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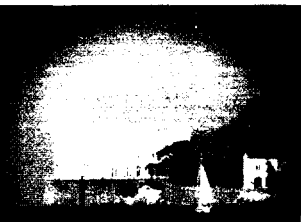

preprint

AN EFFECTIVE FIELD STUDY OF THE MAGNETIC PROPERTIES AND CRITICAL BEHAVIOUR AT THE SURFACE ISING FILM

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Abstract

The influence of corrugation and disorder at the surface on the critical behaviour of a ferromagnetic spin-1/2 Ising film is investigated using mean-field theory and finite cluster approximation. It is found that the critical surface exponent β_1 follows closely the one of a perfect surface, in the two cases: corrugated surface and random equiprobable coupling surface. However, in the case of flat surface with random interactions the surface critical exponent β_1 depends on the concentration p of the strong interaction for $p > p_c = 0.5$, while for $p \leq p_c$, such critical exponent is independent on the value of p and is equal to the one of the perfect surface. Moreover, in the case of corrugated surface, the effective exponent for a layer z, $\beta_{eff}(z,n)$, is calculated as a function of the number of steps at the surface.

I- Introduction:

The influence of the presence of a surface up on the bulk critical behaviour has been the focus of scientific interest during the last decade [1,2]. Since the free surface is the natural break of the translational symmetry in any real system, critical phenomena at perfect surfaces have attracted much interest. Furthermore, the phase diagram for the semi-infinite Ising model is well established [1,3]. Briefly, depending on the values of magnetic coupling constants near the surface region, two different behaviours at the surface can be demonstrated [4]: (i) the surface critical temperature is the same as of the bulk one when the coupling constant on the surface is smaller than a critical value ("ordinary transition"). (ii) The surface critical temperature may be higher than the corresponding bulk one when the surface coupling is larger than the critical value ("extraordinary transition"). At the critical ratio, one encounters the "special transition point", with critical properties of the surface transition deviating from those at the ordinary or the distinct surface transition. Moreover, surface critical exponents have been estimated, both theoretically and experimentally. The general agreement is quite satisfactory [1-3]. The critical behaviour of magnetic materials with variable strength of coupling on the surface is characterised by surface critical exponents independent on the bulk exponents. For their theoretical determination ε-expansion [3], series expansion [5], and Monte Carlo (MC) methods [6,7] were applied.

Furthermore, the magnetic properties depend on several geometric features, which depend on the conditions during the growth. Thus, the connection between experimental and theoretical results requires a detailed study of the dependence of physical quantities

on the geometry of the systems. Some specific properties have already been considered in theoretical works, such as the size and the lattice structure of small clusters [8,9], the thickness of the film [10] and the roughness of one dimensional structures [11], in particular, the role of surface imperfections, unavoidable in real materials, on the critical properties has been studied [12-15].

According to the Harris criterion [16], the relevance of the perturbation is connected to the sign of the specific heat exponent α in the pure system. Small amounts of disorder do not change the nature of the phase transition, if the corresponding pure system's specific heat exponent is negative. Currently, there appears to be widespread consensus that critical exponents of bond disordered (BD) systems in the weak disorder limit are the same as those of the pure system [17-20]. Monte Carlo (MC) simulations of BD systems show logarithmic corrections in the behaviour of the magnetizations which persist even in the strongly disordered regime [20,21]. However, in the 2D spin diluted (SD) Ising model a continuous variation of critical exponents with the spin density has been found [22]. A Monte Carlo study [23] shows that the specific heat does not diverge at the critical point, at least when the concentration of dilution is sufficiently large; the critical behaviour of the magnetic susceptibility and the correlation length are consistent with the pure power-law behaviour with the concentration dependent critical exponents. Moreover, using MC simulations, the site-diluted Ising model in two dimensions has been studied [24] where it is found that such model belongs to the same universality class as the pure one, in spite of the strong logarithmic effects. Besides, the surface critical behaviour of the three-dimensional disordered semi-infinite Ising ferromagnetic has been

studied, using MC simulations [25]; it is found that for J_S less than J_{S_c}, the corresponding surface critical exponent is equal to the surface exponent of perfect one.

The aim of this paper is to study the surface critical behaviour of a spin-1/2 Ising film. In the case of perfect and corrugated surfaces we use a mean-field theory (MFT); while in the case of diluted surface we use a finite cluster approximation [26,27] which is more appropriate than the mean field theory for random system. The paper is organised as follows. In section II, we define the model. The MFT and finite cluster approximation methods are explained in section III. Results and discussion are given in section IV, while section V is reserved for conclusion.

II-Model:

We consider a spin-1/2 Ising model, with ferromagnetic coupling on a three-dimensional cubical lattice consisting of d parallel square lattice layer at spacing to make a "film" of finite thickness, limited by two surfaces top and bottom. Fig. 1 represents a geometry of the model for, at the top of the film, (a) perfect surface, (b) diluted surface and (c) surface with n steps but perfect surface in the bottom.

If we consider only nearest-neighbour interaction, the system is governed by the Hamiltonian:

$$H = -\sum_{\langle ij \rangle} J_{ij} \left(S_{i,z} S_{j,z} + S_{i,z} S_{j,z+1} + S_{i,z} S_{j,z-1} \right) \qquad (1)$$

where $S_{i,z} = \pm 1$ is the spin variable of a site i of the layer z, d is the number of layers and J_{ij} are the exchange interaction between nearest neighbour sites which are equal to J_s when the sites are at the surface, J_1 when a site is being at surface and its neighbour in the bulk and J_b when the two sites are in the bulk.

III- Methods:

III-1 Mean-field theory (MFT):

Mean-field state equation, for the Ising model (1) may be obtained from the variational functional.

$$\begin{aligned} F[m(x,z)] = \sum_{x,z} \Big\{ T\Big[& \frac{1-m(x,z)}{2} \mathrm{Log}(1-m(x,z)) \\ & + \frac{1+m(x,z)}{2} \mathrm{Log}(1+m(x,z)) \Big] \\ & - \frac{1}{2} m(x,z) \Big[2\overline{J_1} m(x,z) + \overline{J_1} m(x+1,z) + \overline{J_1} m(x-1,z) \\ & + \overline{J_2} m(x,z-1) + \overline{J_3} m(x,z+1) \Big] \Big\} \end{aligned} \qquad (2)$$

The couplings $\overline{J_1}$, $\overline{J_2}$ and $\overline{J_3}$ are equal either to J_s or J_1 or J_b depending on the positions of the sites (x,z) and its neighbours.

With the free boundary conditions namely:

$$m(x,0) = m(x,d+1) = m(0,z) = m(j(L-1)+2,z) = 0 \qquad (3)$$

where n is the number of steps and $j \in [1, n+1]$.

At the global minimum of (2), the magnetisation per site in the layer z and at the position x is $\langle\sigma(x,z)\rangle = m^{min}(x,z)$, and the free energy of the system is $F = F\left[m^{min}(x,z)\right]$.

The stationary condition $\frac{\partial F}{\partial m(x,z)} = 0$ leads to mean-field state equations,

$$m(x,z) = \tanh\left[\frac{1}{T}\left(2\overline{J_1}m(x,z) + \overline{J_1}m(x+1,z) + \overline{J_1}m(x-1,z) + \overline{J_2}m(x,z-1) + \overline{J_3}m(x,z+1)\right)\right] \tag{4}$$

Equations (4) are solved numerically by iteration. Different initial guesses can lead to different solutions of the mean-field state equations. The one which makes the variational functional (2) smallest is selected as the global minimum.

III-2 Finite cluster method:

In order to study disordered systems, the finite cluster method with single site approximation [26,27] is more appropriate than the mean field theory.

This method, which still neglects correlation between different spins, but takes into account the correlations relation such as $\left\langle (S(x,z))^2 \right\rangle = 1$ exactly, where $\langle\bullet\rangle$ indicates the thermal average. Such a method, is based on a cluster comprising a single selected site labelled 0 and the neighbouring sites with which it directly interacts. Hence, the Hamiltonian containing the central site 0 is given by:

$$H_{0,z} = -\left(J_{0,z-1}S_{0,z-1} + J_{0,z+1}S_{0,z+1} + \sum_{j=1}^{N-2} J_{0,j}S_{j,z} \right) S_{0,z} \tag{5}$$

$J_{0,j}$ is the exchange coupling between the spin at site 0 and the spin at site j. N is the coordination number. N=6 is the simple cubic lattice case.

The starting point for the single-site cluster approximation is a set of formal identities of the type

$$\left\langle \left\langle S_{0,z} \right\rangle_c \right\rangle = \left\langle \frac{Tr_0 \, S_{0,z} \exp(-\beta H_{0,z})}{Tr_0 \exp(-\beta H_{0,z})} \right\rangle, \qquad (6)$$

where $\left\langle S_{0,z} \right\rangle_c$ denotes the average value of the spin 0 of the layer z for a given configuration c of the nearest-neighbouring spins. $\underset{0}{Tr}$ means the trace performed over $S_{0,z}$ only. $\beta = 1/K_B T$, T is the absolute temperature and K_B is the Boltzmann constant.

By calculating the inner trace in eq. (6), $\left\langle S_{0,z} \right\rangle_c$ is given by :

$$\left\langle S_{0,z} \right\rangle_c = \tanh\left[\frac{1}{K_B T} \left(J_{0,z-1} S_{0,z-1} + J_{0,z+1} S_{0,z+1} + \sum_{j=1}^{N-2} J_{0,j} S_{j,z} \right) \right] \qquad (7)$$

At the surface, the coupling J_S takes two random, ferromagnetic interactions, being either strong J_{S_1}, or weak J_{S_2}, i.e. $J_{S_1} > J_{S_2}$. Note that, random surface fields using Monte Carlo (MC) techniques [28] had been considered.

The two couplings J_{S_1} and J_{S_2} are independent random variables distributed according to the following law:

$$P(J_S) = p\delta(J_S - J_{S_1}) + (1-p)\delta(J_S - J_{S_2}) \tag{8}$$

Then the ratio $R_d = J_{S_2} / J_{S_1}$ measures the degree of dilution: the particular case $R_d = 1$ corresponds to the perfect case.

The magnetisation m_z of the layer z is given by performing the average $\langle \bullet \rangle_D$ over the disorder of the exchange coupling at the surface, and the average $\langle \bullet \rangle$ over the spin configurations, namely

$$m_z = \left\langle \left\langle \left\langle S_{0,z} \right\rangle_c \right\rangle \right\rangle_D \tag{9}$$

with

$$\left\langle \left\langle S_{0,z} \right\rangle_c \right\rangle_D = \int \left\langle S_{0,z} \right\rangle_c \prod_{j=1}^{N-2} P(J_{0,j}) dJ_{0,j} dJ_{o,z-1} dJ_{0,z+1} \tag{10}$$

To develop this expression we use the expansion technique for cluster identities of a spin-1/2 Ising system [29,30].

The magnetisation of the layer z is given by:

At the surface,

$$m_s = \frac{1}{2^{N-2}} \sum_{k_3=0}^{N-2} C_{k_3}^{N-2} \sum_{p_3=0}^{N-2} \sum_{p_2=0}^{1} U_{p_2 p_3 k_3} (m_S)^{p_3} (m_1)^{p_2} C_{p_3}^{N-2} \qquad (11)$$

For z=1,

$$m_1 = \sum_{p_2=0}^{N-2} \sum_{p_1=0}^{1} \sum_{p_3=0}^{1} A_{p_2 p_1 p_3} (m_1)^{p_2} (m_S)^{p_1} (m_2)^{p_3} C_{p_2}^{N-2} \qquad (12)$$

For z=d,

$$m_d = \sum_{p_2=0}^{N-2} \sum_{p_3=0}^{1} B_{p_2 p_1} (m_d)^{p_2} (m_{d-1})^{p_3} C_{p_2}^{N-2} \qquad (13)$$

and for z=2,$\cdots$,d-1

$$m_z = \sum_{p_2=0}^{N-2} \sum_{p_1=0}^{1} \sum_{p_3=0}^{1} V_{p_2 p_1 p_3} (m_z)^{p_2} (m_{z-1})^{p_1} (m_{z+1})^{p_3} C_{p_2}^{N-2} \qquad (14)$$

with

$$\begin{aligned} U_{p_2 p_3 k_3} = \frac{1}{2^{N-1} C_{p_3}^{N-2}} & \sum_{i_2=0}^{1} \sum_{i_3=0}^{k_3} \sum_{j_3=0}^{N-2-k_3} \sum_{\mu_2=0}^{i_2} \sum_{\mu_3=0}^{i_3} \sum_{\upsilon_3=0}^{j_3} (-1)^{\mu_2+\mu_3+\upsilon_3} \\ & \times C_{\mu_2}^{i_2} C_{p_2-\mu_2}^{1-i_2} C_{i_3}^{k_3} C_{j_3}^{N-2-k_3} C_{\mu_3}^{i_3} C_{\upsilon_3}^{j_3} C_{p_3-\mu_3-\upsilon_3}^{N-2-i_3-j_3} \\ & \times \tanh[\beta(J_{S_1}(k_3-2i_3)+J_{S_2}(N-2-k_3-2j_3)+J_1(1-2i_2))] \end{aligned} \qquad (15)$$

$$A_{p_2p_1p_3} = \frac{1}{2^{N-1}C_{p_2}^{N-2}} \sum_{i_2=0}^{N-2} \sum_{i_3=0}^{1} \sum_{i_1=0}^{1} \sum_{\mu_2=0}^{i_2} \sum_{\mu_1=0}^{i_1} \sum_{\mu_3=0}^{i_3} (-1)^{\mu_1+\mu_2+\mu_3}$$
$$\times C_{i_2}^{N-2} C_{\mu_2}^{i_2} C_{p_2-\mu_2}^{N-2-i_2} C_{\mu_1}^{i_1} C_{p_1-\mu_1}^{1-i_1} C_{\mu_3}^{i_3} C_{p_3-\mu_3}^{1-i_3} \qquad (16)$$
$$\times \tanh[\beta(J_1(1-2i_1) + J_b(N-1-2(i_2+i_3)))]$$

$$B_{p_2p_1} = \frac{1}{2^{N-1}C_{p_2}^{N-2}} \sum_{i_2=0}^{N-2} \sum_{i_3=0}^{1} \sum_{\mu_2=0}^{i_2} \sum_{\mu_1=0}^{i_1} C_{i_2}^{N-2} C_{\mu_2}^{i_2} C_{p_2-\mu_2}^{N-2-i_2} C_{p_1-\mu_1}^{1-i_1} \qquad (17)$$
$$\times \tanh[\beta(J_b(N-1-2(i_1+i_2)))]$$

and

$$V_{p_2p_1p_3} = \frac{1}{2^{N-1}C_{p_2}^{N-2}} \sum_{i_2=0}^{N-2} \sum_{i_3=0}^{1} \sum_{i_1=0}^{1} \sum_{\mu_2=0}^{i_2} \sum_{\mu_1=0}^{i_1} \sum_{\mu_3=0}^{i_3} (-1)^{\mu_1+\mu_2+\mu_3}$$
$$\times C_{i_2}^{N-2} C_{\mu_2}^{i_2} C_{p_2-\mu_2}^{N-2-i_2} C_{\mu_1}^{i_1} C_{p_1-\mu_1}^{1-i_1} C_{\mu_3}^{i_3} C_{p_3-\mu_3}^{1-i_3} \qquad (18)$$
$$\times \tanh[\beta(J_b(N-2-2(i_1+i_2+i_3)))]$$

IV- Results and discussion:

IV-1 Case of perfect surface:

Using the mean-field theory, the surface critical behaviour at the ordinary transition, (i.e. $K_BT/J_b \leq K_BT_c/J_b = 6.0$) is investigated.

Typical magnetisation profiles are depicted in Fig. 2a, at $J_S/J_b = 1$ and for several values of $T < T_c$. The magnetisation increases from its surface value to the bulk one.

Moreover, the dependence of the surface magnetisation on the position x is also calculated, it is found that the magnetisation m(x) at the surface decreases when one moves from the middle site of the surface to the boundaries. At fixed temperature T less than T_c, Fig. 2b, shows the dependence of the magnetisation m(x,z) on the position x for different values of z. Moreover, for any position x, the magnetisation decreases when we move from the bulk to the surfaces. Qualitatively, these results are in good agreement with MC simulations [14].

At the vicinity of the critical temperature T_c, the spontaneous magnetisations vanish with a power law given by,

$$m(z,t) \propto t^{\beta_{eff}(z,t)} \tag{19}$$

Then the "effective exponent" $\beta_{eff}(z,t)$ can be defined as:

$$\beta_{eff}(z,t) = d\ln(m(z))/d\ln(t) \tag{20}$$

where $t = \left|T - T_c\right| / T_c$ is the reduced temperature and m(z,t) is the average value of m(x,z,t) on x.

Asymptotically, for sufficiently small values of t, $\beta_{eff}(z,t)$ approaches the asymptotic critical exponent $\beta(z)$. Here we are more interested in the critical properties of the surface so we note $\beta_1 = \beta(z=1)$.

$\beta_{eff}(z,t)$ can be approximated at discrete reduced temperatures t_i and t_{i+1} as:

$$\beta_{eff}(z,t) = \ln(m(z,t_i)/m(z,t_{i+1}))/\ln(t_i/t_{i+1}) \qquad (21)$$

From Fig. 3, it is clear that the effective exponent of the surface $\beta_{eff}(z=1,t)$ increases linearly with the temperature over a wide range of temperatures. In the limit of small t, the asymptotic exponent for the surface is $\beta_1 = 1$, which is in good agreement with the mean-field result of Ref. 9. Furthermore, when we move from the surface into the bulk the effective exponent is usually lowered. Away from criticality T_c and sufficiently deep in the bulk (e.g. z=10), the effective exponent follows closely the behaviour of the effective bulk exponent namely $\beta_b = 1/2$, but when we approach T_c, β_{eff} cross over to its surface critical value.

IV-2 Surface with random exchange interactions:

It is well known that the mean-field theory is inadequate to study disordered systems, so we use the finite cluster method with single site approximation [26,27] in order to study the critical properties of a surface with random interactions. Note that the ratio $R_d = J_{S_2}/J_{S_1}$ measures the degree of dilution at the surface and that the coupling J_{S_1} occurs with a probability p and J_{S_2} with 1-p . When p=0.5, we give in Fig. 4a the dependence of the surface magnetisation on temperature. It is found that for T less than T_c, the dilution has for effect to suppress the magnetisation m_S, and that the transition temperature is independent of the value of R_d, while a special transition depends on the

value of R_d. Moreover, the asymptotic critical exponent β_1 (Fig. 5) follows very closely that of the perfect surface case, leading to $\beta_1 \approx 0.98$ which is better than the one obtained with MFT ($\beta_1 \approx 1$). Furthermore, Fig. 6, gives the surface magnetisation dependence on temperature T for several values of concentration p and for a fixed value of R_d. It is clear that for p less than a critical value p_c=0.5, the transition temperature is independent on the values of p and is equal to the bulk temperature T_c^b; however, for p greater than p_c, the critical temperature increases with increasing the chance of occurring stronger coupling J_{S_1}. Accordingly, from Fig. 7, when the concentration p is higher than the critical value p_c, the critical surface exponent β_1 increases with increasing the probability p. Moreover, far from the critical temperature, the effective exponent $\beta_{eff}(z=1,t)$ decreases with increasing the concentration p i.e. when we increase the chance to have a strong coupling in the surface.

IV-3 Surface with steps:

Using the mean-field theory, we study the effects of corrugation on critical properties of the system. In particular, on surface critical phenomena, we consider the system of Fig. 1c with n steps. Then our aim is to study the effect of the number of steps n, on the critical properties of the surfaces and of the layers near the surface. In Fig. 8, the magnetisation m(x,z) is depicted at a fixed temperature in the ordinary transition for several layers, starting at the top by the surface and moving in the bulk. For a fixed layer, the magnetisation decreases each time the system presents a step, but far away from the

surface (e.g. z=10) the magnetisation keeps a constant value except at the boundaries where it decreases due to the finite size of the system in the x-direction.

In order to examine the impact of the number of steps n ($n \neq 0$) on the dependence of the effective exponent on the layer z, we plot in Fig. 9a $\beta_{eff}(z,n)$ as a function of z for a fixed value of temperature. It is clear that at the surface the effective exponent $\beta_{eff}(z,t)$ remains independent on the number of steps. Furthermore, for any layer z with $n_1 < z < n_2$ such as $n_1 < n_2$, the effective exponent depends strongly on the number of steps, while far away from the range $[n_1, n_2]$, the effective exponent $\beta_{eff}(z,t)$ is insensitive to the steps.

Furthermore, for a fixed number of steps, the effective exponent $\beta_{eff}(z,t)$ of each layer z is given in Fig. 9b for several values of temperatures. Far from the transition temperature, $\beta_{eff}(z,t)$ decreases with increasing z (i.e. when we move from the surface), presents a crossover at z=n, continues to decrease as far as become constant in the depth of the system and increases again in the vicinity of the bottom surface. If we increase the temperature T to approach T_c , the crossover becomes less pronounced. Obviously, as the perfect and diluted cases, $\beta_{eff}(z,t)$ increases with temperature. For a fixed number of steps, Fig. 10 shows the dependence of the effective exponent $\beta_{eff}(z,t)$ on the reduced temperature. Near the critical temperature, β_1 is insensitive to the corrugation in agreement with MC results [15]. To examine the effect of the number of steps on the behaviour of $\beta_{eff}(z,t)$ for a given layer and at a fixed temperature, in the vicinity of the

ordinary transition, Fig. 11 shows the dependence of $\beta_{eff}(z,t)$ on the number of steps. For a fixed layer z, when the number of steps n is sufficiently smaller than the value of z, $\beta_{eff}(z,n)$ is insensitive of the number of steps, when n increases more, $\beta_{eff}(z,n)$ increases and once n overtakes the value of z, then $\beta_{eff}(z,n)$ becomes independent on the number of steps i.e. after a certain degree of corrugation, $\beta_{eff}(z,n)$ will be insensitive to the number of steps and then takes a saturation value, this value varies with varying z.

V- Conclusion:

Using the mean-field theory and the finite cluster method with single site approximation, we have studied the surface critical behaviour of a spin-1/2 Ising film. For a surface with equiprobable random couplings or corrugated surface, our results provide no evidence for critical surface depending neither on the degree of dilution nor on the degree of corrugation. It is found that the surface magnetisation follows a power-law behaviour with the same values of the exponent as the perfect case. Furthermore, the mean-field approximation gives a higher value of the surface critical exponent ($\beta_1 \approx 1$) than the finite cluster approximation ($\beta_1 \approx 0.98$) and the Monte Carlo simulations ($\beta_1 \approx 0.80$) [15]. Nevertheless, for a surface with random coupling and for p greater than a critical value p_c, the transition temperature and the critical exponent of the surface depend strongly on the concentration p of the strong coupling. Besides, it is found that the degree of corrugation has a strong effect on the exponents of layers inside the system.

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References:

[1] K. Binder, in: Phase Transitions and Critical Phenomena, edited by C. Domb and J. L. Lebowitz (Academic Press, London, 1983) Vol. **8**, p. 1.

[2] H. Dosh, Critical Phenomena at Surfaces and Interfaces, Springer tracts in Modern Physics (Springer, Heidelberg, 1992), Vol. **126** H. Dosh, Appl. Phys. A **61**, 475 (1995).

[3] H. W. Diehl, in: Phase Transitions and Critical Phenomena, Vol. **10**, edited by Domb, C. and. J. L. Lebowitz, (London: Academic Press, 1986).

[4]- Q. Hong, Phys. Rev. B.**41**, 9621 (1990).

[5]- K. Binder and P. C. Hohenberg, Phys. Rev. B **6**, 3461 (1972); **9**, 2194 (1974).

[6]- D. P. Landau and K. Binder, Phys. Rev. B **41**, 4633 (1990).

[7]- M. Kikushi and Y. Okabe, Prog. Theor. Phys. **73**: 32 (1985).

[8]- J. Merikoski, J. Timonen, M. Manninen and P. Jena, Phys. Rev. Lett. **66**, 938 (1991).

[9]- B. V. Reddy and S. N. Khamma, Phys. Rev. B **45**, 10103 (1992).

[10]- P. Schilbe, S. Sienbentritt and K. H. Rieder, Phys. Lett. A **216**, 20 (1996).

[11]- F. D. A. Aarao Reis, Phys. Rev. B **55**, 11084 (1997).

[12]- T. Kaneyoshi, Introduction to Surface Magnetism (Boca Raton, Ann Arbor, Boston: CRC Press, 1991).

[13]- H. W. Diehl, A. Nusser, Z. Phys. B **79**, 69 (1990).

[14]- W. Selk, F. Szalma, P. Lajko and F. Igloi, J. Stat. Phys., Vol. **89**, 1079 (1997).

[15]- M. Pleimling and W. Selk, Eur. Phys. J. B **1**, 385 (1998).

[16]- A. B. Harris, J. Phys. C **7**, 1671 (1974).

[17]- Vik. S. Dotsenko and Vl. S. Dotsenko, Adv. Phys. **32**, 129 (1983).

[18]- R. Shankar, Phys. Rev. Lett. **58**, 2466 (1987).

[19]- A. W. W. Luding, Phys. Rev. Lett. **61**, 2388 (1988).

[20]- B. N. shalaev, Sov. Phys. Solid State **26**, 1811 (1984).

[21]- V. B. Andreichenko, Vl. S. Dotsenko, W. Selke and J-S Wang, Nucl. Phys. B **344**, 531 (1990).

[22]- R. Kuhn, Phys. Rev. Lett. **73**, 2268 (1994).

[23]- J. K. Kim and A. Patrascioiu, Phys. Rev. Lett. **72**, 2785 (1994).

[24]- H. G.Ballesteros, L. A. Fernandez, V. Martin-Mayor, A. Munoz Sudupe, G. Parisi and J. J. ruiz Lorenzo, J. Phys. A: Math. Gen. 30, 8379 (1997).

[25]- H. W. Diehl, Eur. Phys. J. B **1**, 401 (1998).

[26]- N. Boccara, Phys. Lett. **94** A, 185 (1983).

[27]- A. Benyoussef and N. Boccara, J. Phys. C **16**, 1143 (1983).

[28]- K. K. Mon, M. P. Nightingale, Phys. Rev. B **37**, 3815 (1988).

[29]- P. Tomczak, E. F Sarmento, A. F. Siqueira and A. R. Ferchemin, Phys. Stat. Sol. B **142**, 551 (1987).

[30]- A. Benyoussef and H. Ez- Zahraouy, Phys. Rev. B **52**, 4245 (1995).

Figure captions

Fig. 1: Geometry and interactions of the model for (a) perfect surface, (b) diluted surface and (c) surface with steps. There are d layers in the z direction. In (a) and (b) each layer contains L sites, while in (c) each layer z contains $z(L+1)+1$ sites if z is less than n+1 and $n(L+1)+1$ sites otherwise. The system is infinite in the direction y.

Fig. 2: Magnetisation profiles for the perfect surface with $J_l = J_S = J_b$ and L=60, d=20. The number accompanying each curve denotes the value of (a) temperature and (b) the value of the layer z.

Fig. 3: The dependence of the effective exponent as a function of reduced temperature t in the perfect case, for $J_l = J_S = J_b$ and L=60, d=20,. The number accompanying each curve denotes the value of the layer z.

Fig. 4: Surface magnetisation profiles for the diluted case with $J_l = J_S = J_b$, p=0.5 and L=60, d=20. The number accompanying each curve denotes the value of R_d.

Fig. 5: The dependence of the effective exponent as a function of reduced temperature t for $J_l = J_S = J_b$, L=60, d=20 and p=0.5. The number accompanying each curve denotes the value of R_d.

Fig. 6: Surface magnetisation versus temperature, for the diluted case with $R_d = 1/10$ and L=60, d=20. The number accompanying each curve denotes the value of the concentration p.

Fig. 7: The dependence of the surface effective exponent in the case of a diluted surface with L=60, d=20 and $R_d = 1/10$. The number accompanying each curve denotes the value of the concentration p.

Fig. 8: Profiles of the magnetisation in the case of corrugated surface, for n=4, $J_1 = J_S = J_b$, T=0.95 T_c, L= 15 and d=60. The number accompanying each curve denotes the value of the layer z.

Fig. 9: The dependence of the effective exponent as a function of layer position z, in the case of corrugated surface, for L=31, d=60, n=8 and $J_1 = J_S = J_b$. The number accompanying each curve denotes, in (a) number of steps n and in (b) the value of temperature.

Fig. 10: The dependence of the effective exponent as a function of reduced temperature t, for L=31, d=60, n=4 and with $J_1 = J_S = J_b$. The number accompanying each curve denotes the value of layer position z.

Fig. 11: The dependence of the effective exponent as a function of the number of steps n for $J_1 = J_S = J_b$, T=0.90 T_c, L= 31 and d=60. The number accompanying each curve denotes the value of layer position z.

Fig.1

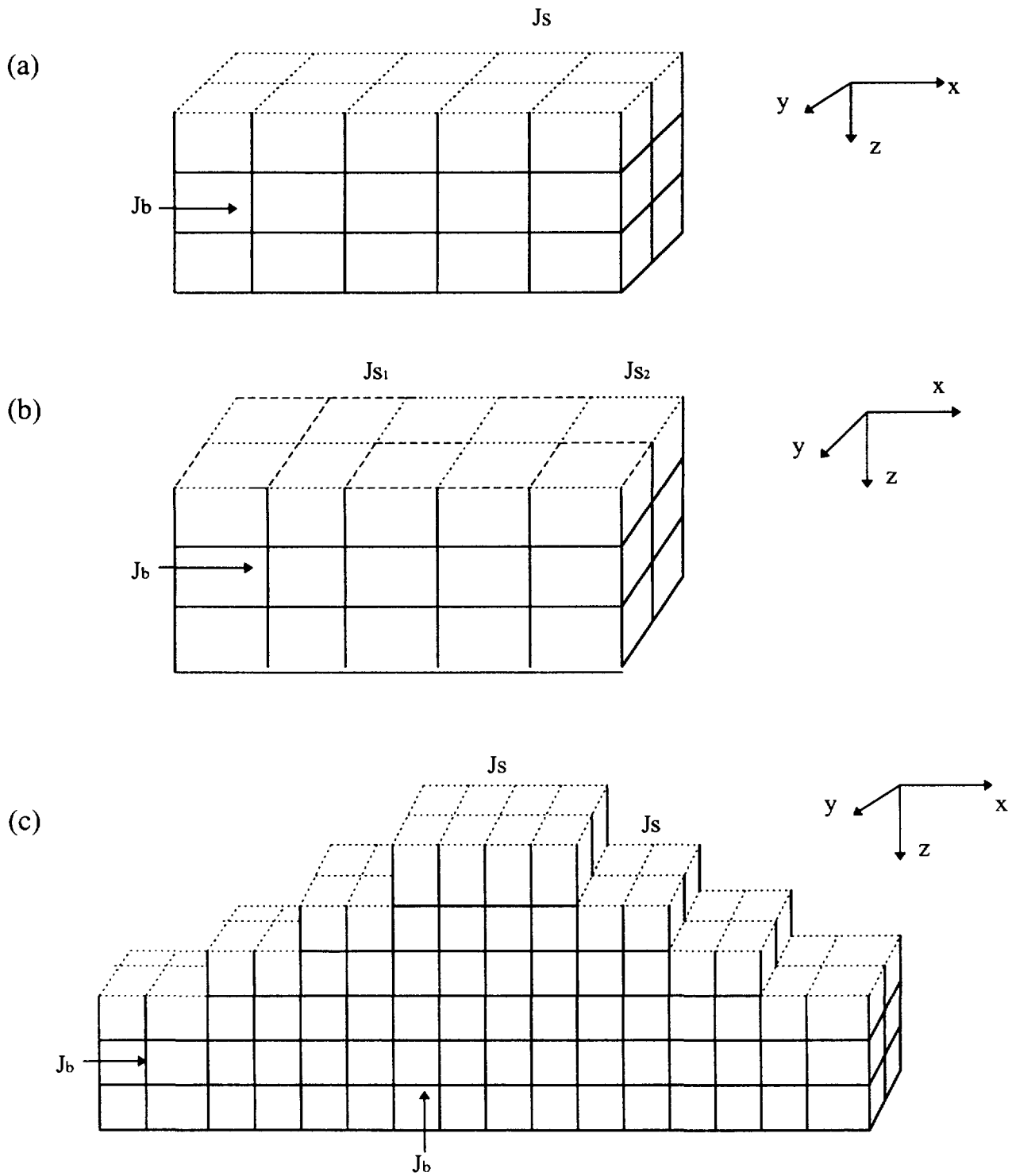
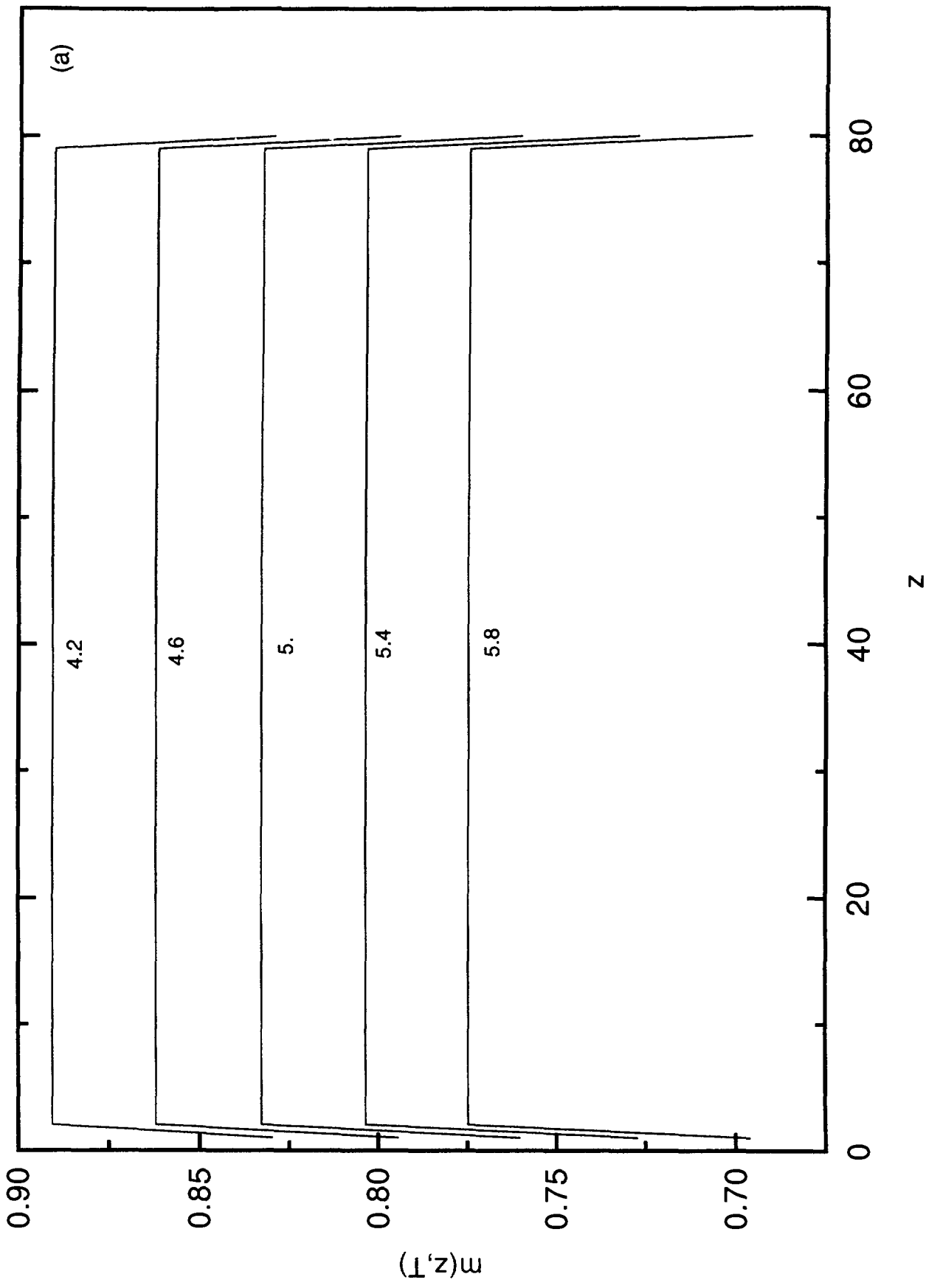


Fig. 2



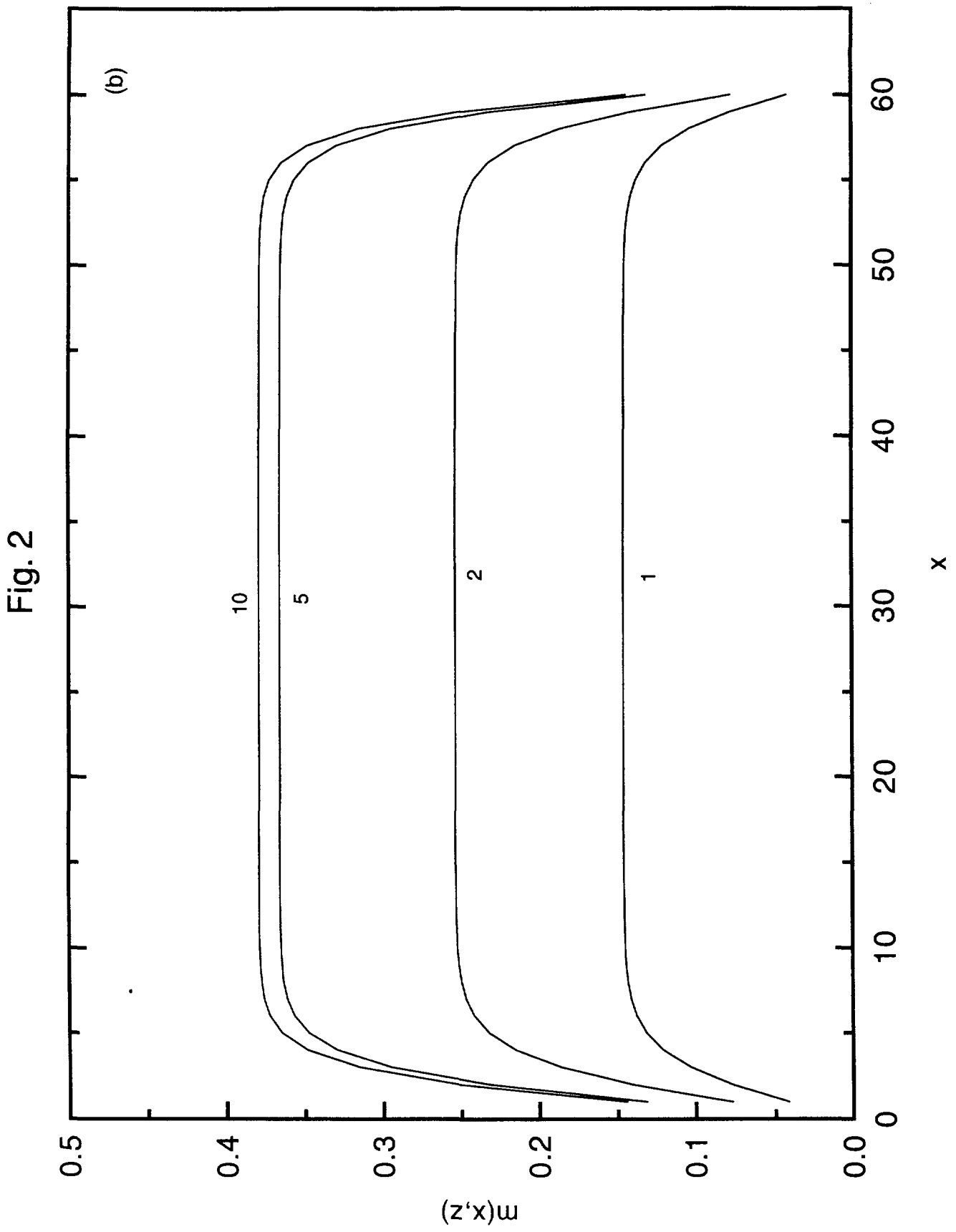


Fig. 2

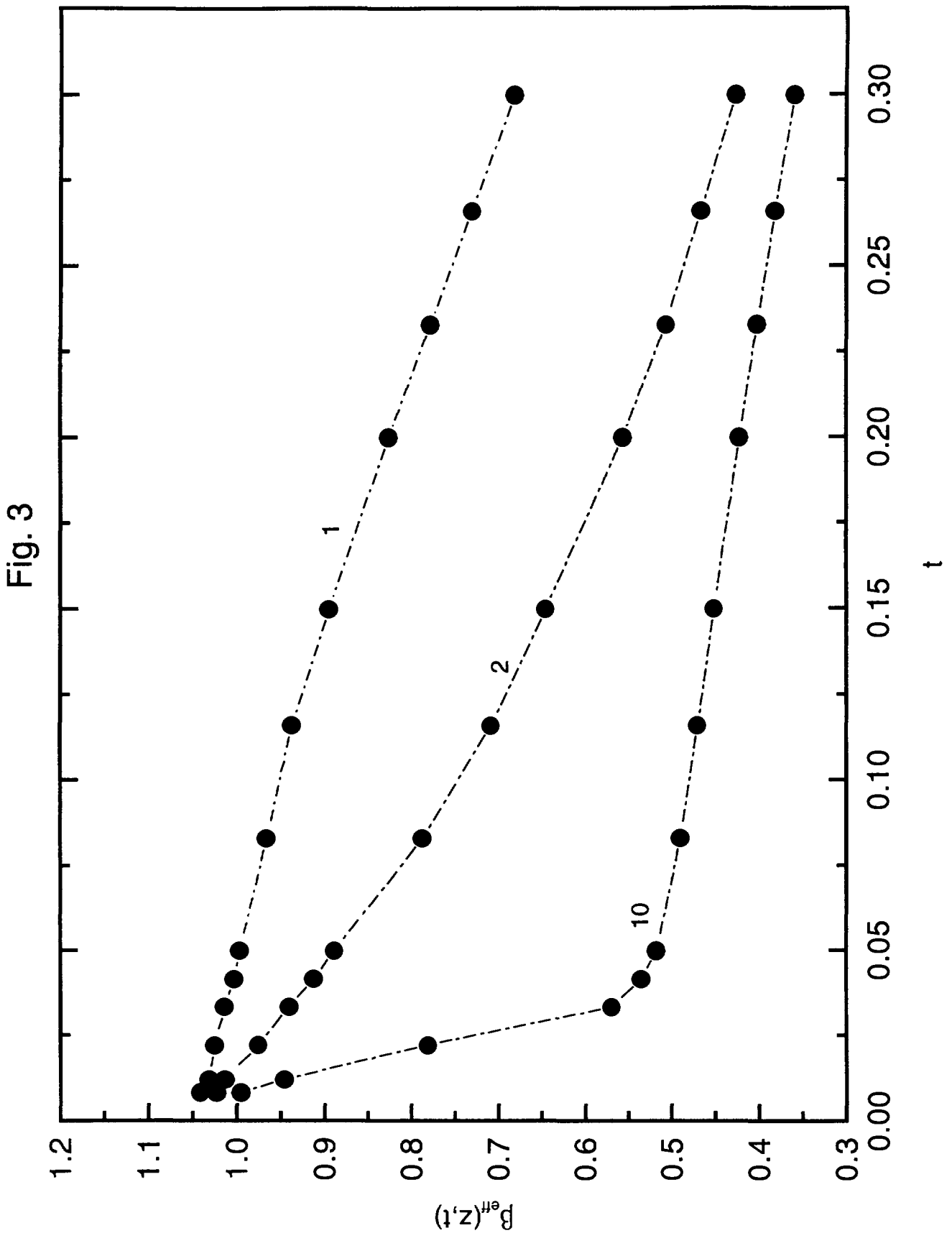


Fig. 3
$\beta_{eff}(z,t)$
t
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.00
0.05
0.10
0.15
0.20
0.25
0.30
1
2
10

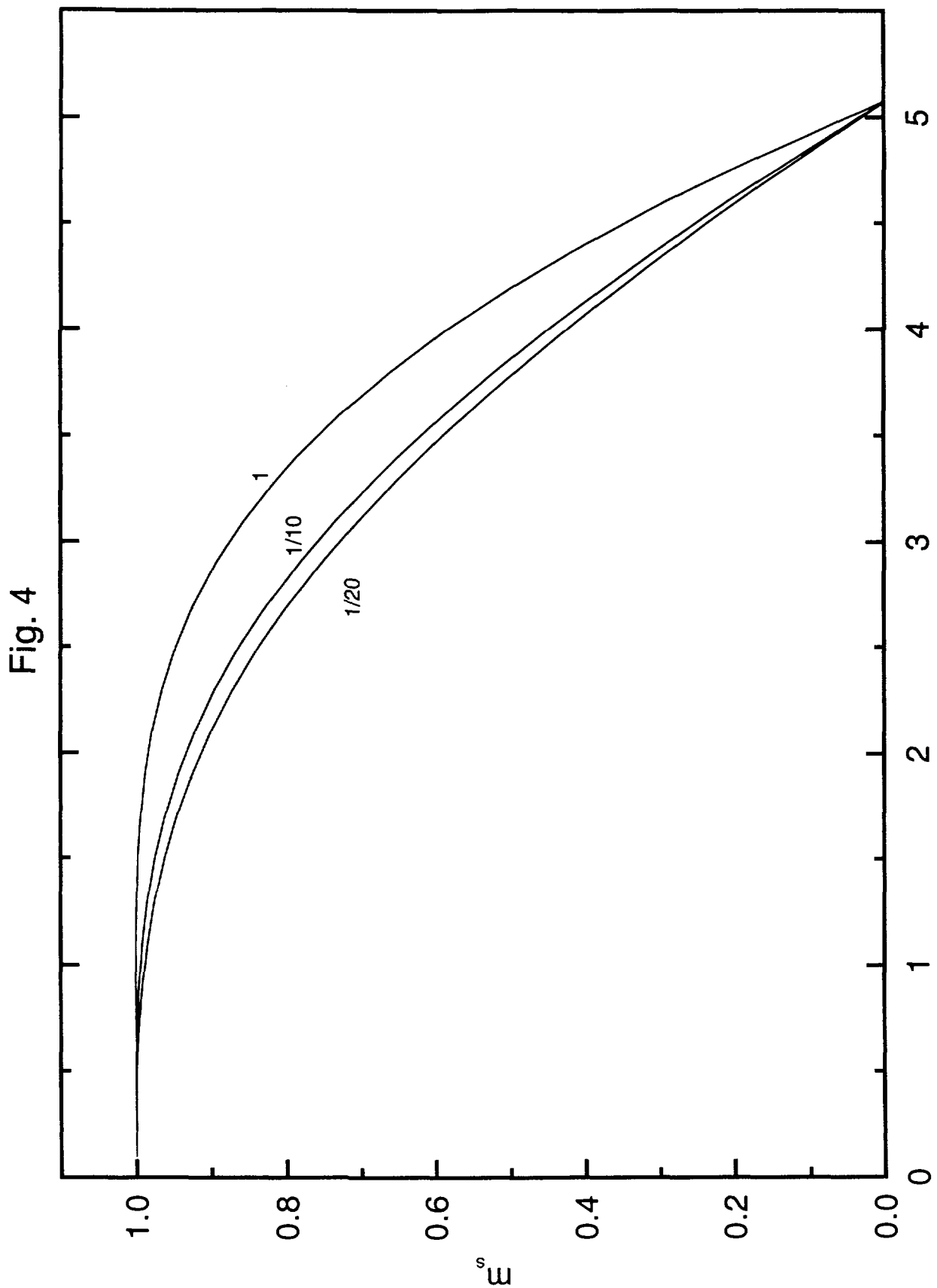


Fig. 4
1
1/10
1/20
1.0
0.8
0.6
0.4
0.2
0.0
m_s
0
1
2
3
4
5
K_BT/J_b

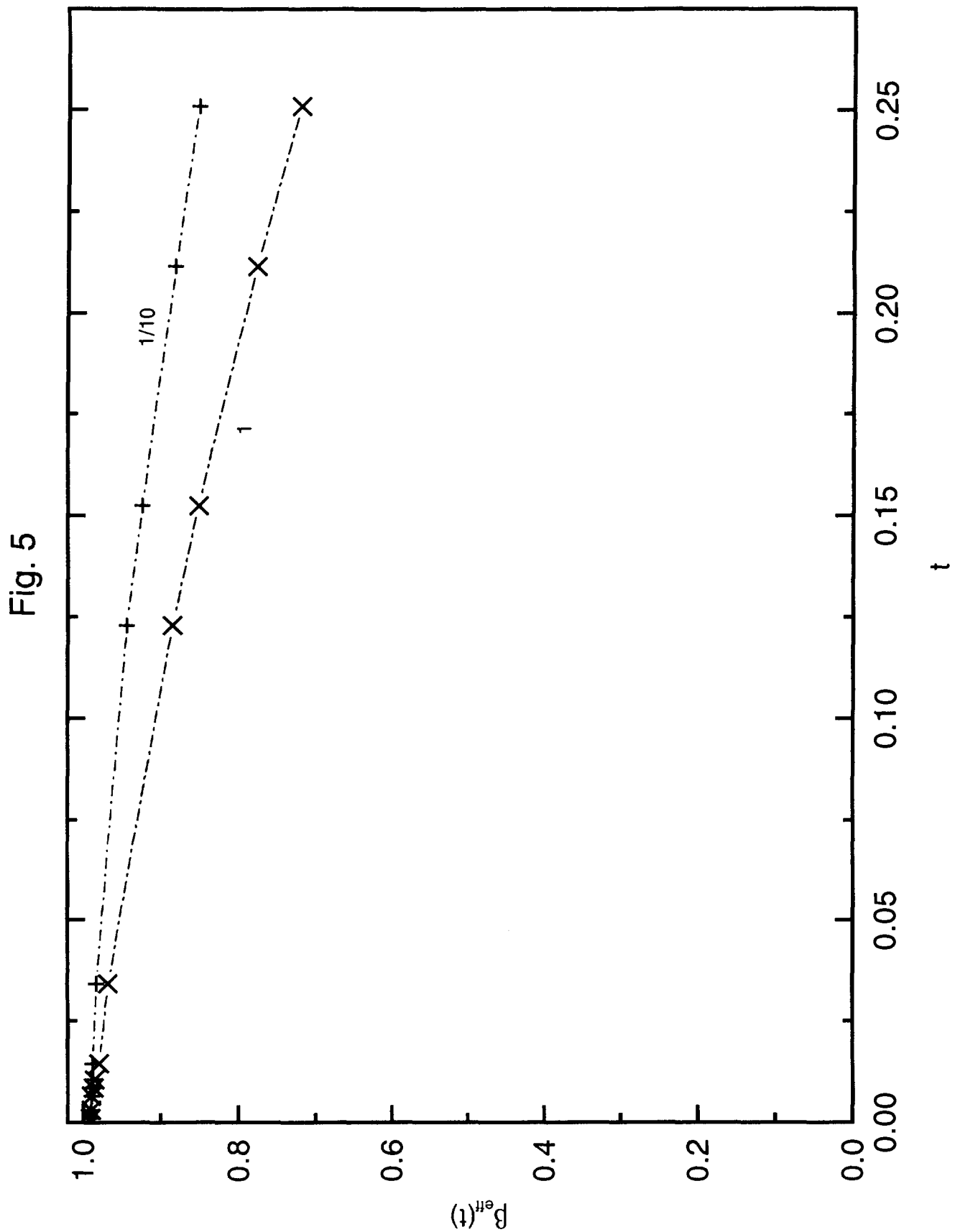


Fig. 5
1/10
1
$\beta_{eff}(t)$
t
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
0.25

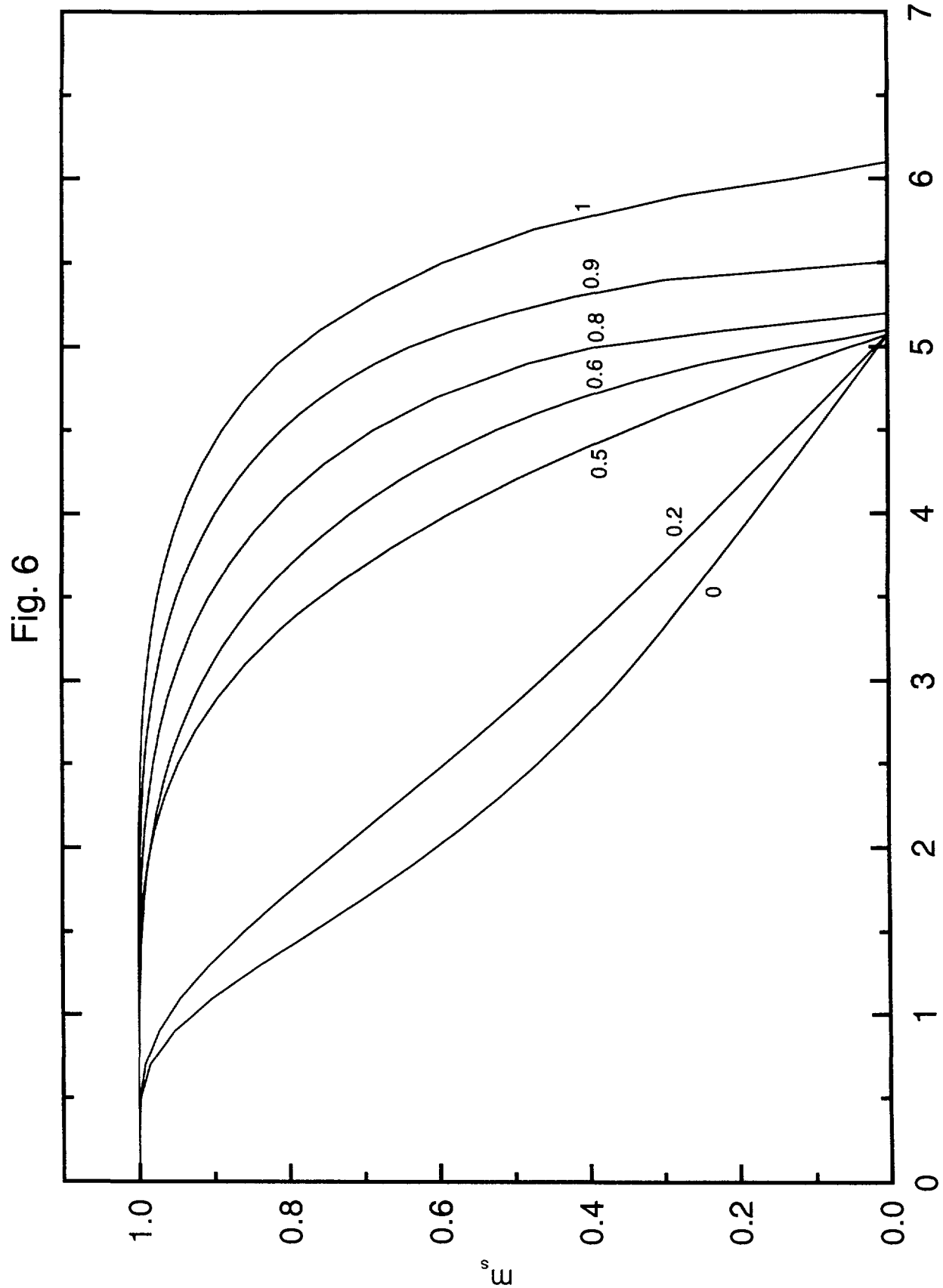


Fig. 6
1.0
0.8
0.6
0.4
0.2
0.0
m_s
0
1
2
3
4
5
6
7
K_BT/J_b
1
0.9
0.8
0.6
0.5
0.2
0

Fig. 7

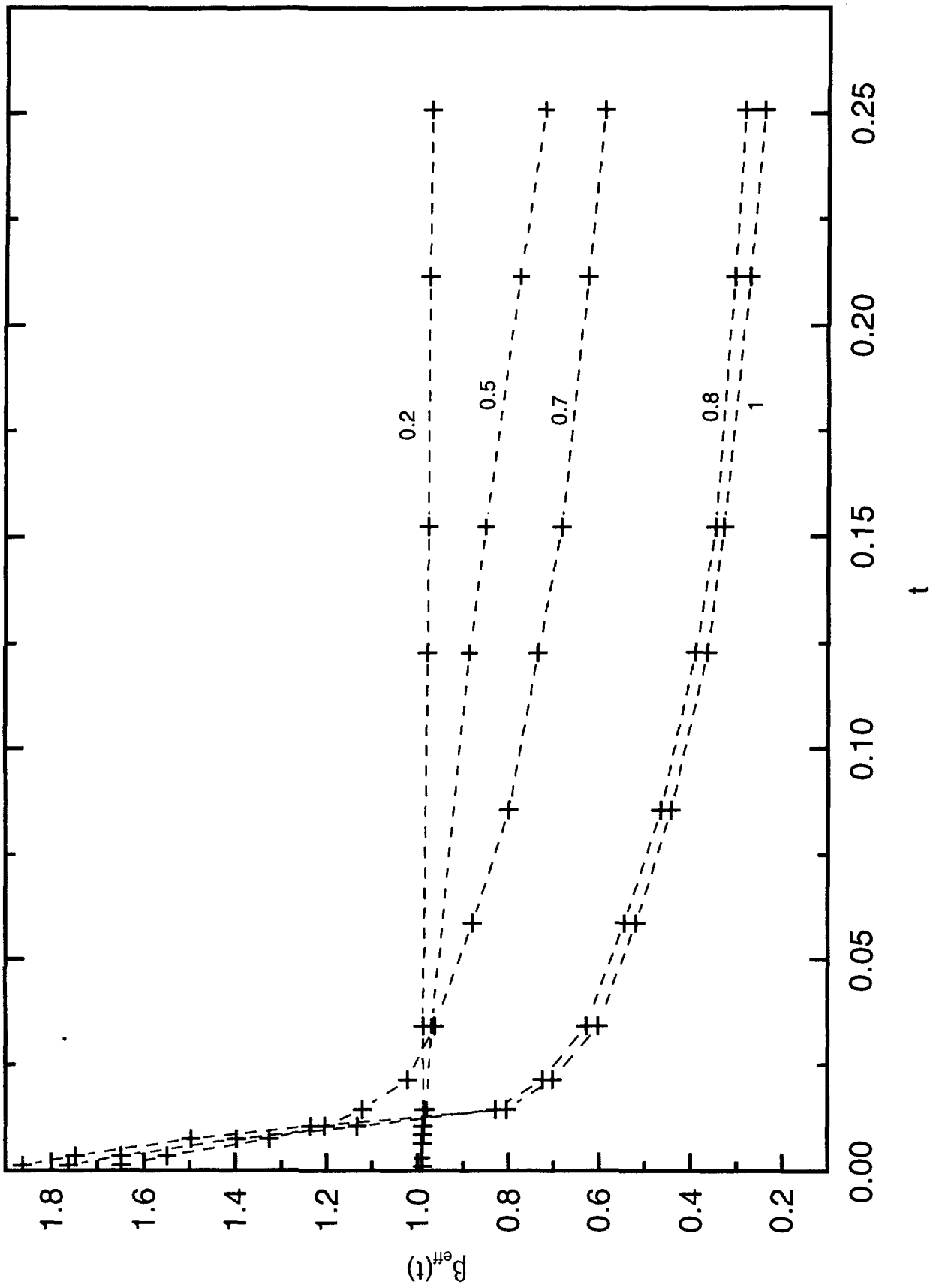
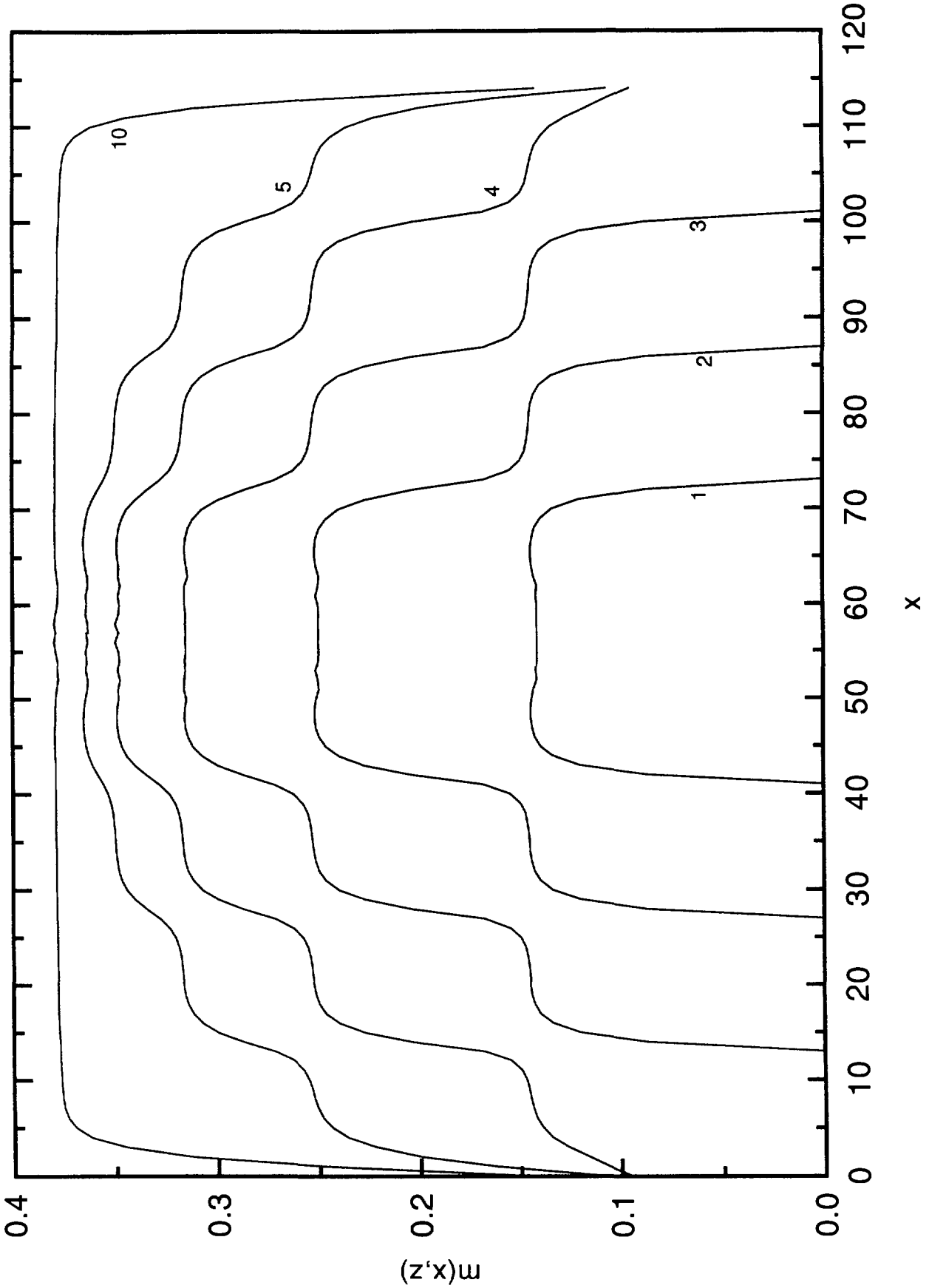

$\beta_{eff}(t)$
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.00
0.05
0.10
0.15
0.20
0.25
t
0.2
0.5
0.7
0.8
1

Fig. 8



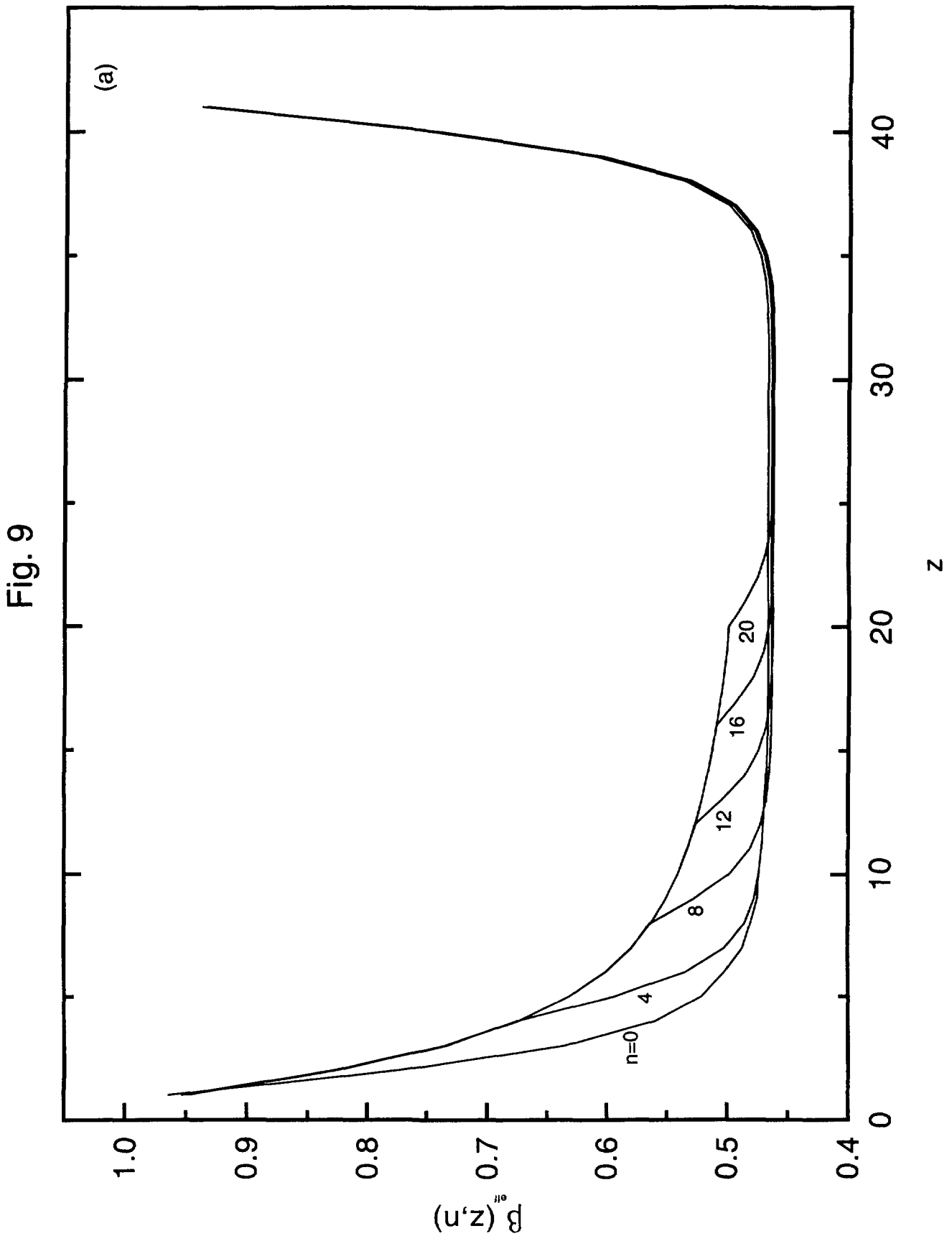


Fig. 9

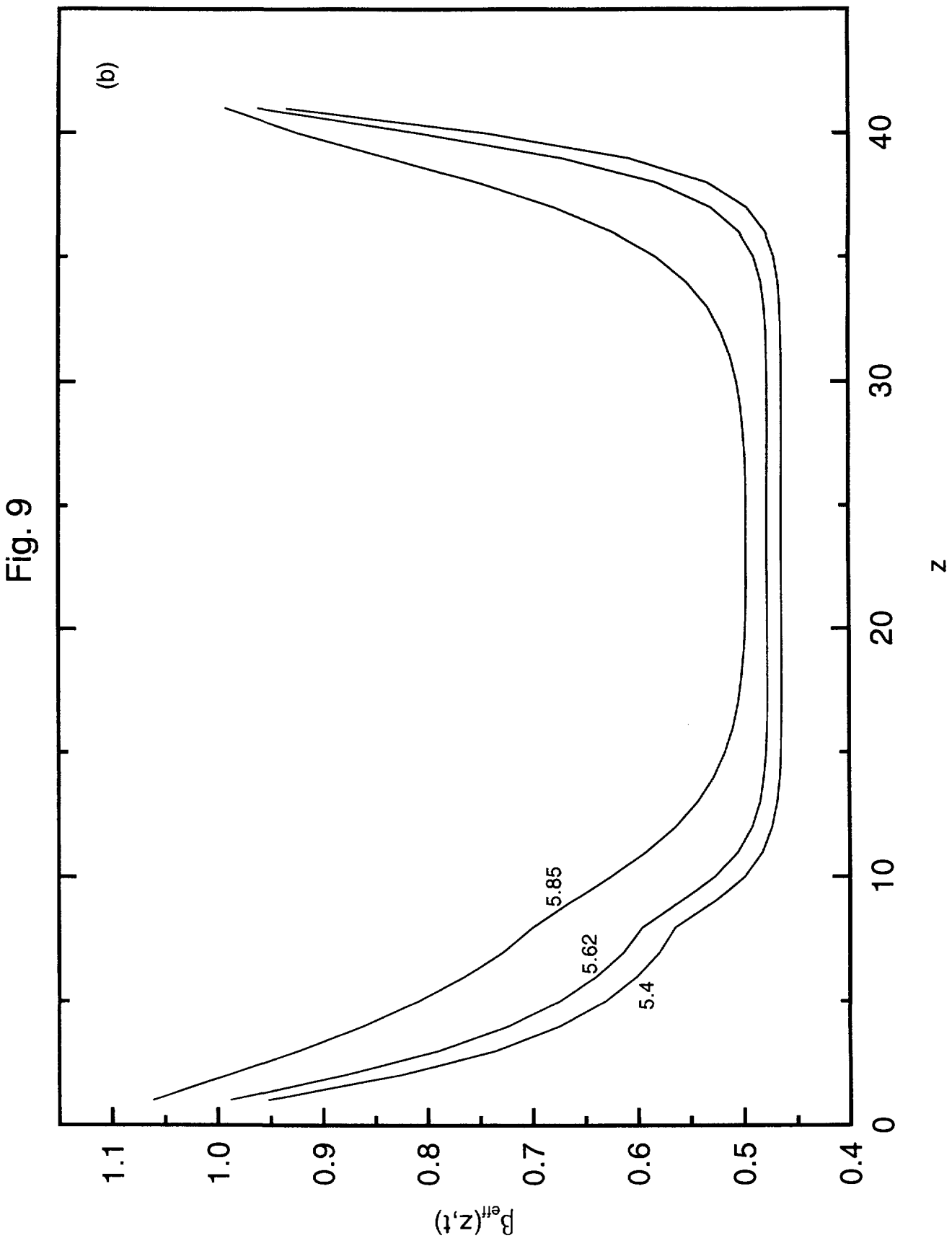


Fig. 9

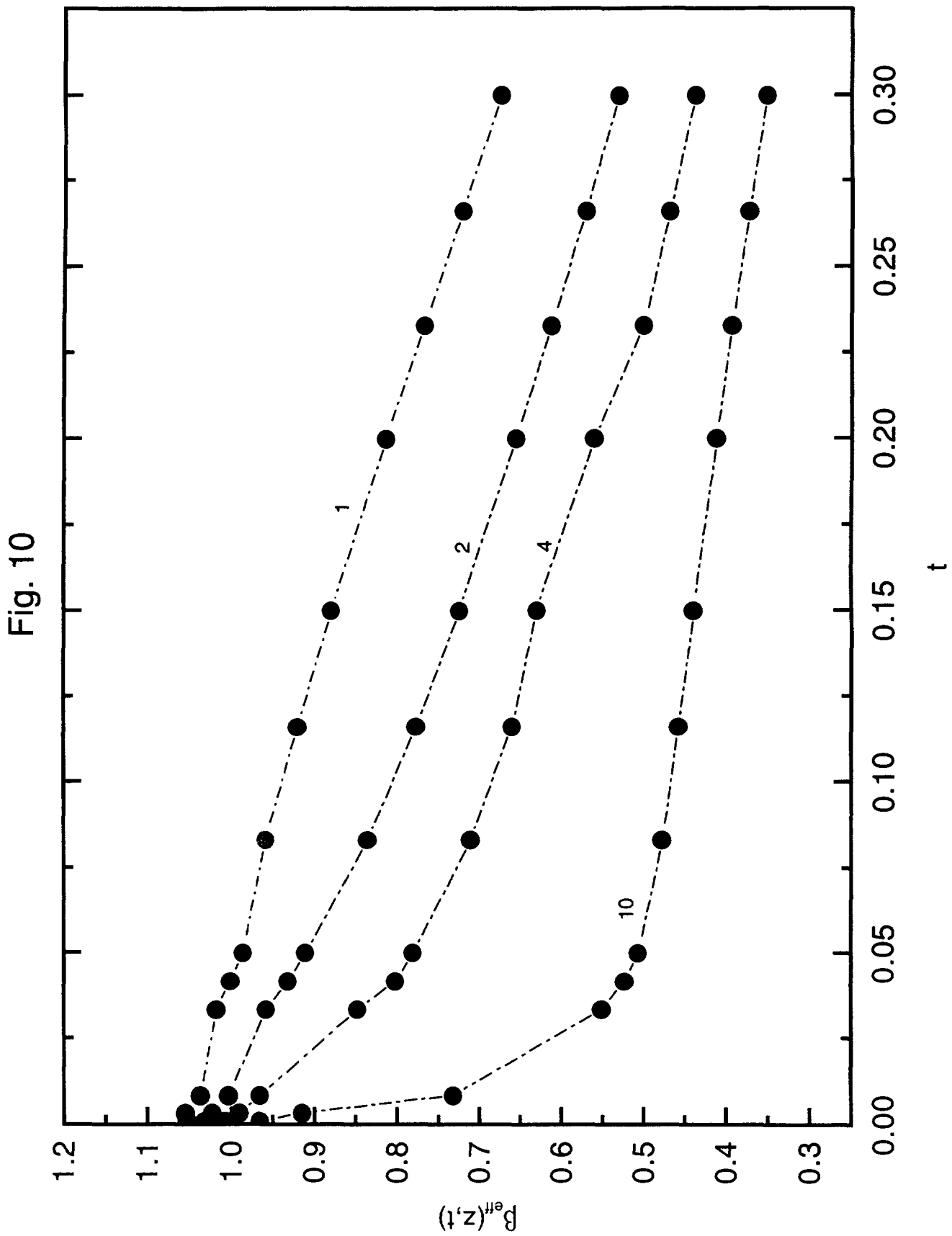
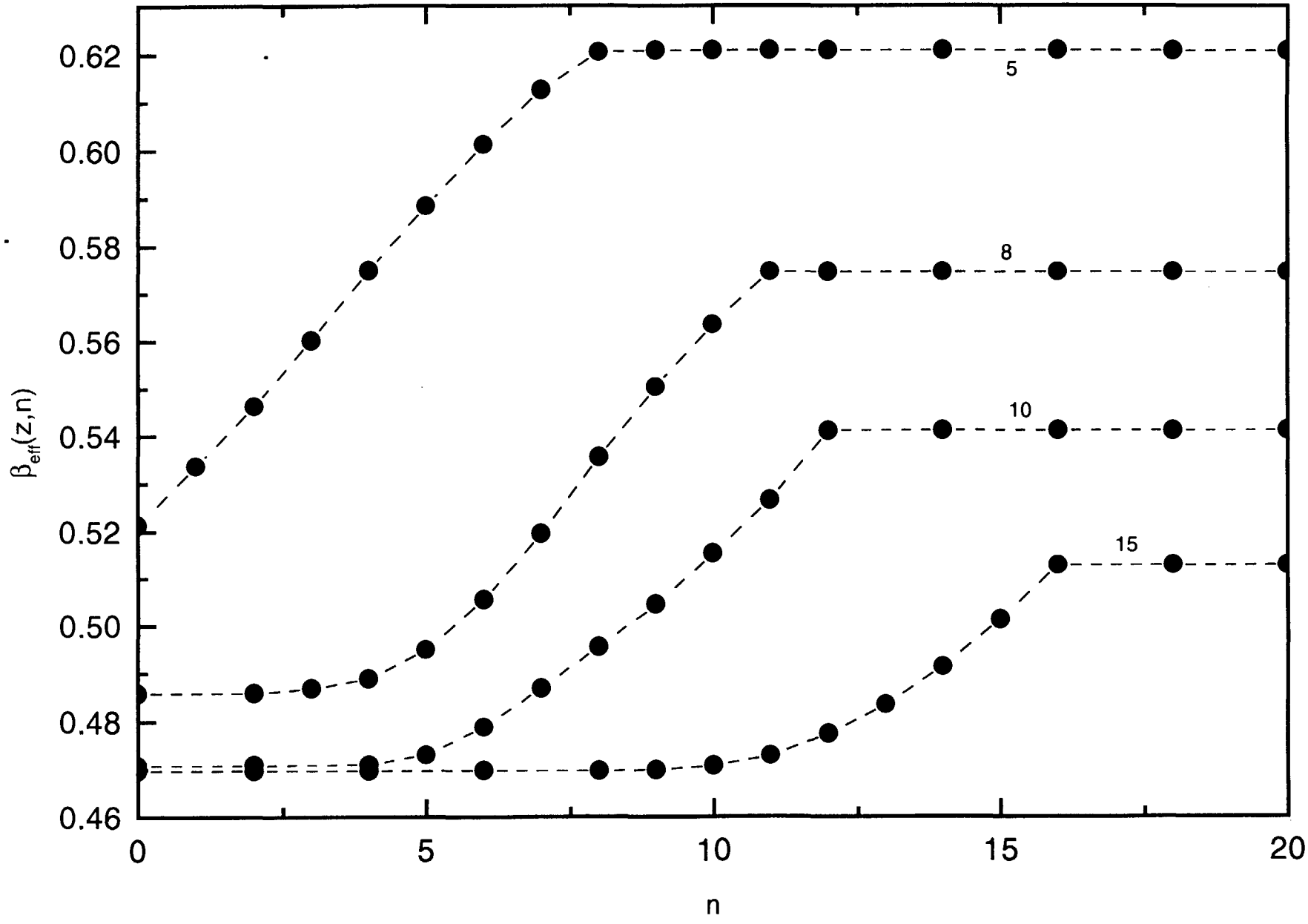


Fig. 10
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
$\beta_{eff}(z,t)$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
t
1
2
4
10

Fig. 11


0.62
0.60
0.58
0.56
0.54
0.52
0.50
0.48
0.46
$\beta_{eff}(z,n)$
0
5
10
15
20
n
5
8
10
15