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DISCUSSION ON MUON COLLIDER PARAMETERS AT CENTER OF MASS ENERGIES FROM 0.1 TEV TO 100 TEV

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Abstract

Some of the potential capabilities and design challenges of muon colliders are illustrated using self-consistent collider parameter sets at center of mass energies ranging from 0.1 TeV to 100 TeV.

1 INTRODUCTION

The main motivation for research and development efforts on muon collider technology is the assertion that affordably priced muon colliders might provide lepton-lepton collisions at much higher center of mass (CoM) energies than is feasible for electron colliders, and perhaps eventually explore the spectrum of elementary particles at mass scales inaccessible even to hadron colliders.

This paper attempts to present some justification for these assertions through discussion and evaluation of the self-consistent muon collider parameter sets given in table 1, at CoM energies ranging from 0.1 to 100 TeV.

The parameter set at 0.1 TeV CoM energy was included as a lower energy reference point and was constrained to essentially reproduce one of the sets of parameters currently under study[1] by the Muon Collider Collaboration (MCC). In contrast, the other parameter sets represent speculation by the author on how the parameters might evolve with CoM energy and they have not been studied or discussed in detail within the MCC.

2 GENERATION OF PARAMETER SETS

The parameter sets in table 1 were generated through iterative runs of a stand-alone FORTRAN program, LUMCALC. The parameter sets are calculated from the input values for several input parameters – namely, the CoM energy (E_{CoM}), the collider ring circumference (C) and depth below the Earth's surface (D), the beam momentum spread (δ) and 6-dimensional invariant emittance (ϵ_{6N}), the reference pole-tip magnetic field for the final focus quadrupoles ($B_{4\sigma}$), and the time until the beams are dumped (t_D) – and from the input of maximum allowable values for several other parameters – namely, the bunch repetition frequency (f_b), the initial number of muons per bunch (N_0), the beam-beam tune disruption parameter ($\Delta\nu$), the beam divergence at the interaction point (σ_θ), the maximum aperture for the final focus quadrupoles ($A_{\pm 4\sigma}$), and maximum allowable neutrino radiation where the plane of the collider ring cuts the Earth's surface.

As a preliminary stage of calculation, LUMCALC makes any parameter adjustments that may be required to satisfy the input constraints. These are, in order: 1) reducing σ_θ to the limit imposed by $A_{\pm 4\sigma}$ (based on scaling to existing final focus designs at 0.1 TeV and 4 TeV[1]), 2) reducing N_0 to attain an acceptable $\Delta\nu$, and 3) reducing f_b until the neutrino radiation is acceptable.

The output luminosity may be derived in terms of the input parameters as:

$$\begin{aligned}\mathcal{L}[\mathrm{cm}^{-2}.\mathrm{s}^{-1}] &= 2.11 \times 10^{33} \times H_B \times (1 - e^{-2t_D[\gamma\tau_\mu]}) \\ &\quad \times \frac{f_b[\mathrm{s}^{-1}](N_0[10^{12}])^2(E_{CoM}[\mathrm{TeV}])^3}{C[\mathrm{km}]} \\ &\quad \times \left(\frac{\sigma_\theta[\mathrm{mr}].\delta[10^{-3}]}{\epsilon_{6N}[10^{-12}]}\right)^{2/3}. \qquad (1)\end{aligned}$$

This formula uses the standard MCC assumption[1] that the ratio of transverse to longitudinal emittances can be chosen in the muon cooling channel to maximize the luminosity for a given ϵ_{6N}. The pinch enhancement factor, H_B, is very close to unity (see table 1), and the numerical coefficient in equation 1 includes a geometric correction factor of 0.76 for the non-zero bunch length, $\sigma_z = \beta^*$ (the "hourglass effect").

3 DISCUSSION

The physics motivation for each of the parameter sets in table 1 is discussed in [2]. Briefly, the number of $\mu\mu \rightarrow ee$ events gives a benchmark estimate of the discovery potential for elementary particles at the full CoM energy of the collider, while the production of hypothesized 100 GeV Higgs particles indicates roughly how the colliders would perform in studying physics at this fixed energy scale.

Further information on the important issue of neutrino radiation can be found in [3]. The numbers given in table 1 come from an analytical calculation that is not intended to be accurate at much better than an order of magnitude level and that is deliberately conservative, i.e. it may well overestimate the radiation levels. The radiation levels are predicted to rise approximately as the cube of the collider energy if other relevant parameters are held fixed (up to some mitigating factors that come into play at the highest energies), rapidly becoming a serious design constraint for colliders at the TeV scale and above.

The 1 TeV parameter set of table 1 would give about the same luminosity as, for example, the design for the proposed NLC linear electron collider at the same energy, and the physics motivation and capabilities might be relatively similar[2,4]. Placement of the collider at 125 meters depth reduces the average neutrino radiation in the collider plane

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[1] web page: http://pubweb.bnl.gov/people/bking/, email: bking@bnl.gov. This work was performed under the auspices of the U.S. Department of Energy under contract no. DE-AC02-98CH10886.

to less than one thousandth of the U.S. federal off-site radiation limit (1 mSv/year, which is of the same order of magnitude as the typical background radiation from natural causes). Nevertheless, attention would still need to be paid to minimizing the length, L, of any straight sections with low beam divergence, since these produce radiation hotspots with intensity proportional to L[3].

The 4 TeV parameter set was chosen as being at about the highest energy that is practical for a "first generation" muon collider on an existing laboratory site, due to neutrino radiation, and the muon current has been reduced to lower the radiation to the same level as the 1 TeV parameter set, accepting the consequent loss in luminosity.

The 4 TeV parameters may be compared to the MCC 4 TeV design presented at Snowmass'96[5], which did not take account of the neutrino radiation issue and hence attained a luminosity higher by more than an order of magnitude. The lower bunch repetition rate of the current 4 TeV parameter set makes some of the design parameters more relaxed than in the Snowmass design, particularly in allowing a "lite front end" with relaxed rate specifications: the proton driver, pion production target and cooling channel. On the other hand, the desire to recover some of the lost luminosity motivates collider ring parameters that are slightly more aggressive, especially β^* (3 mm reduced to 1.2 mm) and σ_θ (0.9 mrad increased to 1.6 mrad). This entails a more difficult final focus design and also a more difficult task to shield the detector region from muon decay backgrounds.

Beyond CoM energies of a few TeV, it is probably necessary to build the colliders at isolated sites where the public would not be exposed to the neutrino radiation disk. These will presumably be "second generation" machines, arriving after the technology of muon colliders has been established in one or more smaller and less expensive machines built at existing HEP laboratories. The gain from being able to relax the neutrino radiation constraint is evident in the 10 TeV parameter set, with an exciting luminosity of $1.0 \times 10^{36}\text{cm}^{-2}.\text{s}^{-1}$ at several times the discovery mass reach of the LHC hadron collider.

From the progression of the parameter sets it is clear that the final focus design will become progressively more difficult with rising CoM energy. Consider, for example, the overall beam demagnification, $\sqrt{\beta_{\text{max}}/\beta^*}$, a dimensionless parameter that should be closely correlated with fractional tolerances in magnet uniformity, residual chromaticity etc. For the 10 TeV example, this has risen to approximately 31 000 in both the x and y coordinates, which – as a very crude comparison that ignores considerable differences in the other final focus parameters – happens to be approaching the y-coordinate value for both the 0.5 TeV (IA) and 1.0 TeV (IIA) designs for the NLC linear collider (i.e. 39 000, with $\beta_{\text{max,y}} = 190$ km and $\beta^*{}_\text{y} = 0.125$ mm) [4]. The spot size – clearly indicative of vibration and alignment tolerances – is also falling, but even at 100 TeV it remains an order of magnitude above the spot size in the y coordinate for the NLC design parameters. For perspective, then, the design of the final focus at 10 TeV CoM energy may well still be less challenging than the design of the muon cooling channel, and the latter task is essentially independent of the collider energy (up to assumed advances for later generation colliders).

The highest energy parameter set in table 1, at 100 TeV, clearly presents the most difficult design challenge, for several reasons: 1) cost reductions will be needed to make a machine of this size affordable, 2) siting will be more difficult than at 10 TeV, since the neutrino radiation is now well above the U.S. federal limit, 3) $\sqrt{\beta_{\text{max}}/\beta^*}$ is almost an order of magnitude larger than at 10 TeV, 4) The assumed ϵ_{6N} is 25 times smaller than for the 10 TeV parameters, albeit with much smaller bunches, so the assumed phase space density is nearly a factor of two larger, and finally 5) the beam power has risen to 170 MW, with synchrotron radiation rising rapidly to contribute a further 110 MW.

Most of these extrapolations correspond to incremental advances in technology, particularly involving magnets: magnetic field strength (for improved cooling and final focus, smaller accelerating rings and collider rings), stability and uniformity (particularly for the final focus) and cost reduction (for the accelerating rings and collider rings). Hence, it is certainly not ruled out that such a parameter set could become achievable after a couple of decades of research and development dedicated to muon collider technology.

4 CONCLUSIONS

It has been shown that muon collider parameter sets at up to 10 TeV CoM energy may well be realistic by today's standards of technology while muon colliders at the 100 TeV energy scale require technological extrapolations that could perhaps be achievable within the relatively near-term future.

5 REFERENCES

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Table 1: Self-consistent parameter sets for muon colliders. The generation of these parameter sets is discussed in the text. Except for the first parameter set, which has been studied in some detail by the Muon Collider Collaboration, these parameters represent speculation by the author on how muon colliders might evolve with energy. The beam parameters at the interaction point are defined to be equal in the horizontal (x) and vertical (y) transverse coordinates.

center of mass energy, E_{CoM}	0.1 TeV	1 TeV	4 TeV	10 TeV	100 TeV
description	MCC para. set	LHC complement	E frontier	2^{nd} gen.	ult. E scale
collider physics parameters:					
luminosity, $\mathcal{L}$ [$cm^{-2}.s^{-1}$]	1.2×10^{32}	1.0×10^{34}	6.2×10^{33}	1.0×10^{36}	4.0×10^{36}
$\int \mathcal{L}dt$ [fb^{-1}/det/year]	1.2	100	62	10 000	40 000
No. of $\mu\mu \rightarrow ee$ events/det/year	10 000	8700	340	8700	350
No. of 100 GeV SM Higgs/det/year	1600	69 000	69 000	1.4×10^7	8.3×10^7
fract. CoM energy spread, σ_E/E [10^{-3}]	0.85	1.6	1.6	1.0	1.0
collider ring parameters:					
circumference, C [km]	0.3	2.0	7.0	15	100
ave. bending B field [T]	3.5	5.2	6.0	7.0	10.5
beam parameters:					
(μ^- or) μ^+/bunch,N_0[10^{12}]	4.0	3.5	3.1	2.4	0.18
(μ^- or) μ^+ bunch rep. rate, f_b [Hz]	15	15	0.67	15	60
6-dim. norm. emittance, ϵ_{6N}[$10^{-12}m^3$]	170	170	170	50	2
x,y emit. (unnorm.) [$\pi.\mu$m.mrad]	210	12	3.0	0.55	0.0041
x,y normalized emit. [π.mm.mrad]	99	57	57	26	1.9
fract. mom. spread, δ [10^{-3}]	1.2	2.3	2.3	1.4	1.4
relativistic γ factor, E_μ/m_μ	473	4732	18 929	47 322	473 220
ave. current [mA]	20	10	0.46	24	4.2
beam power [MW]	1.0	8.4	1.3	58	170
decay power into magnet liner [kW/m]	1.1	0.58	0.03	1.4	1.3
time to beam dump, t_D[$\gamma\tau_\mu$]	no dump	0.5	0.5	no dump	0.5
effective turns/bunch	519	493	563	1039	985
interaction point parameters:					
spot size, $\sigma_x = \sigma_y$[μm]	80	7.6	1.9	0.78	0.057
bunch length, σ_z [mm]	31	4.7	1.2	1.1	0.79
β^* [mm]	31	4.7	1.2	1.1	0.79
ang. divergence, σ_θ [mrad]	2.6	1.6	1.6	0.71	0.072
beam-beam tune disruption parameter, $\Delta\nu$	0.044	0.066	0.059	0.100	0.100
pinch enhancement factor, H_B	1.007	1.040	1.025	1.108	1.134
beamstrahlung fract. E loss/collision	2.1×10^{-14}	1.2×10^{-10}	2.3×10^{-8}	2.3×10^{-7}	3.2×10^{-6}
final focus lattice parameters:					
max. poletip field of quads., $B_{4\sigma}$ [T]	6	10	10	15	20
max. full aperture of quad., $A_{\pm 4\sigma}$[cm]	14	13	30	20	13
β_{max}[km]	1.5	22	450	1100	61 000
final focus demagnification, $\sqrt{\beta_{max}/\beta^*}$	220	2200	19 000	31 000	280 000
synchrotron radiation parameters:					
syn. E loss/turn [MeV]	0.0008	0.01	0.9	17	25 000
syn. rad. power [kW]	0.0002	0.13	0.4	400	110 000
syn. critical E [keV]	0.0006	0.09	1.6	12	1700
neutrino radiation parameters:					
collider reference depth, D[m]	10	125	300	300	300
ave. rad. dose in plane [mSv/yr]	3×10^{-5}	9×10^{-4}	9×10^{-4}	0.66	6.7
str. sect. length for 10x ave. rad., L_{x10}[m]	1.9	1.3	1.1	1.0	2.4
ν beam distance to surface [km]	11	40	62	62	62
ν beam radius at surface [m]	24	8.4	3.3	1.3	0.13