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CHIRAL AND PARITY ANOMALIES AT FINITE TEMPERATURE AND DENSITY

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Two closely related topological phenomena are studied at finite density and temperature. These are chiral anomaly and Chern–Simons term. By using different methods it is shown that $\mu^2 = m^2$ is the crucial point for Chern–Simons term at zero temperature. So when $\mu^2 < m^2$, μ influence disappears and we get the usual Chern–Simons term. On the other hand, when $\mu^2 > m^2$, the Chern–Simons term vanishes because of nonzero density of background fermions. It occurs that the chiral anomaly doesn't depend on density and temperature. The connection between parity anomalous Chern–Simons term and chiral anomaly is generalized on finite density. These results hold in any dimension both in Abelian and in non-Abelian cases.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Киральная аномалия и аномалия четности при конечной температуре и плотности

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Изучаются два близких топологических явления — киральная аномалия и черн-саймоновский член — при произвольной плотности и температуре. При использовании различных методов показано, что $\mu^2 = m^2$ является критической точкой для члена Черн — Саймонса при нулевой температуре. Так, при $\mu^2 < m^2$ μ-зависимость исчезает и получается обычный черн-саймоновский член. С другой стороны, при $\mu^2 > m^2$ черн-саймоновский член исчезает из-за ненулевой плотности фоновых электронов. Киральная аномалия, как оказывается, не зависит от химического потенциала и температуры. Связь между киральной аномалией и черн-саймоновским членом обобщена на случай ненулевой плотности. Полученный результат справедлив в любой размерности как для абелева, так и для неабелева случая.

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1. Introduction

Topological objects in modern physics play a great role. In particular, here we are interested in Chern–Pontriagin and Chern–Simons (CS) secondary characteristic classes. That corresponds to chiral anomaly in even dimensions and to CS (parity anomaly) in odd dimensions. Both phenomena are very important in quantum physics. So, chiral anomalies

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in quantum field theory have direct applications to the decay of π_0 into two photons ($\pi_0 \to \gamma\gamma$), in the understanding and solution of the $U(1)$ problem and so on. On the other hand, there are many effects caused by CS secondary characteristic class. There are, for example, gauge particles mass appearance in quantum field theory, applications to condense matter physics such as the fractional quantum Hall effect and high T_c superconductivity, possibility of free of metric tensor theory construction, etc.

It must be emphasized that these two phenomena are closely related. As it was shown (at zero density) in [1,2] the trace identities connect even dimensional anomaly with the odd dimensional CS. The main goal of this paper is to explore these anomalous objects at finite density and temperature.

It was shown [3,4,5] in a conventional zero density and temperature gauge theory that the CS term is generated in the Eulier–Heisenberg effective action by quantum corrections. Since the chemical potential term $\mu\bar{\psi}\gamma^0\psi$ is odd under charge conjugation we can expect that it would contribute to P and CP nonconserving quantity — CS term. As we will see, this expectation is completely justified. The zero density approach usually is a good quantum field approximation when the chemical potential is small as compared with characteristic energy scale of physical processes. Nevertheless, for investigation of topological effects it is not the case. As we will see below, even a small density could lead to principal effects.

In the excellent paper by Niemi [1] it was emphasized that the charge density at $\mu \neq 0$ becomes nontopological object, i.e., contains both topological part and nontopological one. The charge density at $\mu \neq 0$ (nontopological, neither parity-odd nor parity-even object)* in QED_3 at finite density was calculated and exploited in [6]. It must be emphasized that in [6] charge density (calculated in the constant pure magnetic field) contains parity-odd part corresponding to CS term, as well as parity-even part, which can't be covariantized and doesn't contribute to the mass of the gauge field. Here we are interested in finite density and temperature influence on covariant parity odd form in action leading to the gauge field mass generation — CS topological term. Deep insight on this phenomena at small densities was done in [1,2]. The result for CS term coefficient in QED_3 is $\left[\text{th}\,\frac{1}{2}\,\beta(m-\mu) + \text{th}\,\frac{1}{2}\,\beta(m+\mu)\right]$ (see [2], formulas (10.18)). However, to get this result it was heuristically supposed that at small densities index theorem could still be used and only odd in energy part of spectral density is responsible for parity nonconserving effect. Because of this in [2] it had been stressed that the result holds only for small μ. However, as we'll see below this result holds for any values of chemical potential. Thus, to obtain trustful result at any values of μ one has to use transparent and free of any restrictions on μ procedure, which would allow one to perform calculations with arbitrary non-Abelian background gauge fields.

It was shown at zero chemical potential in [1,2,3] that CS term in odd dimensions is connected with chiral anomaly in even dimensions by trace identities. As we'll see below

*For abbreviation, speaking about parity invariance properties of local objects, we will keep in mind symmetries of the corresponding action parts.

it is possible to generalize a trace identity on nonzero density case. The trace identity connects chiral anomaly with CS term which has μ and T dependent coefficient. Despite chemical potential and temperature give rise to a coefficient in front of CS term, they don't affect the chiral anomaly. Indeed, anomaly is a short-distance phenomenon which should not be affected by medium μ and T effects, or more quantitatively, so as the anomaly has ultraviolet nature, temperature and chemical potential should not give any ultraviolet effect since distribution functions decrease exponentially with energy in the ultraviolet limit.

This paper is organized as follows. In Sec.2 the independence of chiral anomaly from temperature and background fermion density is discussed. It is shown in 2-dimensional Schwinger model that chiral anomaly isn't influenced not only by chemical potential μ, but also by Lagrange multiplier κ at conservation of chiral charge constraint. Besides, we consider CS term appearance at finite density in even dimensional theories. In Sec.3 we obtain CS term in 3-dimensional theory at finite density and temperature by use of a few different methods. In Sec.4 we evaluate CS term coefficient in 5-dimensional theory and generalize this result on arbitrary non-Abelian odd-dimensional theory. In Sec.5 we generalize trace identity on finite density on the basis of the previous calculations. Section 6 is devoted to concluding remarks.

2. Chiral Anomaly and Chern–Simons Term in Even Dimensions

As is well known, chemical potential can be introduced in a theory as Lagrange multiplier at corresponding conservation laws. In nonrelativistic physics this is conservation of full number of particles. In relativistic quantum field theory these are the conserving charges. The ground state energy can be obtained by use of variational principle

$$\langle \psi^* \hat{H} \psi \rangle = \min \tag{1}$$

under charge conservation constraint for relativistic equilibrium system

$$\langle \psi^* \hat{Q} \psi \rangle = \mathrm{const}, \tag{2}$$

where $\hat{H}$ and $\hat{Q}$ are Hamiltonian and charge operators. Instead, we can use method of undetermined Lagrange multipliers and seek absolute minimum of expression

$$\langle \psi^* (\hat{H} - \mu \hat{Q})\, \psi \rangle, \tag{3}$$

wher μ is Lagrange multiplier. Since $\hat{Q}$ commute with the Hamiltonian, $\langle \hat{J_0} \rangle$ is conserved.

On the other hand, we can impose another constraint, which implies chiral charge conservation

$$\langle \psi^* \hat{Q}_5 \psi \rangle = \mathrm{const}, \tag{4}$$

i.e., in Lagrange approach we have

$$\langle \psi^* (\hat{H} - \kappa \hat{Q}_5)\, \psi \rangle = \min, \tag{5}$$

where κ arises as Lagrange multiplier at $\langle \hat{J}_0^5 \rangle = \text{const}$ constraint. Thus, μ corresponds to nonvanishing fermion density (number of particles minus number of antiparticles) in background. Meanwhile, κ is responsible for conserving asymmetry in numbers of left and right handed background fermions.

It must be emphasized that the formal addition of a chemical potential in the theory looks like a simple gauge transformation with the gauge function μt. However, it doesn't only shift the time component of a vector potential but also gives corresponding prescription for handling the Green function poles. The correct introduction of a chemical potential redefines the ground state (Fermi energy), which leads to a new spinor propagator with the correct ε prescription for poles. So, for the free spinor propagator we have (see, for example, [7,8])

$$G(p, \mu) = \frac{\tilde{\not p} + m}{(\tilde{p}_0 + i\varepsilon \,\mathrm{sgn}\, p_0)^2 - \mathbf{p}^2 - m^2}, \tag{6}$$

where $\tilde{p} = (p_0 + \mu, \mathbf{p})$. Thus, when $\mu = 0$ one at once gets the usual ε prescription because of the positivity of $p_0 \,\mathrm{sgn}\, p_0$. In Euclidian metric one has

$$G(p; \mu) = \frac{\tilde{\not p} + m}{\tilde{p}_0^2 + \mathbf{p}^2 + m^2}, \tag{7}$$

where $\tilde{p} = (p_0 + i\mu, \mathbf{p})$. In the presence of a background Yang–Mills field we consequently have for the Green function operator (in Minkovsky's space)

$$\hat{G} = (\gamma\tilde{\pi} - m)\, \frac{1}{(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon(p_0 + \mu)\,\mathrm{sgn}\,(p_0)}, \tag{8}$$

where $\tilde{\pi}_\nu = \pi_\nu + \mu\delta_{\nu 0}$, $\pi_\mu = p_\nu - gA_\nu(x)$.

Now we'll consider chiral anomaly. It was shown in [9], that chiral anomaly doesn't depend on μ and T. In [9] the direct calculations in 4-dimensional gauge theory were performed by use of imaginary and real time formalism, by using the Fujikawa method and perturbation theory. These calculations are rather cumbersome. To clear understand the nature of anomaly μ independence (T independence will be discussed later) we'll consider here the simplest case — 2-dimensional QED and rederive result of [9] by use of the Schwinger nonperturbative method [10]. So, one can write

$$J^\mu = -ig\,\mathrm{tr}\left[\gamma^\mu G(x, x') \exp\left(-ig\int_{x'}^{x} d\xi^\mu A_\mu(\xi)\right)\right]_{x' \to x}, \tag{9}$$

where $G(x, x')$ is the propagator satisfying following equation

$$\gamma^\mu(\partial^x_\mu - igA_\mu(x))\, G(x, x') = \delta(x - x'). \tag{10}$$

Following Schwinger we use anzats

$$G(x, x') = G^0(x, x') \exp [ig(\phi(x) - \phi(x'))], \tag{11}$$

where $G^0(x, x')$ is the free propagator

$$\gamma^\mu \partial^x_\mu G^0(x, x') = \delta(x - x').$$

Thus, for ϕ we can write $\gamma^\mu \partial_\mu \phi = \gamma^\mu A_\mu$. From (6) we have

$$G^0(x, x') = \int \frac{d^2p}{(2\pi)^2} e^{ip(x - x')} \frac{\not p}{p^2 + i\varepsilon(p_0 + \mu) \operatorname{sgn} p_0} = - i\not\partial \Bigg[\int \frac{d^2p}{(2\pi)^2} e^{ip(x - x')} \frac{1}{p^2 + i\varepsilon} -$$

$$- 2 \int\limits_{-\infty}^{+\infty} \frac{dp_1}{2\pi} \int\limits_{-\infty}^{+\infty} \frac{dp_0}{2\pi} \theta(- \tilde{p}_0 \operatorname{sgn} p_0)\, e^{ip(x - x')}\, \Im m \frac{1}{p^2 + i\varepsilon} \Bigg]. \tag{12}$$

So, beside the usual zero density part μ-dependent one appears. Further, we have to take off regularization in the current by use of symmetrical limit $x \to x'$. After some simple calculations it is clearly seen that all μ-dependent terms after taking off the limit disappear. Thus, contribution to the current arises from the μ-dependent part only. So

$$J^\mu = i \frac{g^2}{2\pi} \left(\delta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) A_\nu,$$

$$J^\mu_5 = i \frac{g^2}{2\pi} \left(\varepsilon^{\mu\nu} - \varepsilon^{\mu\alpha} \frac{\partial^\alpha \partial^\nu}{\partial^2} \right) A_\nu \tag{13}$$

and we get the usual anomaly in the chiral current

$$\partial_\mu J^\mu = 0, \qquad \partial_\mu J^\mu_5 = i \frac{g^2}{2\pi} \varepsilon^{\mu\nu} \partial_\mu A_\nu + i \frac{g^2}{4\pi} {}^*F. \tag{14}$$

Let's now consider κ influence on the chiral anomaly. Since, as we've seen above, κ is directly connected to chiral charge, it would be natural to expect some κ effect on chiral anomaly. However, the rather amazing situation occurs. The demand of chiral charge conservation (instead of the usual charge conservation) on the quantum level doesn't influence chiral anomaly. Really, in 2-dimensions introduction of Lagrange multiplier κ at the chiral charge conservation gives the term $\kappa\bar{\psi}\gamma^5\gamma^0\psi = \kappa\bar{\psi}\gamma^1\psi$ in Lagrangian. So, κ affects in the same way as μ, i.e., κ doesn't influence the chiral anomaly (it is also seen from direct calculations which are similar to presented above for the case with μ). That could be explained due to ultraviolet nature of the chiral anomaly, while $\kappa(\mu)$ doesn't introduce new divergences in the theory.

From the above calculations it is clearly seen the principle difference of chiral anomaly and CS. The ultraviolet regulator — P exponent gives rise to the anomaly, but (as we'll see below) doesn't influence CS. Thus, it is natural, that the anomaly doesn't depend on μ, κ and T because it has ultraviolet regularization origin, while neither density nor temperature does affect the ultraviolet behaviour of the theory. The general and clear proof of axial anomaly temperature independence will be presented in Sec.5 on the basis of the trace identities.

We now consider CS in even dimensional theory. From the definition one has

$$\frac{\partial I_{\text{eff}}}{\partial \kappa} = \int d^D x \, \langle J_5^0 \rangle. \tag{15}$$

Since axial anomaly doesn't depend on κ, effective action contains the term proportional to anomalous Q_5 charge with κ as a coefficient. The same is for a chiral theory. There, effective action contains the term proportional to anomalous Q charge with μ as coefficient, see for example [11,12,13]. So, we have

$$\Delta I_{\text{eff}} = -\kappa \int dx_0 W[A] \tag{16}$$

in conventional gauge theory and

$$\Delta I_{\text{eff}}^{\text{chiral}} = -\mu \int dx_0 W[A] \tag{17}$$

in the chiral theory. Here $W[A]$ — CS term. Thus we get CS with Lagrange multiplier as a coefficient.

It is well known that at nonzero temperature in $\beta \to 0$ limit the dimensional reduction effect occurs. So, extra t dependence of CS term in (16) disappears and CS can be treated as a mass term in 3-dimensional theory with $i\kappa/T$ coefficient (the same for chiral theory with μ see [11]). For anomalous part of effective action we have

$$\Delta I_{\text{eff}} = -i\kappa\beta W[A], \qquad \Delta I_{\text{eff}}^{\text{chiral}} = -i\mu\beta W[A] \tag{18}$$

in conventional and chiral gauge theories correspondingly. The only problem arising in a treating of CS as a mass term is that the coefficient is imaginary, see discussions on the theme in [11,13]. One can notice that results (16), (17) and (18) hold in arbitrary even dimension. Let us stress that we don't need any complicated calculations to obtain (16—18). The only thing we need is the knowledge of chiral anomaly independence from μ, κ, and β.

3. CS in 3-Dimensional Theory

3.1. Constant Magnetic Field. Let's first consider a (2 + 1) dimensional Abelian theory. Here we'll use constant magnetic background. We'll evaluate fermion density by performing the direct summation over Landau levels. As a starting point, we'll use the formula for fermion number at finite density and temperature [1]

$$N = -\frac{1}{2}\sum_n \text{th}\left(\frac{1}{2}\beta\lambda_n\right) + \sum_n \left[\frac{\theta(\lambda_n)}{\exp(-\beta(\mu-\lambda_n))+1} - \frac{\theta(-\lambda_n)}{\exp(-\beta(\lambda_n-\mu))+1}\right] =$$

$$= \frac{1}{2}\sum_n \text{th}\,\frac{1}{2}\beta(\mu-\lambda_n) \xrightarrow{\beta\to\infty} \frac{1}{2}\sum_n \text{sgn}\,(\mu-\lambda_n). \tag{19}$$

Landau levels in the constant magnetic field have the form [14]

$$\lambda_0 = -m \text{ sgn } (eB), \qquad \lambda_n = \pm\sqrt{2n|eB| + m^2}, \tag{20}$$

where $n = 1, 2, \ldots$ It is also necessary to take into account in (19) the degeneracy of Landau levels. Namely, the number of degenerate states for each Landau level is $|eB|/2\pi$ per unit area. Even now we can see that only zero modes (because of sgn (eB)) could contribute to the parity odd quantity. So, for zero temperature, by using the identity

$$\text{sgn } (a-b) + \text{sgn } (a+b) = 2 \text{ sgn } (a)\, \theta(|a| - |b|),$$

one gets for zero modes

$$\frac{|eB|}{4\pi} \text{sgn } (\mu + m \text{ sgn } (eB)) =$$
$$= \frac{|eB|}{4\pi} \text{sgn } (\mu)\, \theta(|\mu| - |m|) + \frac{|eB|}{4\pi} \text{sgn } (eB) \text{ sgn } (m)\, \theta(|m| - |\mu|), \tag{21}$$

and for nonzero modes

$$\frac{1}{2}\frac{|eB|}{2\pi}\sum_{n=1}^{\infty} \text{sgn } (\mu - \sqrt{2n|eB| + m^2}) + \text{sgn } (\mu + \sqrt{2n|eB| + m^2}) =$$
$$= \frac{|eB|}{2\pi} \text{sgn } (\mu) \sum_{n=1}^{\infty} \theta(|\mu| - \sqrt{2n|eB| + m^2}). \tag{22}$$

Combining contributions of all modes we get for fermion density

$$\rho = \frac{|eB|}{2\pi} \text{sgn } (\mu) \sum_{n=1}^{\infty} \theta(|\mu| - \sqrt{2n|eB| + m^2}) + \frac{1}{2}\frac{|eB|}{2\pi} \text{sgn } (\mu)(|\mu| - |m|) +$$
$$+ \frac{1}{2}\frac{eB}{2\pi} \text{sgn } (m)\, \theta(|m| - |\mu|) = \frac{|eB|}{2\pi} \text{sgn } (\mu) \left(\text{Int}\left[\frac{\mu^2 - m^2}{2\,|eB|} \right] + \frac{1}{2} \right) \times$$
$$\times\, \theta(|\mu| - |m|) + \frac{eB}{4\pi} \text{sgn } (m)\, \theta(|m| - |\mu|). \tag{23}$$

Here we see that zero modes contribute both to parity-odd and to parity-even part, while nonzero modes contribute to the parity-even part only (note that under parity transformation $B \to -B$). Thus, fermion density contains parity-odd part, leading to CS term in action after covariantization, as well as parity-even part. It is straightforward to generalize the calculations on finite temperature case. Substituting zero modes into (19) one gets

$$N_0 = \frac{|eB|}{2\pi}\,\frac{1}{2}\left[\frac{1}{2}\, \beta(\mu + m \text{ sgn } (eB)) \right] =$$

$$= \frac{|eB|}{4\pi}\left[\frac{\text{sh } (\beta\mu)}{\text{ch } (\beta\mu) + \text{ch } (\beta m)} + \text{sgn } (eB)\, \frac{\text{sh } (\beta m)}{\text{ch } (\beta\mu) + \text{ch } (\beta m)} \right], \tag{24}$$

so, extracting parity-odd part, one gets for CS at finite temperature and density

$$N_{CS} = \frac{eB}{4\pi}\,\frac{\mathrm{sh}\,(\beta m)}{\mathrm{ch}\,(\beta\mu) + \mathrm{ch}\,(\beta m)} = \frac{eB}{4\pi}\,\mathrm{th}\,(\beta\mu)\,\frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}\,. \tag{25}$$

So, the result coincides with the result for CS term coefficient by Niemi [2] obtained for small μ $\left[\ \mathrm{th}\,\frac{1}{2}\,\beta(m-\mu) + \mathrm{th}\,\frac{2}{2}\,\beta(m+\mu)\ \right]$. It is obviously the limit to zero temperature. The lack of this method is that it works only for Abelian and constant field case.

This result at zero temperature can be obtained by use of Schwinger proper-time method. Consider (2 + 1) dimensional theory in the Abelian case and choose background field in the form

$$A^{\mu} = \frac{1}{2}\,x_{\nu}F^{\nu\mu}, \qquad F^{\nu\mu} = \mathrm{Const.}$$

To obtain the CS term in this case, it is necessary to consider the background current

$$\langle J^{\mu}\rangle = \frac{\delta S_{\mathrm{eff}}}{\delta A_{\mu}}$$

rather than the effective action itself. This is because the CS term formally vanishes for such the choice of A^{μ} but its variation with respect to A^{μ} produces a nonvanishing current. So, consider

$$\langle J^{\mu}\rangle = -\,ig\ \mathrm{tr}\,[\gamma^{\mu}G(x, x')]_{x\to x'} \tag{26}$$

where

$$G(x, x') = \exp\left(-\,ig\int_{x'}^{x} d\zeta_{\mu}A^{\mu}(\zeta)\right)\langle x\,|\,\hat{G}\,|\,x'\rangle. \tag{27}$$

Let's rewrite the Green function (8) in a more appropriate form

$$\hat{G} = (\gamma\tilde{\pi} - m)\left[\frac{\theta((p_0 + \mu)\,\mathrm{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon} + \frac{\theta(-\,(p_0 + \mu)\,\mathrm{sgn}(p_0))}{(\gamma\tilde{\pi})^2 - m^2 - i\varepsilon}\right]. \tag{28}$$

Now, we use the well-known integral representation of denominators

$$\frac{1}{\alpha \pm i0} = \mp\, i\int_0^{\infty} ds\ \mathrm{e}^{\pm\, i\alpha s},$$

which corresponds to introducing the «proper-time» s into the calculation of the Eulier–Heisenberg Lagrangian by the Schwinger method [15]. We obtain

$$\hat{G} = (\gamma\tilde{\pi} - m)\left[-\,i\int_0^{\infty} ds\,\exp\,(is\,[(\gamma\tilde{\pi})^2 - m^2 + i\varepsilon])\,\theta((p_0 + \mu)\,\mathrm{sgn}\,(p_0)) + \right.$$

$$\left. +\, i\int_0^{\infty} ds\,\exp\,(-\,is\,[(\gamma\tilde{\pi})^2 - m^2 - i\varepsilon])\,\theta(-\,(p_0 + \mu)\,\mathrm{sgn}\,(p_0))\right]. \tag{29}$$

For simplicity, we restrict ourselves only to the magnetic field case, where $A_0 = 0$, $[\tilde{\pi}_0, \tilde{\pi}_\mu] = 0$. Then we easily can factorize the time dependent part of the Green function. By using the obvious relation

$$(\gamma\tilde{\pi})^2 = (p_0 + \mu)^2 - \pi^2 + \frac{1}{2} g\sigma_{\mu\nu}F^{\mu\nu} \tag{30}$$

one gets

$$G(x, x')\big|_{x \to x'} = -i \int \frac{dp_0}{2\pi} \frac{d^2p}{(2\pi)^2} (\gamma\tilde{\pi} - m) \int_0^\infty ds \Big[e^{is(\tilde{p}_0^2 - m^2)} e^{-is\pi^2} e^{isg\sigma F/2} -$$

$$- \theta(-(p_0 + \mu)\,\mathrm{sgn}\,(p_0)) \left(e^{is(\tilde{p}_0^2 - m^2)} e^{-is\pi^2} e^{isg\sigma F/2} + e^{-is(\tilde{p}_0^2 - m^2)} e^{+is\pi^2} e^{-isg\sigma F/2} \right)\Big]. \tag{31}$$

Here the first term corresponds to the usual μ-dependent case and there are two additional μ-dependent terms. In the calculation of the current the following trace arises:

$$\mathrm{tr}\,[\gamma^\mu(\gamma\tilde{\pi} - m)\, e^{isg\sigma F/2}] =$$

$$= 2\pi^\nu g^{\nu\mu} \cos(g\,|{}^*F|\,s) + 2\,\frac{\pi^\nu F^{\nu\mu}}{|{}^*F|} \sin(g\,|{}^*F|\,s) - 2im\,\frac{{}^*F^\mu}{|{}^*F|} \sin(g\,|{}^*F|\,s),$$

where ${}^*F^\mu = \varepsilon^{\mu\alpha\beta}F_{\alpha\beta}/2$ and $|{}^*F| = \sqrt{B^2 - E^2}$. Since we are interested in calculation of the parity-odd part (CS term) it is enough to consider only terms proportional to the dual strength tensor ${}^*F^\mu$. On the other hand, the term $2\pi^\mu g^{\nu\mu} \cos(g\,|{}^*F|\,s)$ at $\nu = 0$ (see expression for the trace, we take in mind that here there is only magnetic field) also gives nonzero contribution to the current J^0 [6]

$$J^0_{\mathrm{even}} = g\,\frac{|gB|}{2\pi} \left(\mathrm{Int}\left[\frac{\mu^2 - m^2}{2\,|gB|} \right] + \frac{1}{2} \right) \theta(|\mu| - |m|). \tag{32}$$

This part of current is parity invariant because under parity $B \to -B$. It is clear that this parity-even object does contribute neither to the parity anomaly nor to the mass of the gauge field. Moreover, this term has magnetic field in the argument's denominator of the cumbersome function — integer part. So, the parity even term seems to be «noncovariantizable», i.e., it can't be converted in covariant form in effective action. Since we explore the parity anomalous topological CS term, we won't consider this parity even term. So, only the term proportional to the dual strength tensor ${}^*F^\mu$ gives rise to CS. The relevant part of the current after spatial momentum integration reads

$$J^{\mu}_{CS} = \frac{g^2}{4\pi^2} m^* F^{\mu} \int_{-\infty}^{+\infty} dp_0 \int_0^{\infty} ds \left[e^{is(\tilde{p}_0^2 - m^2)} - \right.$$

$$\left. - \theta(-\tilde{p}_0 \,\text{sgn}\,(p_0)) \left(e^{is(\tilde{p}_0^2 - m^2)} + e^{-is(\tilde{p}_0^2 - m^2)} \right) \right]. \qquad (33)$$

Thus, we get besides the usual CS part [4], also the μ-dependent one. It is easy to calculate it by use of the formula

$$\int_0^{\infty} ds\, e^{is(x^2 - m^2)} = \pi \left(\delta(x^2 - m^2) + \frac{i}{\pi} \mathcal{P} \frac{1}{x^2 - m^2} \right)$$

and we get eventually

$$J^{\mu}_{CS} = \frac{m}{|m|} \frac{g^2}{4\pi} {}^*F^{\mu}[1 - \theta(-(m+\mu)\,\text{sgn}\,(m)) - \theta(-(m-\mu)\,\text{sgn}\,(m))] =$$

$$= \frac{m}{|m|} \theta(m^2 - \mu^2) \frac{g^2}{4\pi} {}^*F^{\mu}. \qquad (34)$$

Let's now discuss the non-Abelian case. Then $A^{\mu} = T_a A^{\mu}_a$ and

$$\langle J^{\mu}_a \rangle = -ig \,\text{tr}\, [\gamma^{\mu} T_a G(x, x')]_{x \to x'}.$$

It is well known [4,16] that there exist only two types of the constant background fields. The first is the «Abelian» type (it is easy to see that the self-interaction $f^{abc} A^{\mu}_b A^{\nu}_c$ disappears under that choice of the background field)

$$A^{\mu}_a = n_a \frac{1}{2} x_{\nu} F^{\nu\mu}, \qquad (35)$$

where n_a is an arbitrary constant vector in the color space, $F^{\nu\mu}$ = Const. The second is the pure «non-Abelian» type

$$A^{\mu} = \text{Const.} \qquad (36)$$

Here the derivative terms (Abelian part) vanish from the strength tensor and it contains only the self-interaction part $F^{\mu\nu}_a = gf^{abc} A^{\mu}_b A^{\nu}_c$. It is clear that to catch Abelian part of the CS term we should consider the background field (35), whereas for the non-Abelian (derivative noncontaining, cubic in A) part we have to use the case (36).

Calculations in the «Abelian» case reduce to the previous analysis, except the trivial adding of the color indices in the formula (34):

$$J^{\mu}_a = \frac{m}{|m|} \theta(m^2 - \mu^2) \frac{g^2}{4\pi} {}^*F^{\mu}_a. \qquad (37)$$

In the case (36) all calculations are similar. The only difference is that the origin of term $\sigma_{\mu\nu}F^{\mu\nu}$ in (30) is not the linearity A in x (as in Abelian case) but the pure non-Abelian A^{μ} = Const. Here term $\sigma_{\mu\nu}F^{\mu\nu}$ in (30) becomes quadratic in A and we have

$$J^{\mu}_{a} = \frac{m}{|m|}\,\theta(m^2 - \mu^2)\,\frac{g^3}{4\pi}\,\varepsilon^{\mu\alpha\beta}\,\mathrm{tr}\,[T_a A^{\alpha} A^{\beta}]. \tag{38}$$

Combining formulas (37) and (38) and integrating over field A^{μ}_{a} we obtain eventually

$$S^{\mathrm{CS}}_{\mathrm{eff}} = \frac{m}{|m|}\,\theta(m^2 - \mu^2)\,\pi W[A], \tag{39}$$

where $W[A]$ is the CS term

$$W[A] = \frac{g^2}{8\pi^2}\int d^3x\varepsilon^{\mu\nu\alpha}\,\mathrm{tr}\left(F_{\mu\nu}A_{\alpha} - \frac{2}{3}\,gA_{\mu}A_{\nu}A_{\alpha}\right).$$

In conclusion note, that it may seem that covariant notation used through this section is rather artificial. However, the covariant notation is useful here because it helps us to extract Levi–Chivita tensor corresponding to parity anomalous CS term.

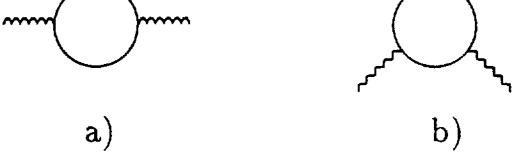
3.2. Arbitrary Gauge Field Background. One can see that the procedures we've used above to calculate CS are noncovariant. Indeed, both of them use the constant magnetic background. Here we'll use completely covariant free of any restriction on gauge field procedure, which allows us to perform calculations at once in non-Abelian case. We'll employ the perturbative expansion. The zero temperature case within this procedure has been explored in [17].

Let's first consider non-Abelian 3-dimensional gauga theory. The only graphs whose P-odd parts contribute to the parity anomalous CS term are shown in Fig.1.

So, the part of effective action containing the CS term looks as

$$I^{\mathrm{CS}}_{\mathrm{eff}} = \frac{1}{2}\int_x A_{\mu}(x)\int_p \mathrm{e}^{-ixp}\,A_{\nu}(p)\,\Pi^{\mu\nu}(p) +$$

$$+\frac{1}{3}\int_x A_{\mu}(x)\int_{p,r} \mathrm{e}^{-ix(p+r)}\,A_{\nu}(p)\,A_{\alpha}(r)\,\Pi^{\mu\nu\alpha}(p, r), \tag{40}$$



a) b)

Fig.1. Graphs whose P-odd parts contribute to the CS term in non-Abelian $3D$ gauge theory

where polarization operator and vertices have a standard form

$$\Pi^{\mu\nu}(p) = g^2 \int_k \mathrm{tr}\, [\gamma^\mu S(p+k;\mu)\, \gamma^\nu S(k;\mu)],$$
$$\Pi^{\mu\nu\alpha}(p, r) = g^3 \int_k \mathrm{tr}\, [\gamma^\mu S(p + r + k;\mu)\, \gamma^\nu S(r + k;\mu)\, \gamma^\alpha S(k;\mu)], \tag{41}$$

where $S(k;\mu)$ is the Euclidean fermion propagator at finite density and temperature (7) and the following notation is used $\int_x = i\int_0^\beta dx_0 \int d\mathbf{x}$ and $\int_k = \frac{i}{\beta}\sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^2}$. First consider the second order term (Fig.1, graph (a)). It is well known that the only object giving us the possibility to construct P- and T-odd form in action is Levi–Chivita tensor*. Thus, we will drop all terms noncontaining Levi–Chivita tensor. Signal for the mass generation (CS term) is $\Pi^{\mu\nu}(p^2 = 0) \neq 0$. So we get

$$\Pi^{\mu\nu} = g^2 \int_k (-i2me^{\mu\nu\alpha}p_\alpha)\frac{1}{(\tilde{k}^2 + m^2)^2}. \tag{42}$$

After some simple algebra one obtains

$$\Pi^{\mu\nu} = -i2mg^2 e^{\mu\nu\alpha} p_\alpha \frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int \frac{d^2k}{(2\pi)^2}\frac{1}{(\tilde{k}^2 + m^2)^2} =$$
$$= -i2mg^2 e^{\mu\nu\alpha} p_\alpha \frac{i}{\beta}\sum_{n=-\infty}^{\infty}\frac{i}{4\pi}\frac{1}{\omega_n^2 + m^2}, \tag{43}$$

where $\omega_n = (2n + 1)\pi/\beta + i\mu$. Performing summation we get

$$\Pi^{\mu\nu} = i\frac{g^2}{4\pi} e^{\mu\nu\alpha} p_\alpha\, \mathrm{th}\,(\beta m)\frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}. \tag{44}$$

It is easily seen that in $\beta \to \infty$ limit we'll get zero temperature result [17]

$$\Pi^{\mu\nu} = i\frac{m}{|m|}\frac{g^2}{4\pi} e^{\mu\nu\alpha} p_\alpha\, \theta(m^2 - \mu^2). \tag{45}$$

In the same manner handling the third order contribution (Fig.1b) one gets

$$\Pi^{\mu\nu\alpha} = -2g^3 i e^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{m(\tilde{k}^2 + m^2)}{(\tilde{k}^2 + m^2)^3} =$$
$$= -i2mg^3 e^{\mu\nu\alpha}\frac{i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^2k}{(2\pi)^2}\frac{1}{(\tilde{k}^2 + m^2)^2} \tag{46}$$

*In three dimensions it arises as a trace of three γ matrices (Pauli matrices) and further all calculations are identical to the second order

$$\Pi^{\mu\nu\alpha} = i \frac{g^3}{4\pi} e^{\mu\nu\alpha} \text{th}\,(\beta m) \frac{1}{1 + \text{ch}\,(\beta\mu)/\text{ch}\,(\beta m)}\,. \tag{47}$$

Substituting (44), (47) in the effective action (40) we get eventually

$$I_{\text{eff}}^{\text{CS}} = \text{th}\,(\beta m) \frac{1}{1 + \text{ch}\,(\beta\mu)/\text{ch}\,(\beta m)} \frac{g^2}{8\pi} \int d^3x e^{\mu\nu\alpha}\, \text{tr}\left(A_\mu \partial_\nu A_\alpha - \frac{2}{3} g A_\mu A_\nu A_\alpha \right). \tag{48}$$

Thus, we've got CS term with temperature and density dependent coefficient.

4. Chern–Simons in Arbitrary Odd Dimension

Let's now consider 5-dimensional gauge theory. Here the Levi–Chivita tensor is 5-dimensional $e^{\mu\nu\alpha\beta\gamma}$ and the relevant graphs are shown in Fig.2.

The part of effective action containing CS term reads

$$I_{\text{eff}}^{\text{CS}} = \frac{1}{3} \int\limits_x A_\mu(x) \int\limits_{p,r} e^{-ix(p+r)} A_\nu(p)\, A_\alpha(r)\, \Pi^{\mu\nu\alpha}(p, r) +$$

$$+ \frac{1}{4} \int\limits_x A_\mu(x) \int\limits_{p,r,s} e^{-ix(p+r+s)} A_\nu(p)\, A_\alpha(r)\, A_\beta(s)\, \Pi^{\mu\nu\alpha\beta}(p, r, s) +$$

$$+ \frac{1}{5} \int\limits_x A_\mu(x) \int\limits_{p,r,s\,q} e^{-ix(p+r+s+q)} A_\nu(p)\, A_\alpha(r)\, A_\beta(s)\, A_\gamma(s)\, \Pi^{\mu\nu\alpha\beta\gamma}(p, r, s, q). \tag{49}$$

All calculations are similar to 3-dimensional case. First cosider third-order contribution (Fig.2a)

$$\Pi^{\mu\nu\alpha}(p, r) = g^3 \int\limits_k \text{tr}\, [\gamma^\mu S(p + r + k; \mu)\, \gamma^\nu S(r + k; \mu)\, \gamma^\alpha S(k; \mu)]. \tag{50}$$

Taking into account that trace of five γ matrices in 5-dimensions is

$$\text{tr}\, [\gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta \gamma^\rho] = 4i e^{\mu\nu\alpha\beta\rho},$$

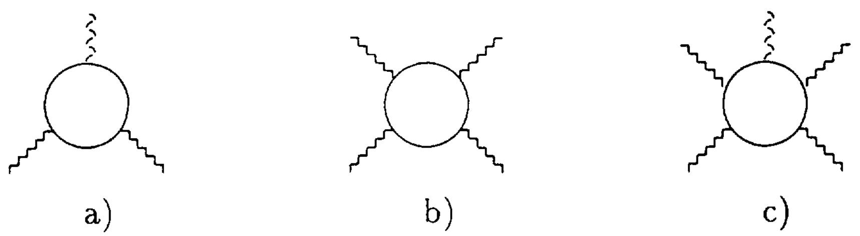


Fig.2. Graphs whose P-odd parts contribute to the CS term in non-Abelian $5D$ theory

we extract the parity-odd part of the vertices

$$\Pi^{\mu\nu\alpha} = g^3 \frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} (i4me^{\mu\nu\alpha\beta\sigma} p_\beta r_\sigma) \frac{1}{(\tilde{k}^2 + m^2)^3}, \tag{51}$$

or in more transparent way

$$\Pi^{\mu\nu\alpha} = i4mg^3 e^{\mu\nu\alpha\beta\sigma} p_\alpha r_\sigma \frac{i}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(\omega_n^2 + \mathbf{k}^2 + m^2)^3} =$$

$$= i4mg^3 e^{\mu\nu\alpha\beta\sigma} p_\alpha r_\sigma \frac{i}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{-i}{(64\pi)^2} \frac{1}{\omega_n^2 + m^2}. \tag{52}$$

Performing summation one comes to

$$\Pi^{\mu\nu\alpha} = i \,\mathrm{th}\, (\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^3}{16\pi^2} e^{\mu\nu\alpha\beta\sigma} p_\alpha r_\sigma. \tag{53}$$

In the same way operating graphs (b) and (c) (Fig.2) one will obtain

$$\Pi^{\mu\nu\alpha\beta} = i \,\mathrm{th}\, (\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^4}{8\pi^2} e^{\mu\nu\alpha\beta\sigma} s_\sigma \tag{54}$$

and

$$\Pi^{\mu\nu\alpha\beta\gamma} = i \,\mathrm{th}\, (\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^5}{16\pi^2} e^{\mu\nu\alpha\beta\sigma}. \tag{55}$$

Substituting (53)—(55) in the effective action (49) we get the final result for CS in 5-dimensional theory

$$I_{\mathrm{eff}}^{\mathrm{CS}} = \mathrm{th}\, (\beta m) \frac{1}{1 + \mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)} \frac{g^3}{48\pi^2} \int_x e^{\mu\nu\alpha\beta\gamma} \times$$

$$\times \mathrm{tr} \left(A_\mu \partial_\nu A_\alpha \partial_\beta A_\gamma + \frac{3}{2} g A_\mu A_\nu A_\alpha \partial_\beta A_\gamma + \frac{3}{5} g^2 A_\mu A_\nu A_\alpha A_\beta A_\gamma \right). \tag{56}$$

It is remarkable that all parity odd contributions are finite both in 3-dimensional and in 5-dimensional cases. Thus, all values in the effective action are renormalized in a standard way, i.e., the renormalizations are determined by conventional (parity even) parts of vertices.

From the above direct calculations it is clearly seen that the chemical potential and temperature dependent coefficient is the same for all parity odd parts of diagrams and doesn't depend on space dimension. So, the influence of finite density and temperature on CS term generation is the same in any odd dimension:

$$I_{\rm eff}^{\rm CS} = {\rm th}\,(\beta m)\,\frac{1}{1+{\rm ch}\,(\beta\mu)/{\rm ch}\,(\beta m)}\,\pi W[A] \xrightarrow{\beta\to\infty} \frac{m}{|m|}\,\theta(m^2-\mu^2)\,\pi W[A], \qquad (57)$$

where $W[A]$ is the CS secondary characteristic class in any odd dimension. Since only the lowest orders of perturbative series contribute to CS term at finite density and temperature (the same situation is well known at zero density), the result obtained by using formally perturbative technique appears to be nonperturbative. Thus, the μ- and T-dependent CS term coefficient reveals the amazing property of universality. Namely, it does depend on neither dimension of the theory nor Abelian or non-Abelian gauge theory is studied.

The arbitrariness of μ gives us the possibility to see CS coefficient behaviour at any masses. It is very interesting that $\mu^2 = m^2$ is the crucial point for CS at zero temperature. Indeed, it is clearly seen from (57) that when $\mu^2 < m^2$, μ-influence disappears and we get the usual CS term $I_{\rm eff}^{CS} = \pi W[A]$. On the other hand, when $\mu^2 > m^2$, the situation is absolutely different. One can see that here the CS term disappears because of nonzero density of background fermions. We'd like to emphasize the important massless case $m = 0$ considered in many a papers, see for example [2,4,18]. Here even negligible density or temperature, which always takes place in any physical processes, leads to vanishing of the parity anomaly. Let us stress again that we nowhere have used any restrictions on μ. Thus we not only confirm result in [2] for CS in QED_3 at small density, but also expand it on arbitrary μ, non-Abelian case and arbitrary odd dimension.

5. Trace Identity

Here, we'll consider trace identity at finite temperature and density. First of all, by using well-known trace identity at finite temperature [1,2], we'll present the simple reasons that chiral anomaly doesn't depend on temperature in any even dimension. Indeed, at finite temperature and zero density trace identity still holds and one has [1,2]

$$\langle N\rangle_\beta = -\frac{1}{2\beta}\sum_{-\infty}^{+\infty}\frac{m}{m^2+\omega_n^2}\left(\int dx\,({\rm anomaly}) + \int dx\partial_i\,{\rm tr}\,\langle x\,|\,i\Gamma_i\Gamma^c\,\frac{1}{H_0+i\sqrt{m^2+\omega_n^2}}\rangle\right). \qquad (58)$$

The second term at the left-hand side is a surface term, which doesn't contribute to topological part of the trace identity [1,2]. Thus, for topological part, we are interested in, trace identity takes the form

$$\langle N\rangle_\beta^{\rm topological} = -\frac{1}{2\beta}\sum_{-\infty}^{+\infty}\frac{m}{m^2+\omega_n^2}\,(\int dx\,({\rm anomaly})). \qquad (59)$$

The result for the left-hand side of Eq.(59) we know in arbitrary odd dimension. Really, substituting (57) in

$$\langle N\rangle_\beta^{\rm CS} = \langle N\rangle_\beta^{\rm topological} = \frac{\delta I_{\rm eff}^{\rm CS}}{g\delta A_0}, \tag{60}$$

and taking into account that

$$\frac{1}{2\beta}\sum_{n=-\infty}^{+\infty}\frac{m}{\omega_n^2+m^2} = \frac{1}{4}\,\frac{\mathrm{sh}\,(\beta m)}{1+\mathrm{ch}\,(\beta m)}, \tag{61}$$

one can see that the only possibility to reconcile left and right sides of Eq.(59) is to put temperature independence of anomaly. Thus, we've got that axial anomaly doesn't depend on temperature in any even-dimensional theory.

Further, we can generalize trace identity for topological part on arbitrary finite density. Really, from (57) and (60) we get

$$\langle N\rangle_{\beta,\mu}^{\rm CS} = -\frac{1}{4}\,\mathrm{th}\,(\beta m)\,\frac{1}{1+\mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}\int dx\,(\text{anomaly}), \tag{62}$$

where $\langle N\rangle_{\beta,\mu}^{\rm CS}$ — odd part of fermion number in D-dimensional theory at finite density and temperature, (anomaly) — axial anomaly in $(D-1)$-dimensional theory. On the other hand, as we have seen above, the anomaly doesn't depend on μ in 2 and 4 dimensions (and doesn't depend on T in any even-dimensional theory). Our comprehension of the problem allows us to generalize this on arbitrary even dimension. Indeed, anomaly is the result of ultraviolet regularization, while μ (and T) don't effect on ultraviolet behavior of a theory. Taking in mind (62) and that at finite density

$$\frac{1}{2\beta}\sum_{n=-\infty}^{+\infty}\frac{m}{\omega_n^2+m^2} = \frac{1}{4}\,\mathrm{th}\,(\beta m)\,\frac{1}{1+\mathrm{ch}\,(\beta\mu)/\mathrm{ch}\,(\beta m)}, \tag{63}$$

we can identify $\langle N\rangle_{\beta,\mu}^{\rm topological}$ and $\langle N\rangle_{\beta,\mu}^{\rm CS}$. So, we get generalized on finite density trace identity for topological part of fermion number

$$\langle N\rangle_{\beta,\mu}^{\rm CS} = \langle N\rangle_{\beta,\mu}^{\rm topological} = -\frac{1}{2\beta}\sum_{-\infty}^{+\infty}\frac{m}{m^2+\omega_n^2}\left(\int dx\,(\text{anomaly})\right). \tag{64}$$

The physical underground of formula (64) could be more clearly understood if we remember calculations we've performed in Sec.3.1 by use of summation over Landau levels. Really, we've seen that only zero modes contribute to P-odd part in contrast to P-even part which is contributed by all modes. Therefore, index theorem and trace identities hold only for parity-odd (topological) part of fermion number at finite density.

Thus, Eq.(64) connects CS term and chiral anomaly in arbitrary dimensional theory at finite density and temperature.

6. Conclusions

The finite temperature and density influence on CS term generation is obtained in any odd dimensional theory both for Abelian and for non-Abelian cases. It is of interest that $\mu^2 = m^2$ is the crucial point for CS at zero temperature. Indeed, it is clearly seen from (57) that when $\mu^2 < m^2$, μ-influence disappears and we get the usual CS term $I_{\text{eff}}^{\text{CS}} = \pi W[A]$. On the other hand, when $\mu^2 > m^2$, the CS term disappears because of nonzero density of background fermions.

The μ and T-dependent CS term coefficient reveals the amazing property of universality. Namely, it does depend on neither dimension of the theory nor Abelian or non-Abelian gauge theory is studied. It must be stressed that at $m = 0$ even negligible density or temperature, which always take place in any physical processes, leads to vanishing of the parity anomaly.

The medium effects such as finite density and temperature influence on chiral anomaly have been studied. The simple and general arguments that chiral anomaly is independent of temperature have been presented. It is shown that even if we introduce conservation of chiral charge as the constraint, the chiral anomaly isn't effected. By using the fact that chiral anomaly doesn't depend on temperature and density we explore the CS number appearance of CS number in even-dimensional theories under two type of constraints. These are charge conservation with Lagrange multiplier μ (conventional chemical potential) and chiral charge conservation with Lagrange multiplier κ, what corresponds to conservation of the left (right)-handed fermions asymmetry in the background.

On the other hand, the chiral anomaly independence of density and temperature together with our direct calculations of CS coefficient permit us the simple generalization of trace identity on finite density case. Thus, the connection between CS term and chiral anomaly at finite density and temperature is obtained in artibrary dimensional theory.

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