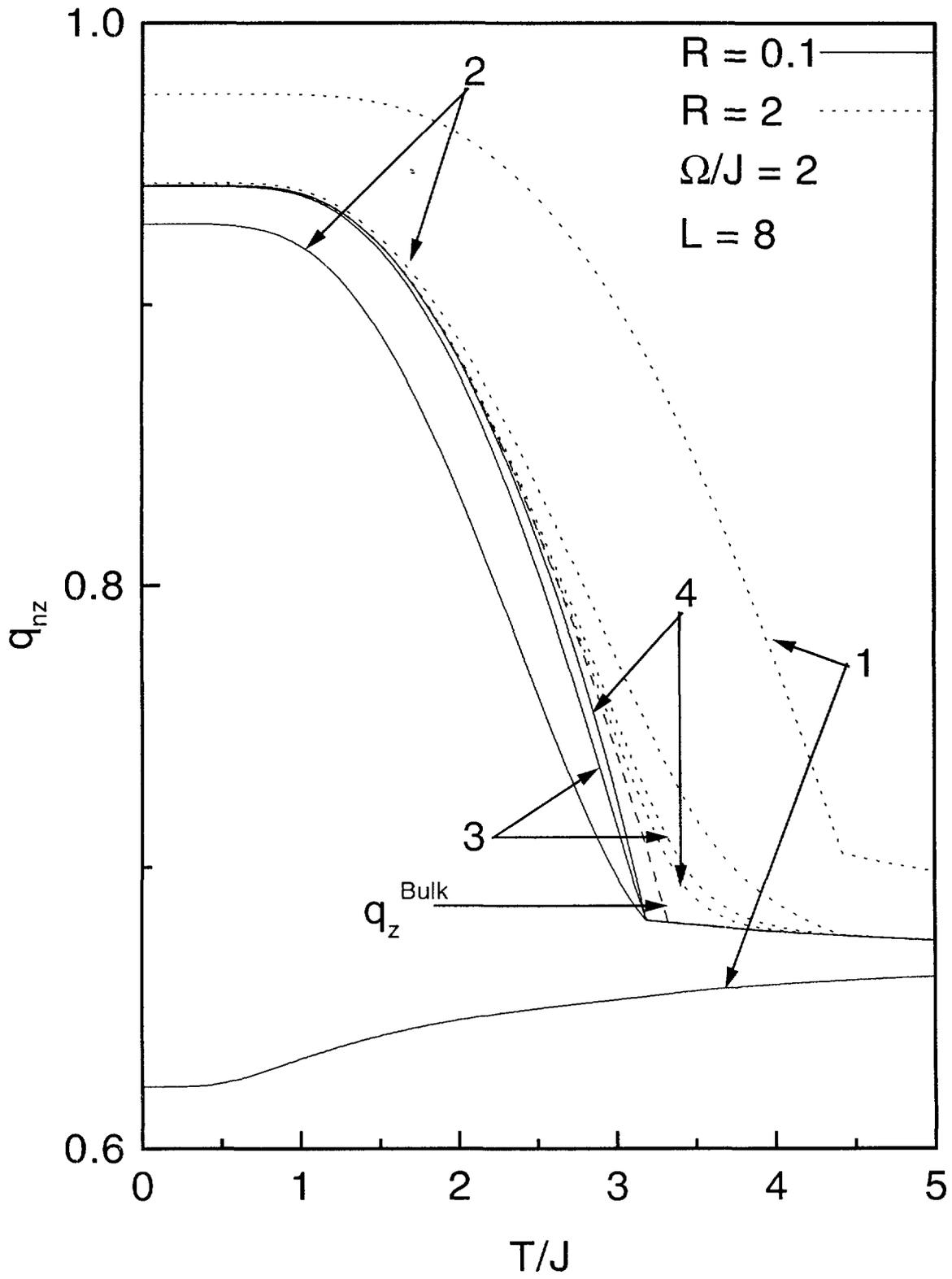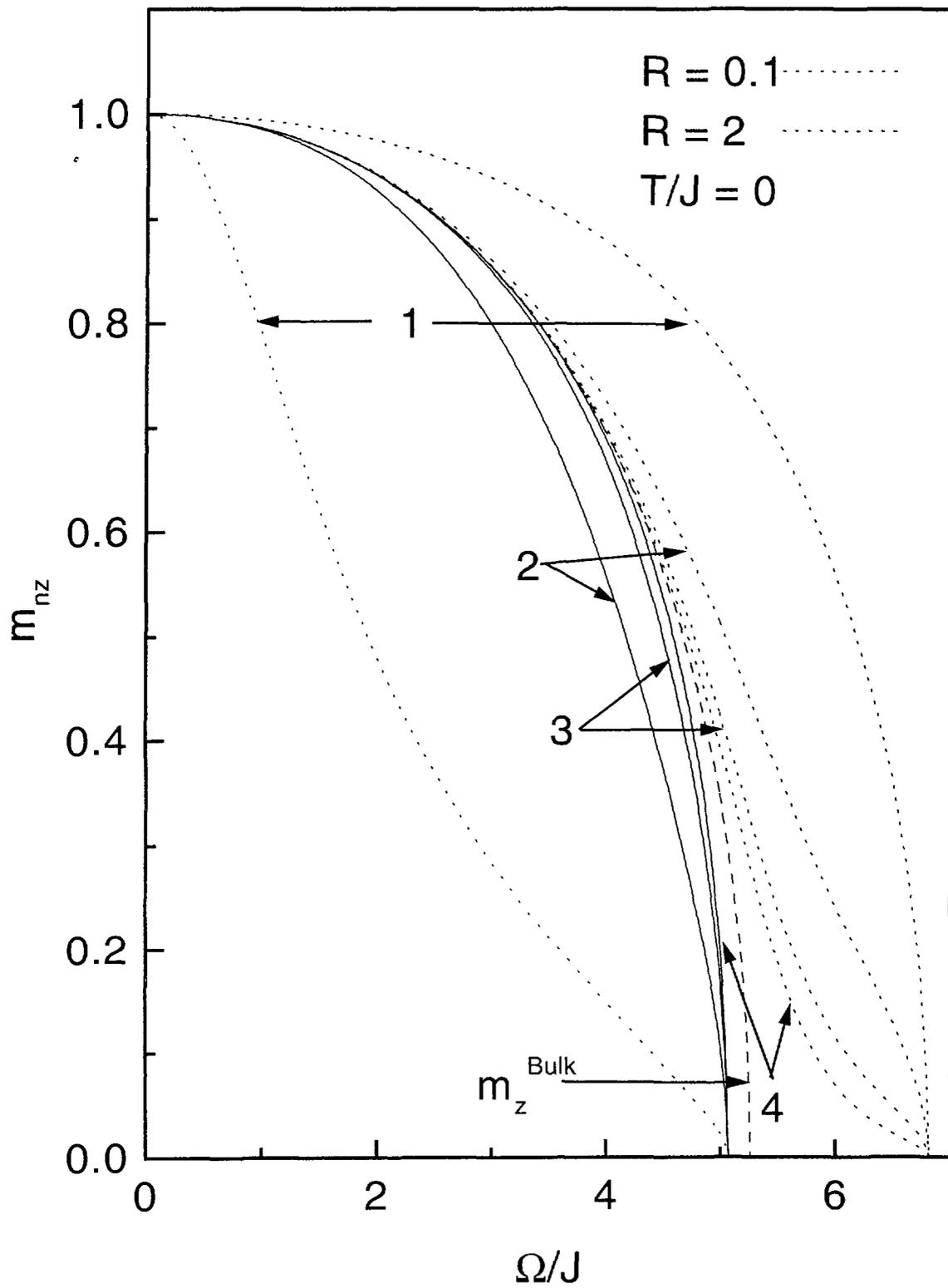
Fig. 1a
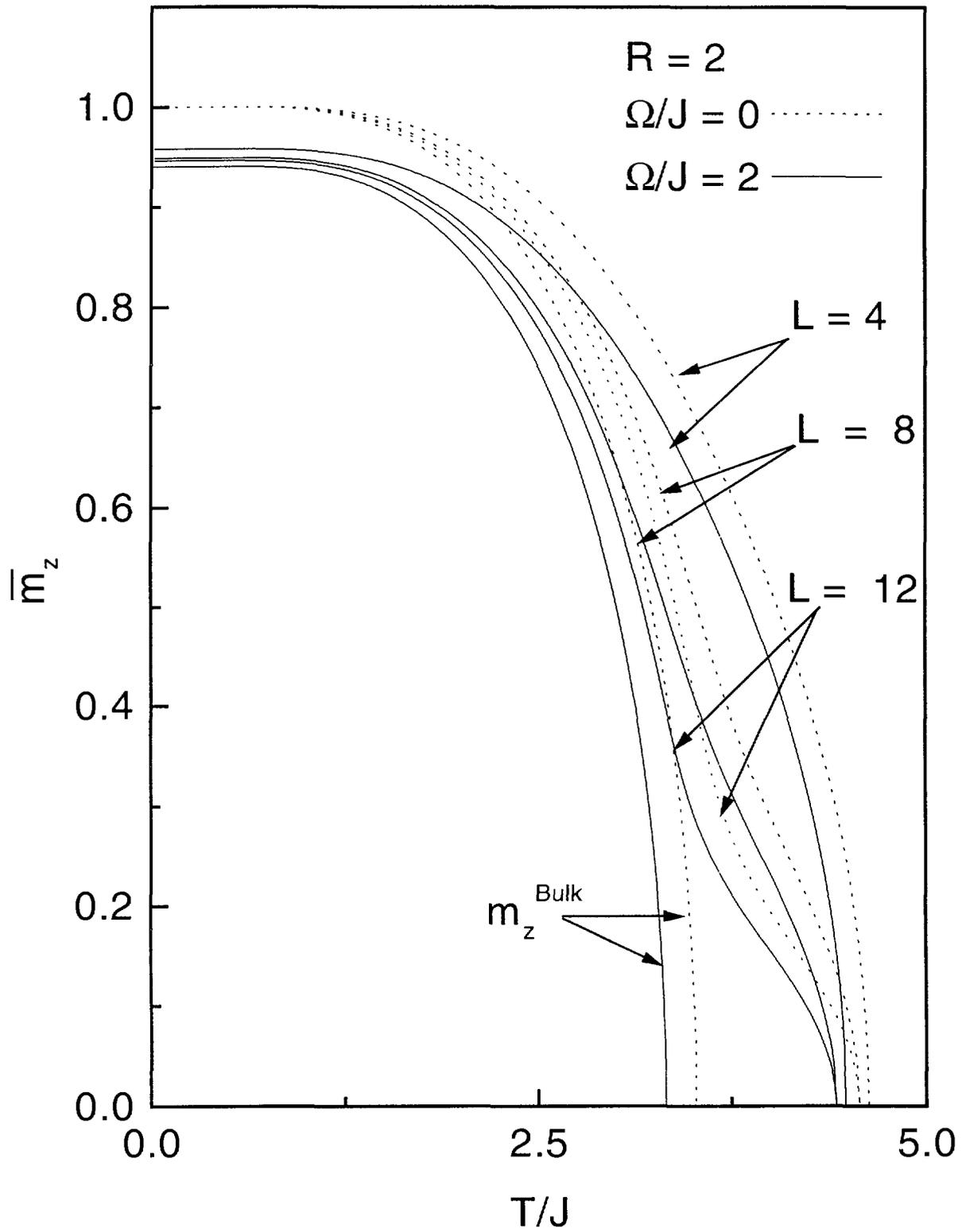
Fig. 1b
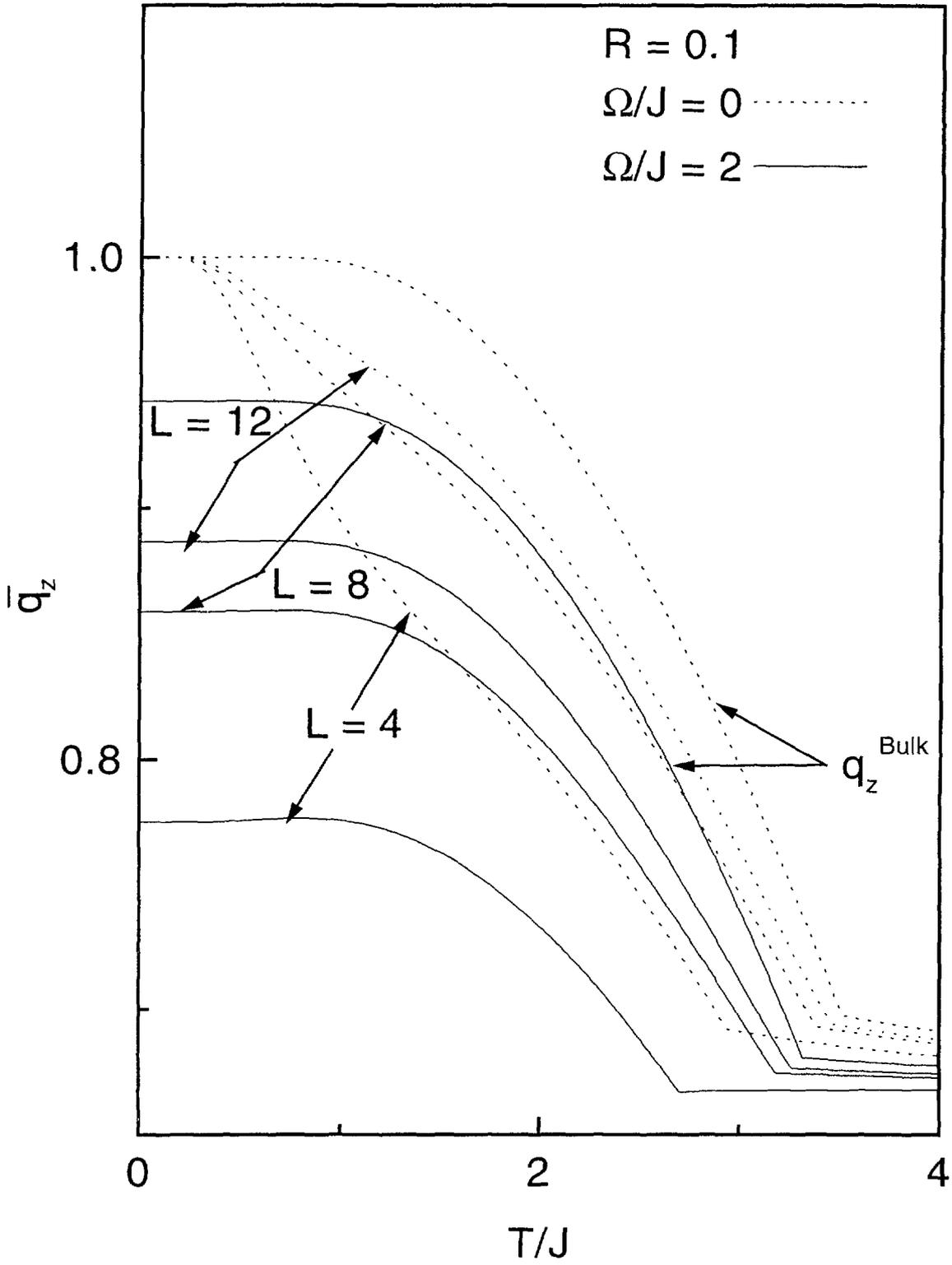
Fig. 2a

Fig. 2b

Fig. 3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 6a

Fig. 6b