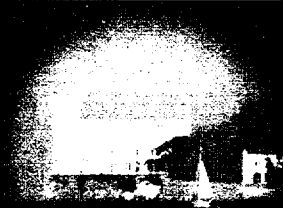




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XA9848694



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FROM A POINT-SOURCE
ABOVE GROUND LEVEL**

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preprint

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United Nations Educational Scientific and Cultural Organization
and
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**DIFFUSION OF DUST PARTICLES FROM A POINT-SOURCE
ABOVE GROUND LEVEL**

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MIRAMARE - TRIESTE

October 1998

Abstract

A pollutant of small particles is emitted by a point source at a height h above ground level in an atmosphere in which an uni-directional wind speed, U , is prevailing. The pollutant is subjected to diffusion in all directions in the presence of advection and settling due to gravity. The equation governing the concentration of the pollutant is studied when the wind speed and the different components of diffusion tensor are proportional to the distance above ground level and the source has a uniform strength.

Adopting a Cartesian system of coordinates in which the x -axis lies along the direction of the wind velocity, the z -axis is vertically upwards and the y -axis completes the right-hand triad, the solution for the concentration $c(x, y, z)$ is obtained in closed form. The relative importance of the components of diffusion along the three axes is discussed. It is found that for any plane $y = \text{constant}$ ($= A$), $c(x, y, z)$ is concentrated along a curve of "extensive pollution". In the plane $A = 0$, the concentration decreases along the line of extensive pollution as we move away from the source. However, for planes $A \neq 0$, the line of extensive pollution possesses a point of accumulation, which lies at a nonzero value of x . As we move away from the plane $A = 0$, the point of accumulation moves laterally away from the plane $x = 0$ and towards the plane $z = 0$. The presence of the point of accumulation is entirely due to the presence of lateral diffusion.

1. INTRODUCTION

The transport of small particles by wind is relevant to many geophysical (see,e.g., El Baz et al, 1991) and environmental (see,e.g., Sharan et al, 1996) problems. The dynamics of such transport are governed by three main factors: the mean wind speed, the size of the particles and the turbulent eddies that disperse and diffuse the particles. For large particles, of diameter of 0.5 mm or more, the influence of gravity is dominant and the particles are transported by rolling and sliding along the surface, a phenomenon referred to as creep. For intermediate particle sizes, of diameter in the range 0.1 – 0.4 mm, transport is observed to take place by saltation i.e., bouncing along the surface (Abbott and Francis, 1977). In the case of smaller particles, of diameter 0.1 mm or less, the particles are transported as a suspension (Skidmore, 1986). This latter case includes smoke from industrial chimneys and small snow flakes (Smith,1957; Takeuchi, 1980; Schmidt, 1982).

The present study deals with the transport of particles in suspension and is an extension of two earlier studies (Hassan and Eltayeb, 1991; Eltayeb and Hassan, 1992), which will be referred to as HE and EH, respectively. HE examined the concentration of dust due to a line source situated along the vertical z -axis when the mean flow U is proportional to z^m for some $m > 0$. The study led to an analytic expression for the concentration at every point of the $x-z$ plane. However, the study neglected the horizontal component of diffusion. EH examined the situation in which the source lies along the horizontal x -axis. Ignoring horizontal diffusion, the solution for the concentration at every point of the $x-z$ plane was obtained in closed form. Both studies investigated the relative importance of vertical diffusion and gravitational effects.

In the present study, we consider the concentration of a pollutant emitted by a source of given strength situated at a fixed height h above ground level in the presence of a prevailing wind U in a horizontal direction. Both vertical and horizontal components of diffusion are included. It is found that if the mean flow and all components of diffusion are proportional to the vertical distance then the concentration at every point of space can be obtained in closed form. This solution permitted the investigation of the relative importance of the different components of diffusion and gravitational effects. Diffusion in the horizontal plane has two components: along and normal to the direction of the prevailing wind. It is shown that the presence of the horizontal component of diffusion normal to the flow has a profound effect on the distribution of the pollutant in the cross plane. Although the concentration decays laterally, the effect of lateral diffusion and the flow is to lead to the formation of a point of accumulation of dust in lateral planes. The point of accumulation always lies on a surface of “ extensive pollution” in the (x, y, z) space. As we move away from the plane of flow and source the point of accumulation descends and moves away from the source always remaining on the surface of extensive pollution..

In section2, we define the model and solve the equation and boundary conditions to obtain an expression for the concentration in closed form. In section3, we discuss the results and in section 4 we make some concluding remarks.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Define a Cartesian system of coordinates $O(x^*, y^*, z^*)$ in which Oz^* is vertically upwards and Ox^* and Oy^* are horizontal. The concentration $c(x^*, y^*, z^*)$ of pollutant particles is governed by

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{u}) = \nabla \cdot (\mathbf{D} \cdot \nabla c) - \mathbf{w} \cdot \nabla c \quad (2.1)$$

in which \mathbf{u} is the velocity, \mathbf{w} is the settling velocity and \mathbf{D} is the spatial tensor. We assume the diffusion tensor has the three non-zero components (so that $D_{ij} = 0$, for $i \neq j$). We define the x -axis along the direction of the prevailing wind so that the other components of velocity in the horizontal plane can be neglected. The vertical settling velocity is also assumed to be larger than the eddy vertical velocity. Thus

$$\mathbf{u} = (U, 0, 0), \quad U = \alpha z^*, \quad \mathbf{w} = (0, 0, -W) \quad (2.2)$$

and (2.1) takes the form

$$U \frac{\partial c}{\partial x^*} = \frac{\partial}{\partial x^*} \mathbf{D}_x \cdot \frac{\partial c}{\partial x^*} + \frac{\partial}{\partial y^*} \mathbf{D}_y \cdot \frac{\partial c}{\partial y^*} + \frac{\partial}{\partial z^*} \mathbf{D}_z \cdot \frac{\partial c}{\partial z^*} + W \frac{\partial c}{\partial z^*} \quad (2.3)$$

in which we have assumed a steady state. Further, we take

$$U = \alpha z^*, \quad (D_x, D_y, D_z) = (\gamma, \beta, \alpha) z^* \quad (2.4)$$

and write (2.3) as

$$\alpha z^* \frac{\partial c}{\partial x^*} = \gamma z^* \frac{\partial^2 c}{\partial x^{*2}} + \beta z^* \frac{\partial^2 c}{\partial y^{*2}} + \lambda z^* \frac{\partial^2 c}{\partial z^{*2}} + (W + \lambda) \frac{\partial c}{\partial z^*} \quad (2.5)$$

If we define

$$a = \beta/\lambda, \quad b = \gamma\lambda/\alpha^2 = \frac{\gamma}{\lambda} \left(\frac{\lambda}{\alpha} \right)^2, \quad v = W/2\lambda,$$

$$x = (\lambda/\alpha) x^*, \quad z = z^*, \quad y = y^*, \quad (2.6)$$

we can write (2.5) in the neat form

$$\frac{\partial c}{\partial x} = \frac{2\nu+1}{z} \frac{\partial c}{\partial z} + \frac{\partial^2 c}{\partial z^2} + b \frac{\partial^2 c}{\partial x^2} + a \frac{\partial^2 c}{\partial y^2} \quad (2.7)$$

The parameters in (2.7) have the following representation: ν represents the effect of the settling velocity W , b represents the effect of longitudinal diffusion (i.e., along the flow) and a represents the latitudinal (across the flow) diffusion. Thus (2.7) is a generalization of the equation studied in HE to include diffusion in the x and y directions. We will see in section 3 below that the presence of such components of diffusion has a profound effect on the distribution of the concentration in the (x, y, z) space.

The equation (2.7) governs the distribution of the concentration $c(x, y, z)$ due to a source of strength Q situated at $(0, 0, h)$. The relevant boundary conditions then are :

$$\begin{aligned} \text{(i)} \quad & c(x, y, z_0) = 0 \\ \text{(ii)} \quad & c(x, y, z) \rightarrow 0 \quad \text{as } x, |y|, z \rightarrow \infty \\ \text{(iii)} \quad & c(0, y, z) = \frac{Q}{U} \delta(y) \delta(z-h) \end{aligned} \quad (2.8)$$

where z_0 is the roughness height.

We shall now proceed to solve (2.7) subject to (2.8). Define $q(x, y, z)$ by

$$q(x, y, z) = z^\nu c(x, y, z) \quad (2.9)$$

and reduce (2.7) and (2.8) to

$$\frac{\partial q}{\partial x} = \frac{1}{z} \frac{\partial}{\partial z} \left(z \frac{\partial q}{\partial z} \right) - \frac{\nu^2}{z} q + b \frac{\partial^2 q}{\partial x^2} + a \frac{\partial^2 q}{\partial y^2} \quad (2.10)$$

and

$$\begin{aligned} \text{(i)} \quad & q(x, y, z_0) = 0 \\ \text{(ii)} \quad & z^{-\nu} q(x, y, z) \rightarrow 0 \quad \text{as } x, |y|, z \rightarrow \infty \\ \text{(iii)} \quad & q(0, y, z) = \frac{Q}{\alpha} z^{\nu-1} \delta(y) \delta(z-h) \end{aligned} \quad (2.11)$$

Equations (2.10) and (2.11) can be solved by adopting the Weber's Transform defined by

$$\tilde{q}(x, y, z) = \int_0^\infty q(x, y, z) H_\nu(pz) z dz, \quad (2.12)$$

in which

$$H_\nu(pz) = J_\nu(pz)Y_\nu(pz_0) - J_\nu(pz_0)Y_\nu(pz), \quad (2.13)$$

together with its inverse transform

$$q(x,y,z) = \int_0^\infty \frac{\tilde{q}(x,y,z) H_\nu(pz)}{J_\nu^2(pz_0) + Y_\nu^2(pz_0)} p dp \quad (2.14)$$

Here $J_\nu(x)$ and $Y_\nu(x)$ are Bessel functions of the First Kind with order ν and argument x .

If we apply the transform to equation (2.10), use the boundary conditions (2.11i,ii) and note that $H_\nu(pz_0) = 0$, we obtain

$$\frac{\partial \tilde{q}}{\partial x} = a \frac{\partial^2 \tilde{q}}{\partial y^2} + b \frac{\partial^2 \tilde{q}}{\partial x^2} - p^2 \tilde{q} \quad (2.15)$$

The application of the transform to (2.11,iii) yields

$$\tilde{q}(x,y,z) = \frac{Q}{\alpha} h^\nu H_\nu(ph) \delta(y) \quad (2.16)$$

We now proceed to solve (2.15) and (2.16). We note that the method of solution will depend on whether (i) $\alpha = 0$ or (ii) $\alpha \neq 0$. We must consider the two cases separately.

(i) $\alpha = 0$

Here the y -derivative disappears and the concentration is independent of the lateral coordinate y . Equations (2.15) and (2.16) now take the form

$$\frac{\partial \tilde{q}}{\partial x} + p^2 \tilde{q} - b \frac{\partial^2 \tilde{q}}{\partial x^2} = 0 \quad (2.17)$$

$$\tilde{q}(0,p) = \frac{Q}{\alpha} h^\nu H_\nu(ph) \quad (2.18)$$

The solution is simply

$$\tilde{q}(x, p) = \frac{Q}{\alpha} h^\nu H_\nu(ph) \exp\left(\frac{x}{2b} \left[1 - \sqrt{1 + 4bp^2}\right]\right) \quad (2.19)$$

We now use the inverse transform to get

$$q(x, z) = \frac{Q}{\alpha} h^\nu e^{x/2b} \int_0^\infty F_\nu(z_o, h; p, z) \cdot \exp\left(-\frac{x}{2b} \sqrt{1 + 4bp^2}\right) p dp \quad (2.20)$$

in which

$$F_\nu(z_o, h; p, z) = \frac{H_\nu(ph) H_\nu(pz)}{J_\nu^2(pz_o) + Y_\nu^2(pz_o)} \quad (2.21)$$

and hence

$$c(x, z) = \frac{Q}{\alpha} (h/z)^\nu e^{x/2b} \int_0^\infty F_\nu(z_o, h; p, z) \cdot \exp\left(-\frac{x}{2b} \sqrt{1 + 4bp^2}\right) p dp \quad (2.22)$$

In the simpler case in which $b=0$ also, the expression (2.21) reduces to

$$c(x, z) = \frac{Q}{\alpha} (h/z)^\nu \int_0^\infty F_\nu(z_o, h; p, z) \cdot e^{-p^2 x} p dp \quad (2.23)$$

In the absence of roughness ($z_o = 0$) this last expression reduces to

$$\alpha(x, z) = \frac{Q}{2\alpha x} (h/z)^\nu \cdot \exp\left(-\frac{z^2 + h^2}{4x}\right) \cdot I_\nu\left(\frac{zh}{2x}\right) \quad (2.24)$$

where $I_\nu(x)$ is the modified Bessel function. The expression (2.24) agrees with the well known result derived earlier by Smith (1957) and more recently by Lin and Hildemann (1997) using Green's function methods.

(ii) $a \neq 0$

Here diffusion in all directions is potent and we must consider (2.15) and (2.16) in their entirety. Before we apply the Weber's Transform, we must apply Fourier Transform in y . This is defined by

$$\hat{q}(x, \omega, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{q}(x, y, p) e^{i\omega y} dy \quad (2.25)$$

Equations (2.15) and (2.16) then become

$$b \frac{\partial^2 \hat{q}}{\partial x^2} - \frac{\partial \hat{q}}{\partial x} - (p^2 + a\omega^2) \hat{q} = 0 \quad (2.26)$$

$$\hat{q}(0, \omega, p) = \frac{Q}{\alpha} h^\nu H_\nu(ph) \quad (2.27)$$

It then follows that

$$\hat{q}(x, \omega, p) = \frac{Q}{\alpha} h^\nu H_\nu(ph) \cdot \exp\left(\frac{x}{2b} \left\{1 - \sqrt{1 + 4b(p^2 + a\omega^2)}\right\}\right) \quad (2.28)$$

We now use the inverse Fourier Transform to obtain

$$\tilde{q}(x, y, p) = \frac{Q}{\alpha\pi} h^\nu H_\nu(ph) e^{x/2b} \int_0^\infty \exp\left(-\frac{x}{2b} \sqrt{1 + 4b(p^2 + a\omega^2)}\right) \cos(\omega y) d\omega \quad (2.29)$$

This integral can be evaluated (see Appendix) to get

$$\tilde{q}(x, y, p) = \frac{Q}{2\alpha\pi\sqrt{b}} h^\nu H_\nu(ph) e^{x/2b} \cdot \frac{x\sqrt{1+4bp^2}}{\sqrt{by^2+x^2a}} \cdot K_1\left(\frac{\sqrt{1+4bp^2}}{2b\sqrt{a}} \cdot \sqrt{by^2+x^2a}\right) \quad (2.30)$$

where $K_1(x)$ is the Modified Bessel function of order 1 and argument x . Then

$$q(x, y, z) = \frac{Q}{2\alpha\pi\sqrt{b}} \frac{xe^{x/2b}}{\sqrt{by^2+x^2a}} h^\nu \cdot \int_0^\infty F_\nu(z_0, h; p, z) \cdot \sqrt{1+4bp^2} \cdot K_1\left(\frac{\sqrt{1+4bp^2}}{2b\sqrt{a}} \cdot \sqrt{by^2+x^2a}\right) pdp \quad (2.31)$$

The general expression for the concentration $c(x, y, z)$ in the presence of diffusion in the directions of x, y and z is then

$$c(x, y, z) = \frac{Q}{2\alpha\pi\sqrt{b}} \frac{xe^{x/2b}}{\sqrt{by^2 + x^2a}} (h/z)^{\nu} \cdot \int_0^{\infty} F_{\nu}(z_0, h; p, z) \cdot \sqrt{1 + 4bp^2} \cdot K_1 \left(\frac{\sqrt{1 + 4bp^2}}{2b\sqrt{a}} \cdot \sqrt{by^2 + x^2a} \right) p dp \quad (2.32)$$

The case in which latitudinal diffusion acts in the absence of longitudinal diffusion (i.e., when $a \neq 0, b = 0$) can be obtained from (2.30) by taking the limit $b \rightarrow 0$. Thus

$$c(x, y, z) = \frac{Q}{2\alpha} \left(\frac{1}{\pi ax} \right)^{1/2} (h/z)^{\nu} \exp(-y^2/4ax) \int_0^{\infty} F_{\nu}(z_0, h; p, z) \cdot e^{-p^2x} p dp \quad (2.33)$$

In the limit $z_0 \rightarrow 0$, the integral in (2.33) can easily be manipulated to obtain

$$c(x, y, z) = \frac{Q}{4\alpha} \left(\frac{1}{\pi ax^3} \right)^{1/2} (h/z)^{\nu} \cdot \exp \left\{ \frac{1}{4x} \left(\frac{y^2}{a} + z^2 + h^2 \right) \right\} \cdot I_{\nu} \left(\frac{zh}{2x} \right) \quad (2.34)$$

The expression (2.32) provides the concentration at every point of space in closed form. However, the integral (2.32) cannot be evaluated analytically and we must resort to numerical integration. The expressions (2.22), (2.23), (2.24), (2.33) and (2.34) supply simpler expressions for the evaluation of the concentration in those special cases. We find it convenient to normalize the concentration for the numerical computations by

$$\bar{c}(x, y, z) = (\alpha/Q)c(x, y, z) \quad (2.35)$$

3. DISCUSSION OF THE RESULTS

The expressions (2.22), (2.23), (2.24), (2.32), (2.33) and (2.34) have been evaluated numerically and the results are summarized in figures 1-9. We find it informative to discuss the cases (i) $a = b = 0$, (ii) $a = 0, b \neq 0$, (iii) $a \neq 0, b = 0$ and (iv) $a, b \neq 0$ separately in order to identify the roles played by the various components of diffusion.

(i) Vertical diffusion only

Here both longitudinal (along x) and latitudinal (along y) diffusions are absent so that $a=b=0$. The only operative factors are the settling under gravity and vertical diffusion. This case then corresponds to that discussed in HE with the difference that the source is here situated at a height h above ground level. The concentration $c(x,z)$ is independent of y and therefore it has the same distribution in planes $y=\text{constant}$. Figure 1 illustrates the influence of the settling velocity W as represented by v . As the settling velocity increases, the concentration of the dust tends to accumulate near the vertical through the source. The extent of the spread of the dust in the (x,z) plane also depends on the height h (Figure 2). As the height of the source increases, the particles spread over a larger area even if v is moderately large so that the increase in height tends to offset the influence of the settling velocity. The influence of the roughness height is much weaker and a change in z_0 does not lead to significant changes in the distribution of the dust (see Figure 3).

A distinct feature of the distribution of the dust particles in the (x,z) plane is the appearance of a "line of extensive pollution". The position of this curve depends strongly on v and to a lesser extent on h . The location of the line of extensive pollution is identified as follows: For any fixed x ($=x_m$, say), $c(x,z)$ has a maximum (over z) at a certain height z_m . The curve traced by (x_m, z_m) is the line of extensive pollution because it represents the maximum dust flux in the (x,z) plane (see Figure 4). An increase in z_0 raises the level of the curve while an increase in h flattens it and moves it up towards the level of the source.

(ii) The influence of longitudinal diffusion

Here $a=0$ but $b \neq 0$ and there is no diffusion in the y direction. The concentration is again independent of y . The distribution of dust is here modified by the addition of longitudinal diffusion. As the strength of longitudinal diffusion is increased (i.e., as b is increased from zero), the concentration $c(x,z)$ along the line of extensive pollution is enhanced while its values elsewhere are diminished. This is illustrated in Figure 5. The concentration downstream is enhanced by the presence of longitudinal diffusion. The addition of longitudinal diffusion also affects the position of the line of extensive pollution. As b is increased, the line is flattened and raised towards the level of the source at h . This indicates that the presence of longitudinal diffusion acts to inhibit the influence of the settling velocity and promote the spread of dust.

(iii) The influence of lateral diffusion

When $b=0$ and $a \neq 0$, diffusion along the y direction is present in the absence of diffusion in the x direction. The concentration $c(x,y,z)$ then depends on y also. We note from (2.31) that the concentration is even in y and decays exponentially as $|y|$ increases. The line of extensive pollution now becomes a surface in the (x,y,z) space. The shapes of the contours of $c(x,y,z)$ in planes $y = \text{constant}$ do not change much with the increase of a (see Figure 6(a), (b)). However, the point of maximum concentration, which lies at the source at $(0,0,h)$ in the plane $y = 0$, is blown by the flow and lateral diffusion to position (x_m, y_m, z_m) . As $|y|$ is increased, x_m increases while z_m decreases with $|y|$ (see Figure 6(d)) and the concentration in planes $y = \text{constant}$ tends to spread out further downstream.

(iv) The general case

The expression (2.32) can be evaluated numerically for any set of values of the parameters a, b, v, z_o and h . The resulting concentration $c(z, b, v, z_o, h; x, y, z)$ will depend on the relative values of these parameters each of which represents the effects discussed above. The relative importance of the lateral diffusion is illustrated in Figures 8 and 9. As a increases, with all other parameters fixed but non-zero, the concentration tend to accumulate at a location whose position (x_m, y_m, z_m) depends on a and y_m . As a increases, x_m decreases and z_m increases towards h . If a, z_o, b and v remain fixed, the increase of $|y|$ is associated with increased x_m and decreased z_m , as we have seen in case (iii) above.

4. CONCLUDING REMARKS

The concentration $c(x,y,z)$ of a pollutant emitted by a source, of uniform strength, situated at a height h above ground level in a stream of uniform unidirectional wind has been studied. The relative importance of gravity, diffusion and the height of the source on the distribution of the dust in space has been investigated. It is found that lateral diffusion (i.e., horizontal component of the diffusion tensor normal to the wind direction) has a strong influence on the distribution of dust. The dust tend to accumulate on a surface, S , whose shape depends on the settling velocity as well as on diffusion. We call this surface "the surface of extensive pollution". The surface depends in a complicated way on the

lateral diffusion and the height of the source. However, some prominent properties of this surface can be identified. In the absence of lateral diffusion, it does not depend on the lateral coordinate y and hence its shape in the cross-sections $y = \text{constant}$ ($= A$, say) are the same. In this case the shape of the interface is primarily affected by the strength of the settling velocity (see Figure 4). The presence of lateral diffusion destroys the uniformity of the surface in lateral planes. Furthermore, for sufficiently large values of A (about 2.0 or more) the curve of extensive pollution in the cross-section $y=A$ possesses an “accumulation point” at a non-zero value of x . As A increases, the point of accumulation (i.e., the point where the concentration has a local maximum), which occurs at the source for $A=0$, is displaced and lies at a point (x_m, y_m, z_m) whose coordinates are determined by the relative importance of lateral diffusion, A and to a lesser extent by the longitudinal diffusion (See Figure 8).

The model discussed here adopts certain forms for the wind speed, U , and diffusion tensor \mathbf{D} . See equation (2.4) above). These forms may not be very realistic and more complicated expressions may be more appropriate for real life problems. However, we believe that the central result of the existence of a surface of extensive pollution will survive in the case of other complicated forms for U and \mathbf{D} , although the shape of the surface may be modified. The advantage of the simple forms taken here is that we have been able to solve the governing equation and boundary conditions analytically to provide an expression for the concentration in closed form and thereby a thorough study of the importance of the various parameters of the problem was made relatively simple.

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APPENDIX

To evaluate the integral (2.27), we appeal to standard results. From Gradshteyn and Rizhik, page 498, item 3.96(2), we have

$$\int_0^{\infty} \exp\left(-\beta\sqrt{\gamma^2+x^2}\right)\cos(ax)\frac{dx}{\sqrt{\gamma^2+x^2}} = K_0\left(\gamma\sqrt{a^2+\beta^2}\right)$$

$$\operatorname{Re}(\beta) > 0, \operatorname{Re}(\gamma) > 0, a > 0 \quad (\text{A.1})$$

and $K_\nu(x)$ is the modified Bessel function of order ν and argument x . Note the symbols a, β, γ are not the same as those used in text but are any parameters in the standard result (A.1). Now consider the integral

$$I(\gamma) = \int_0^{\infty} \exp\left(-\beta\sqrt{\gamma^2+x^2}\right)\cos(ax)dx \quad (\text{A.2})$$

Then

$$\frac{\partial I(\gamma)}{\partial \gamma} = -\beta\gamma \int_0^{\infty} \exp\left(-\beta\sqrt{\gamma^2+x^2}\right)\cos(ax)\frac{dx}{\sqrt{\gamma^2+x^2}} \quad (\text{A.3})$$

and hence from (A.1)

$$\frac{\partial I(\gamma)}{\partial \gamma} = -\beta\gamma K_0\left(\gamma\sqrt{a^2+\beta^2}\right) \quad (\text{A.4})$$

Now items 9.6.26 and 9.6.28 on page 376 of Abramowitz and Stegun (1965) give

$$-zK_0(z) = \frac{d}{dz}(zK_1(z)) \quad (\text{A.5})$$

so that

$$-\beta\gamma K_0\left(\gamma\sqrt{a^2+\beta^2}\right) = \frac{\beta}{\sqrt{a^2+\beta^2}} \frac{d}{d\gamma}\left(\lambda K_1\left(\gamma\sqrt{a^2+\beta^2}\right)\right) \quad (\text{A.6})$$

Equations (A.4) – (A.6) then give

$$\frac{d}{d\gamma} \left(I(\gamma) - \frac{\beta\gamma}{\sqrt{a^2 + \beta^2}} K_1(\gamma\sqrt{a^2 + \beta^2}) \right) = 0 \quad (\text{A.7})$$

which indicates that

$$I(\gamma) = \frac{\beta\gamma}{\sqrt{a^2 + \beta^2}} K_1(\gamma\sqrt{a^2 + \beta^2}) \quad (\text{A.8})$$

We now define

$$\beta = x\sqrt{\frac{a}{b}}, \quad \gamma = \frac{1}{2}\sqrt{\frac{1+4bp^2}{ab}}, \quad x = \omega, a = y \quad (\text{A.9})$$

Substitute from (A.9) into (A.2) and use (A.8) to find that

$$\begin{aligned} & \int_0^\infty \exp\left(-\frac{x}{2b}\sqrt{1+4b(p^2+a\omega^2)}\right) \cos(y\omega) d\omega \\ &= \frac{x\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{1+4bp^2}}{\sqrt{ab}} \cdot \frac{1}{\sqrt{y^2+x^2a/b}} \cdot K_1\left(\frac{\sqrt{1+4bp^2}}{2\sqrt{ab}} \cdot \sqrt{y^2+x^2a/b}\right) \\ &= \frac{x}{2\sqrt{b}} \cdot \frac{\sqrt{1+4bp^2}}{\sqrt{by^2+x^2a}} \cdot K_1\left(\frac{\sqrt{1+4bp^2}}{2b\sqrt{a}} \cdot \sqrt{by^2+x^2a}\right) \end{aligned} \quad (\text{A.10})$$

This expression is used in (2.27) to obtain (2.28).

FIGURE CAPTIONS

FIGURE 1. The contours of the concentration $\bar{c}(x, z)$ when $a=b=0, h=5.0$ and $z_o=0.5$ for three different values of the settling velocity: (a) $\nu=0.5$, (b) $\nu=1.0$ and (c) $\nu=2.0$.

FIGURE 2. Contours of the concentration when $a=b=0, h=10.0, \nu=0.5$ and $z_o=0.5$. Compare with Figure 1(a).

FIGURE 3. Contours of the concentration when $a=b=0, h=5.0, \nu=0.5$ and $z_o=0.1$. Compare with Figure 1(a).

FIGURE 4. Illustration of the dependence of the line of extensive pollution on h, z_o and ν in the absence of horizontal diffusion ($a=b=0$) for (a) $h=5.0, z_o=0.1$, (b) $h=5.0, z_o=0.5$, (c) $h=10.0, z_o=0.1$. The values on the curves refer to ν .

FIGURE 5. The influence of longitudinal diffusion ($b \neq 0$) on the concentration when $h=5.0, z_o=0.5$ and $\nu=0.5$: (a) $b=0.5$, (b) $b=1.0$ (c) $b=2.0$. Include Figure 1(a) in comparing the contours for different values of b .

FIGURE 6. The dependence of concentration on lateral diffusion ($a \neq 0$) when $h=5.0, z_o=0.5, b=0, \nu=0.5$ and (a) $a=0.5, y=0.5$, (b) $a=1.0, y=0.5$, (c) $a=1.0, y=1.0$, (d) $a=0.5, y=2.0$. Compare (a) and (b) for increasing a , (b) and (c) for changing y . Note the appearance of the point of accumulation for larger y .

FIGURE 7. Contours of the concentration in the $y-z$ plane when $h=5.0, z_o=0.5, b=0, \nu=0.5, x=1.0$ and (a) $a=0.5$, (b) $a=1.0$. Note the lateral extent of concentration with increasing a .

FIGURE 8. Illustration of the importance of the horizontal components of diffusion. Here $h=5.0, z_o=0.5, \nu=0.5$ and (a) $a=0.5, b=1.0, y=1.0$ (b) $a=2.0, b=1.0, y=1.0$ (c) $a=0.5, b=0.5, y=2.5$ (d) $a=0.5, b=1.0, y=2.5$. Compare (a) with (b) for change in a , (a) with (c) for change in y and (c) with (d) for change in b .

FIGURE 9. The dependence of the line of extensive pollution on y for $h=5.0, z_o=0.5, \nu=0.5$ and (a) $a=1.0, b=0.5$, (b) $a=2.0, b=0.5$ (c)

$a=1.0, b=1.0$. The labels $y=A$ on the curves refer to the cross section at $y=A$. The surface of extensive pollution can be visualized by displacing the curves $y=A$ to be at their y position, and noting that the concentration is even in y , to form a sheet in the (x,y,z) space.

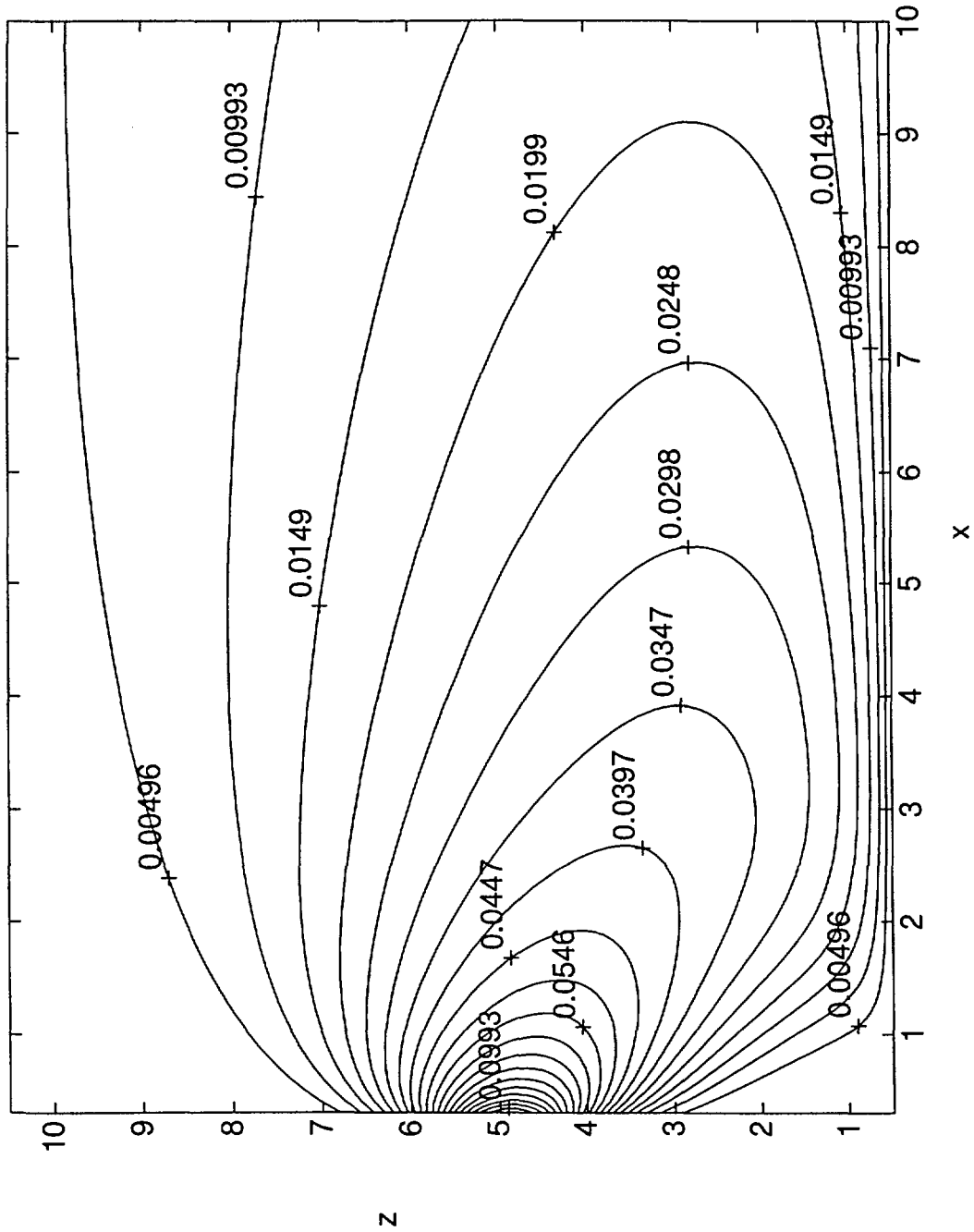


Figure 1a: $h = 5.0, z_0 = 0.5, a = 0.0, b = 0.0, v = 0.5$

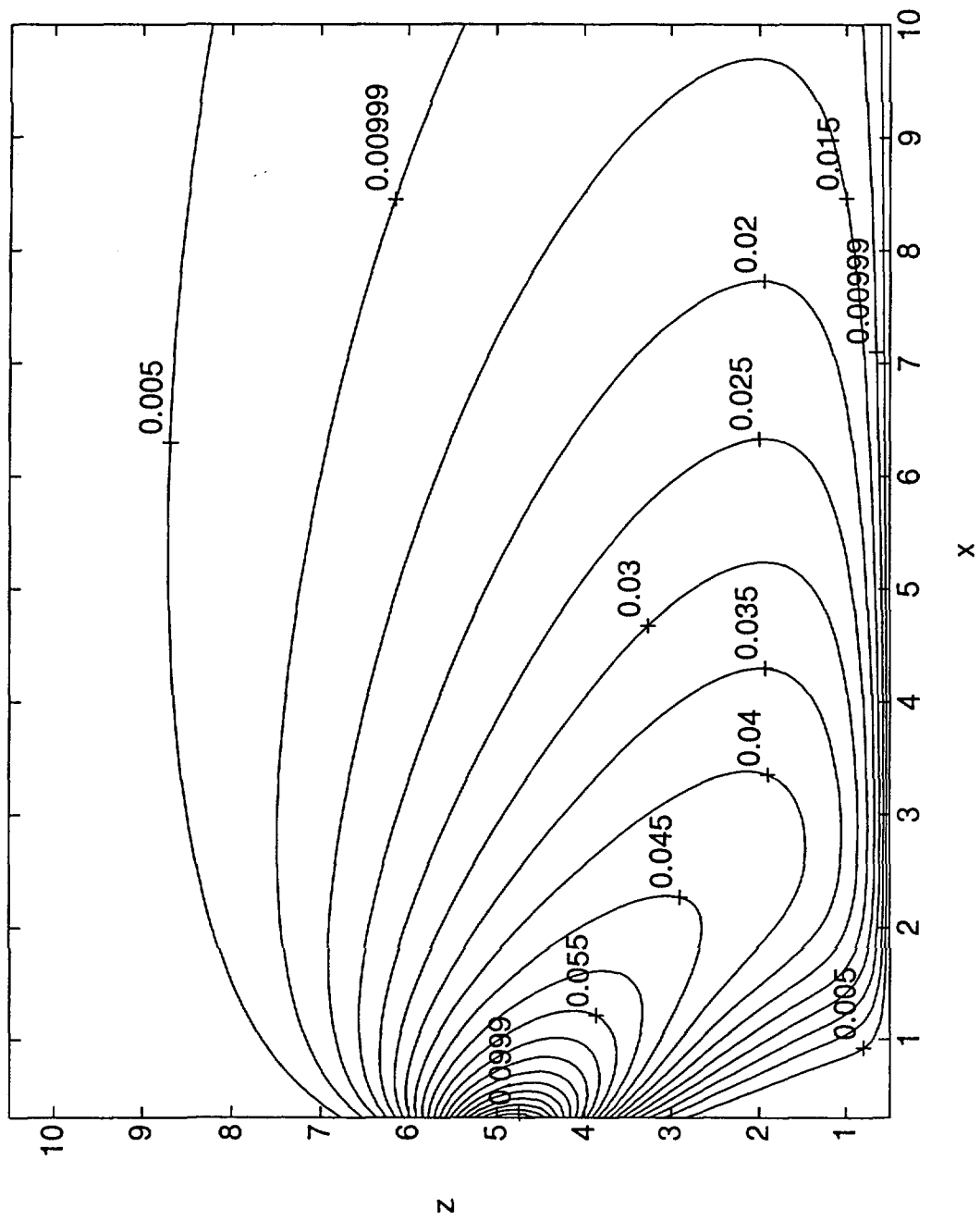


Figure 1b: $h = 5.0, z_0 = 0.5, a = 0.0, b = 0.0, v = 1.0$

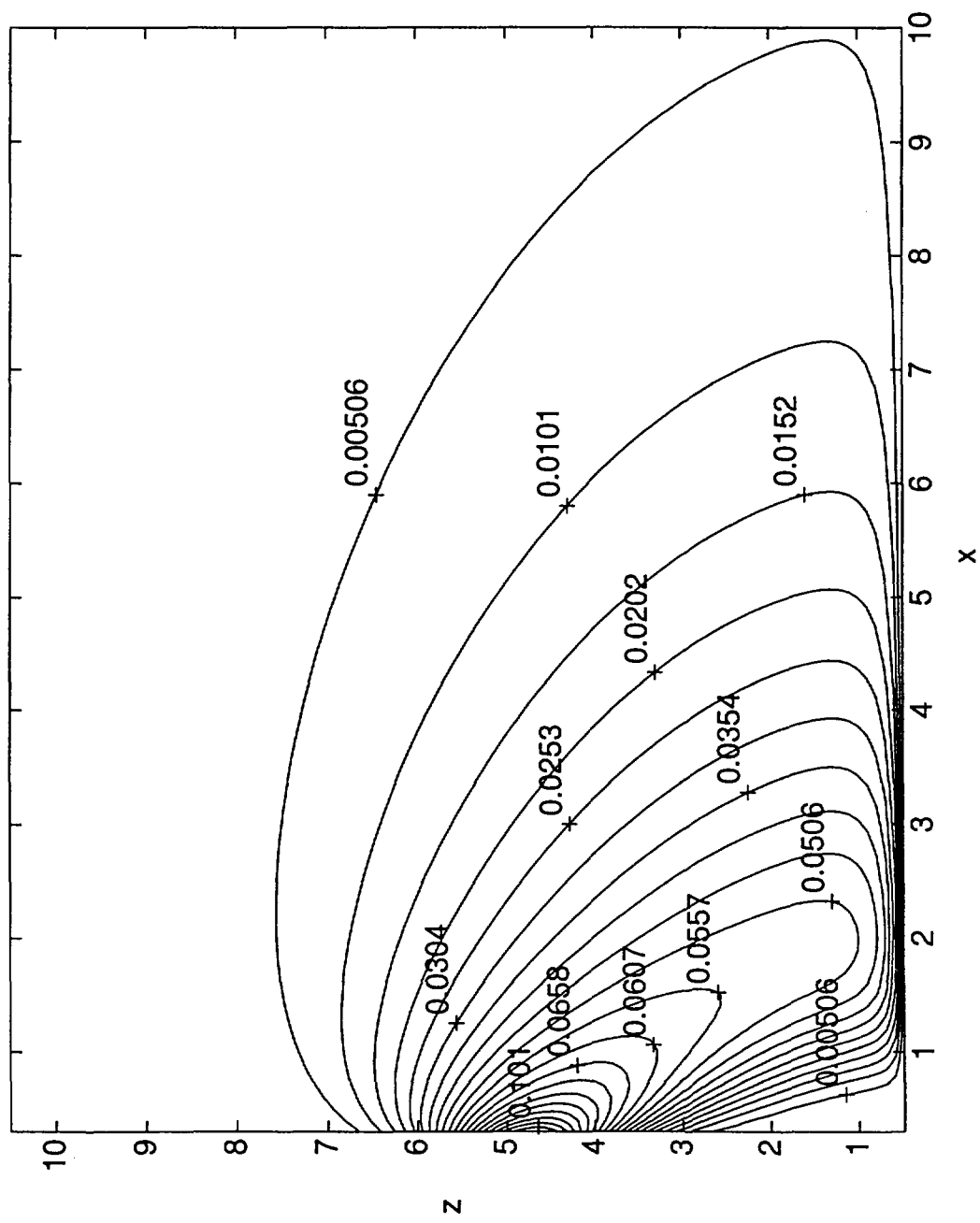


Figure 1c: $h = 5.0, z_0 = 0.5, a = 0.0, b = 0.0, v = 2.0$

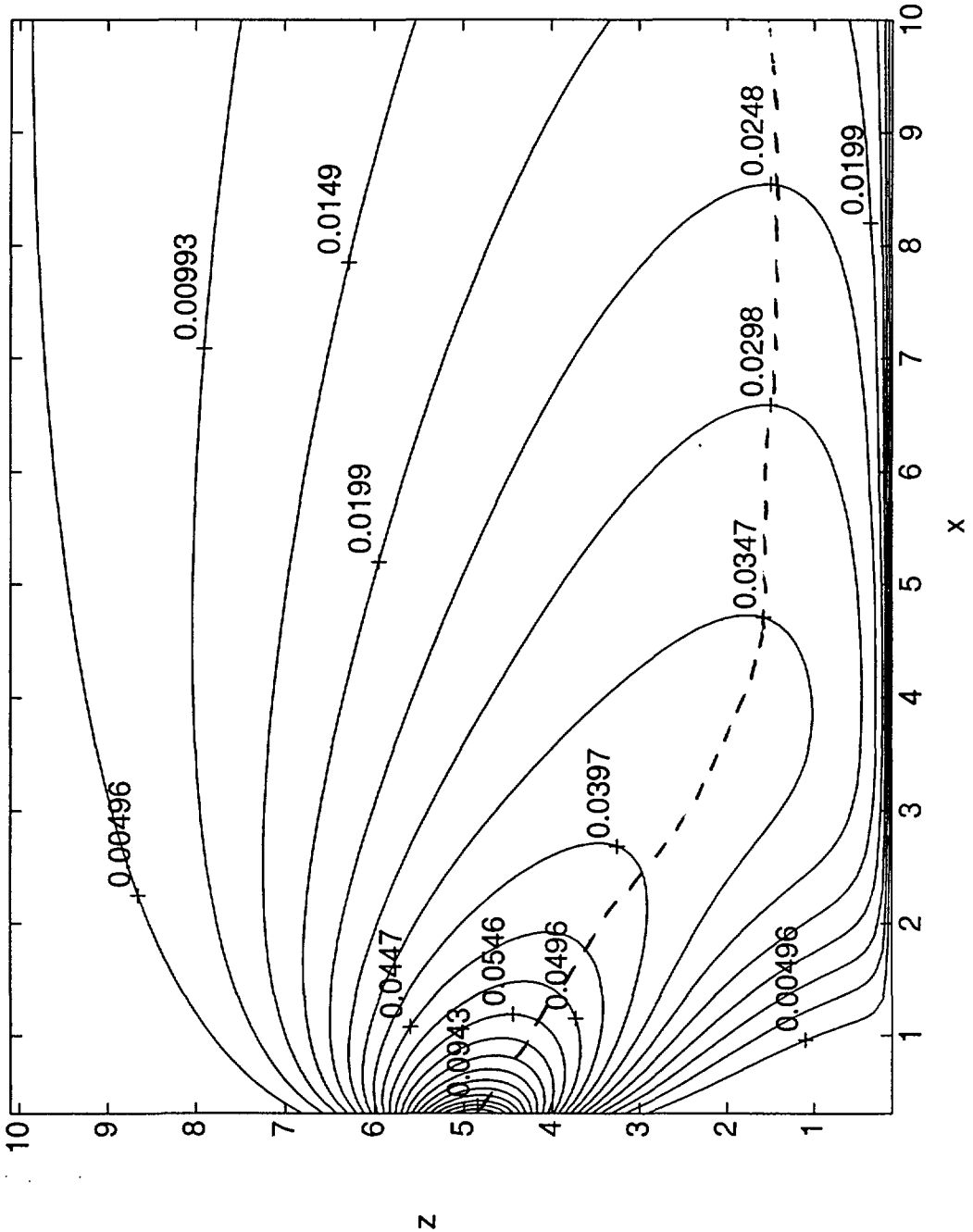


Figure 2: $h = 5.0, z_0 = 0.1, a = 0.0, b = 0.0, v = 0.5$

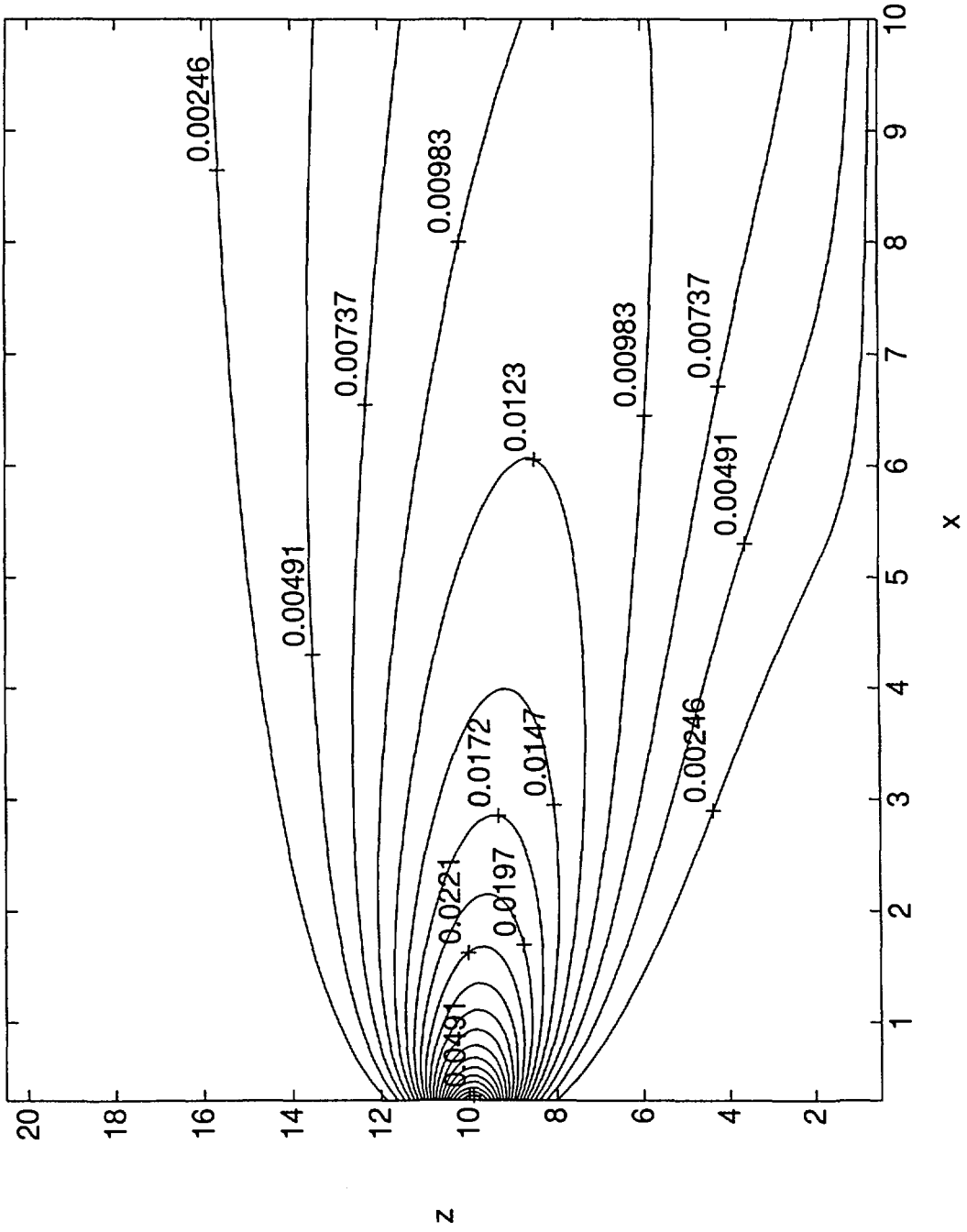


Figure 3: $h = 10.0$, $z_0 = 0.5$, $a = 0.0$, $b = 0.0$, $v = 0.5$

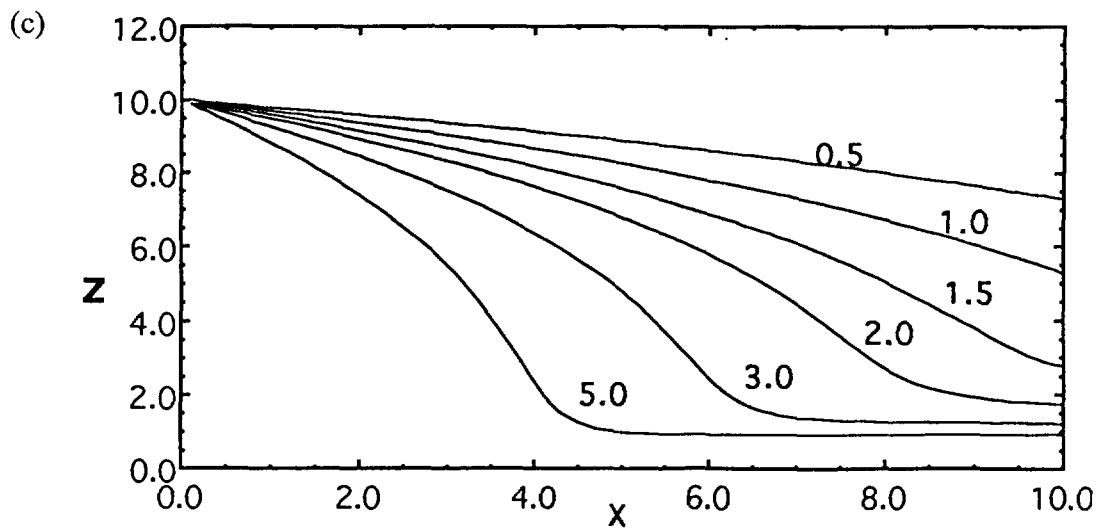
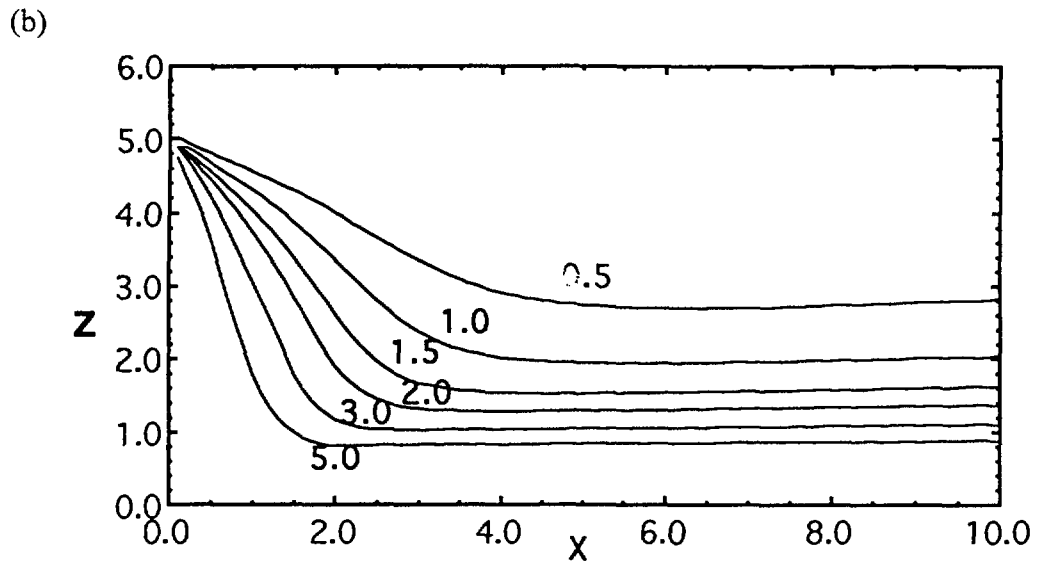
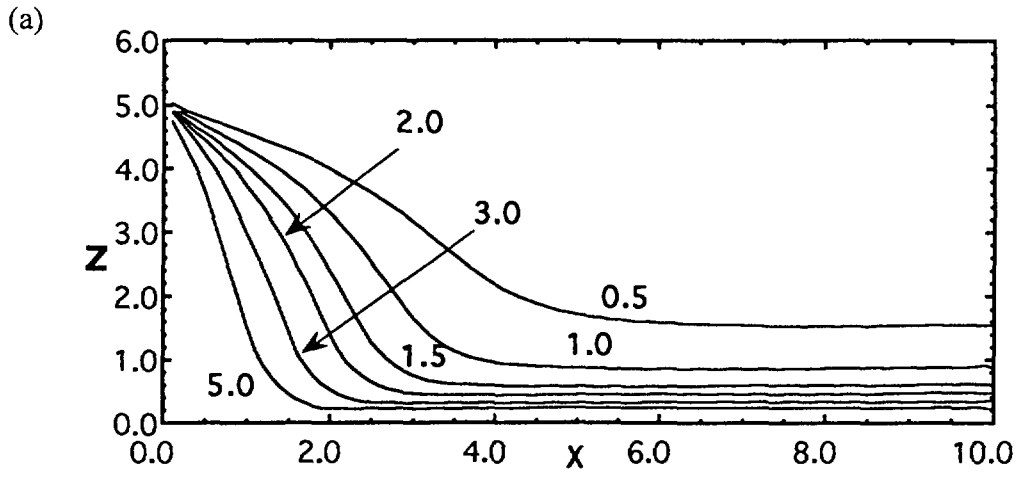


Figure 4

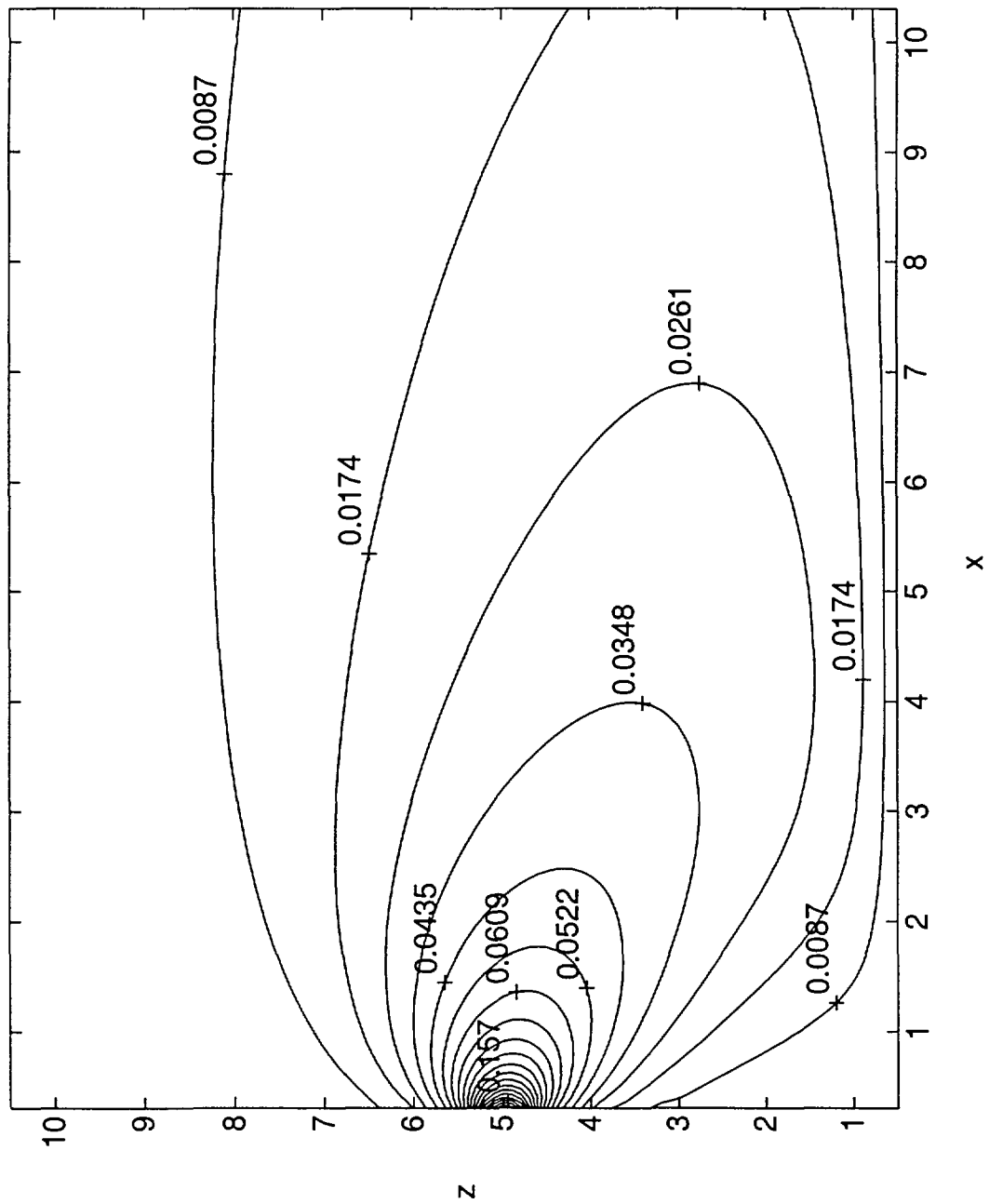


Figure 5a: $h = 5.0, z_0 = 0.5, a = 0.0, b = 0.5, v = 0.5$

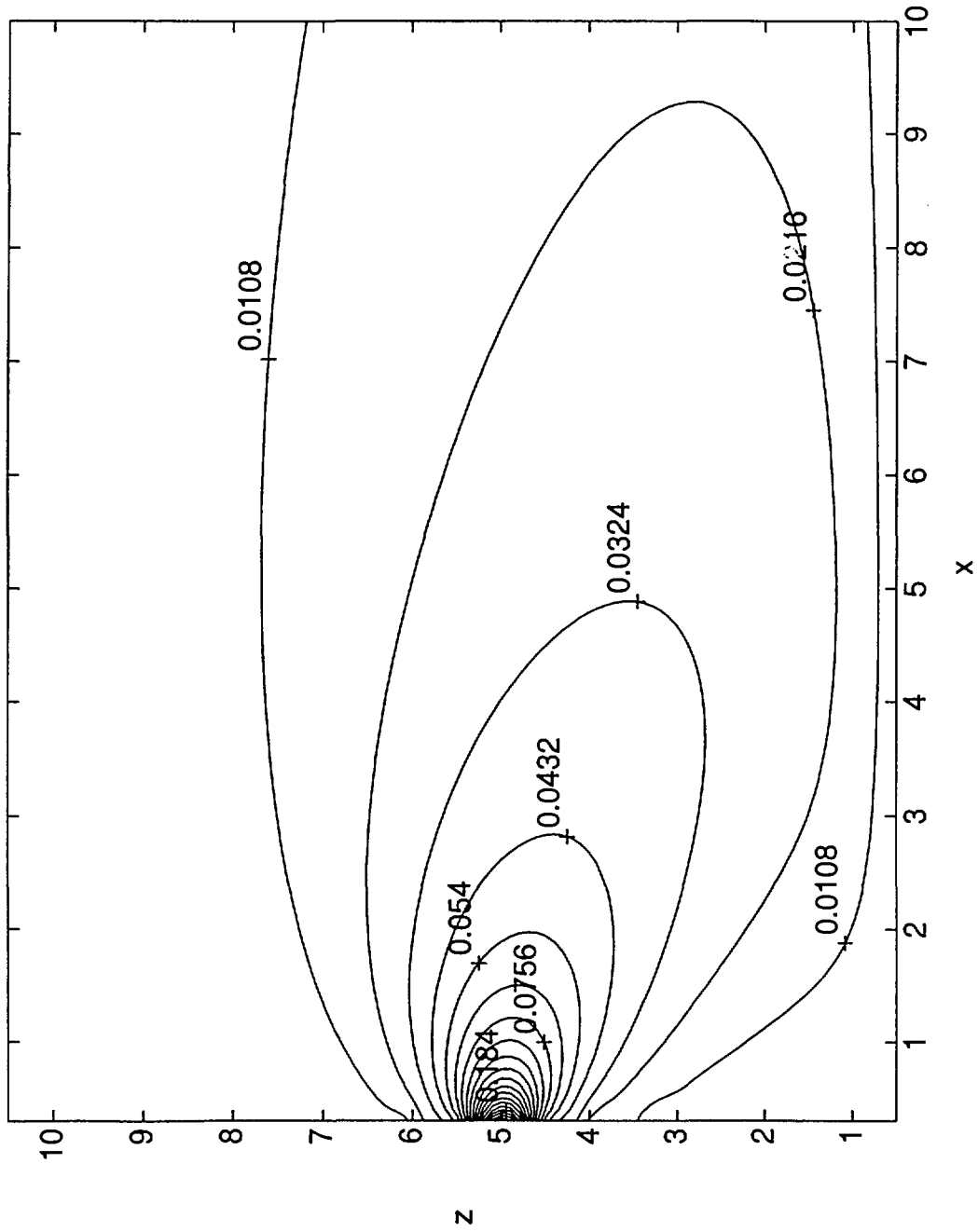


Figure 5b: $h = 5.0, z_0 = 0.5, a = 0.0, b = 1.0, v = 0.5$

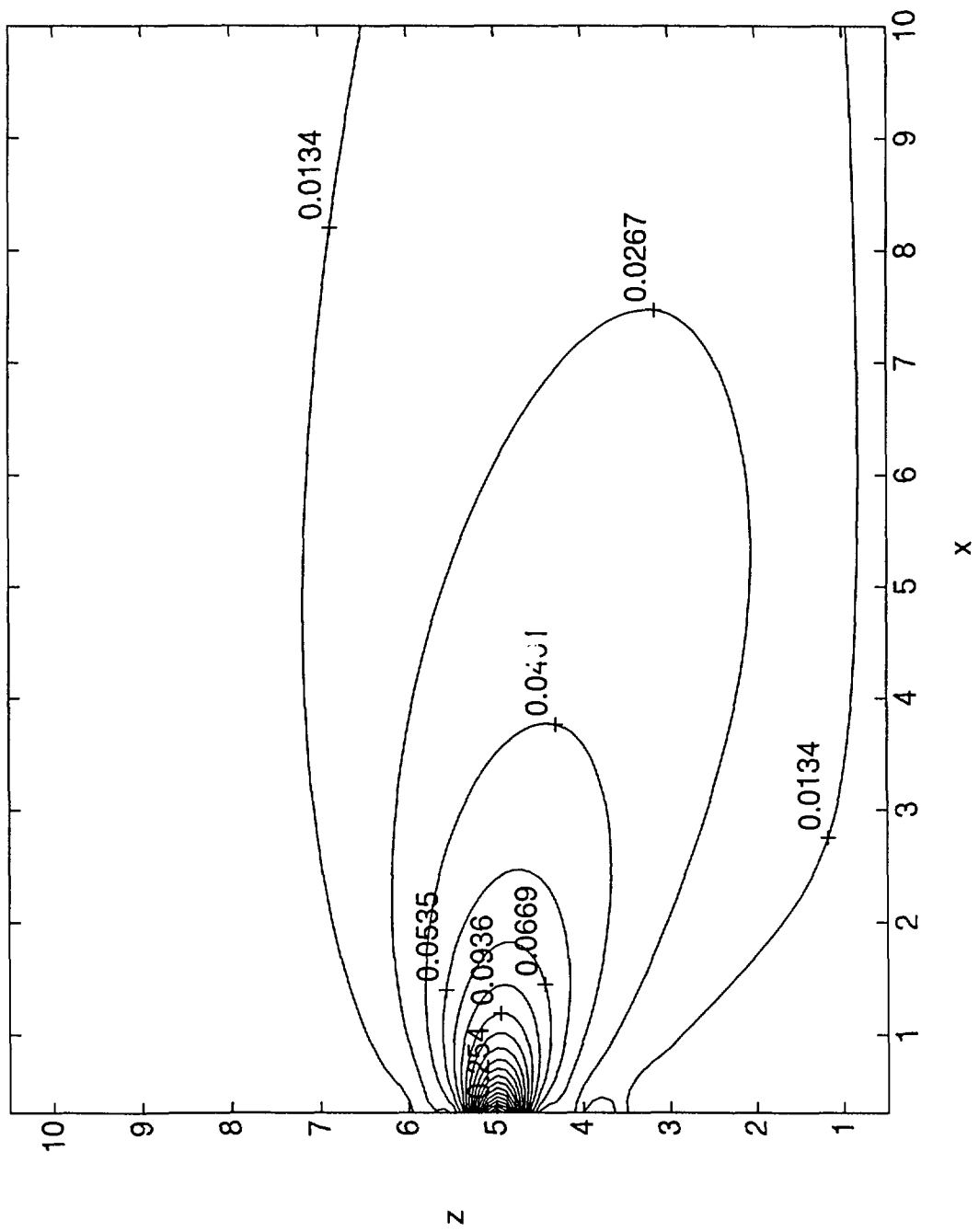


Figure 5c: $h = 5.0, z_0 = 0.5, a = 0.0, b = 2.0, v = 0.5$

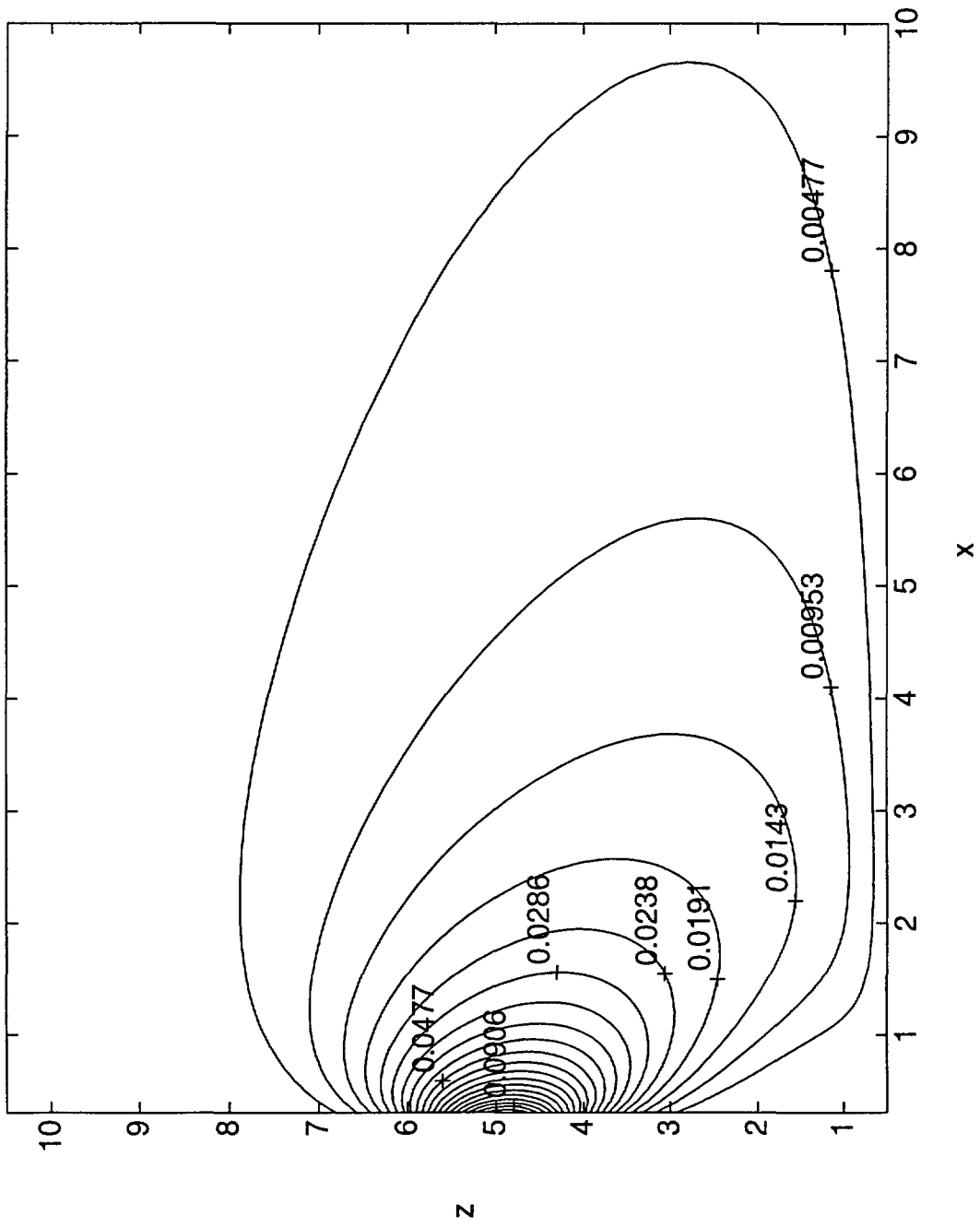


Figure 6a: $h = 5.0, z_0 = 0.5, a = 0.5, b = 0.0, v = 0.5, y = 0.5$

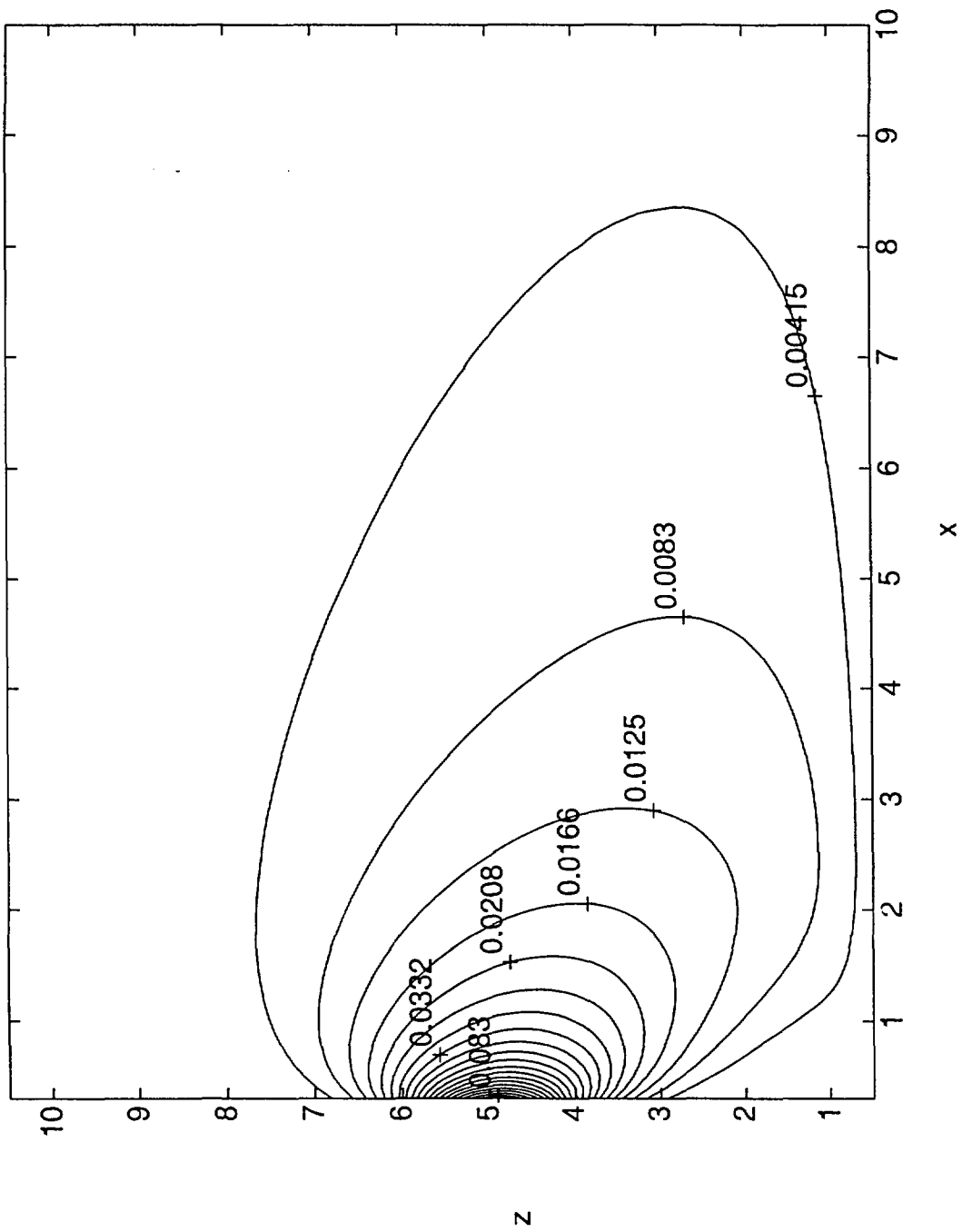


Figure 6b: $h = 5.0, z_0 = 0.5, a = 1.0, b = 0.0, v = 0.5, y = 0.5$

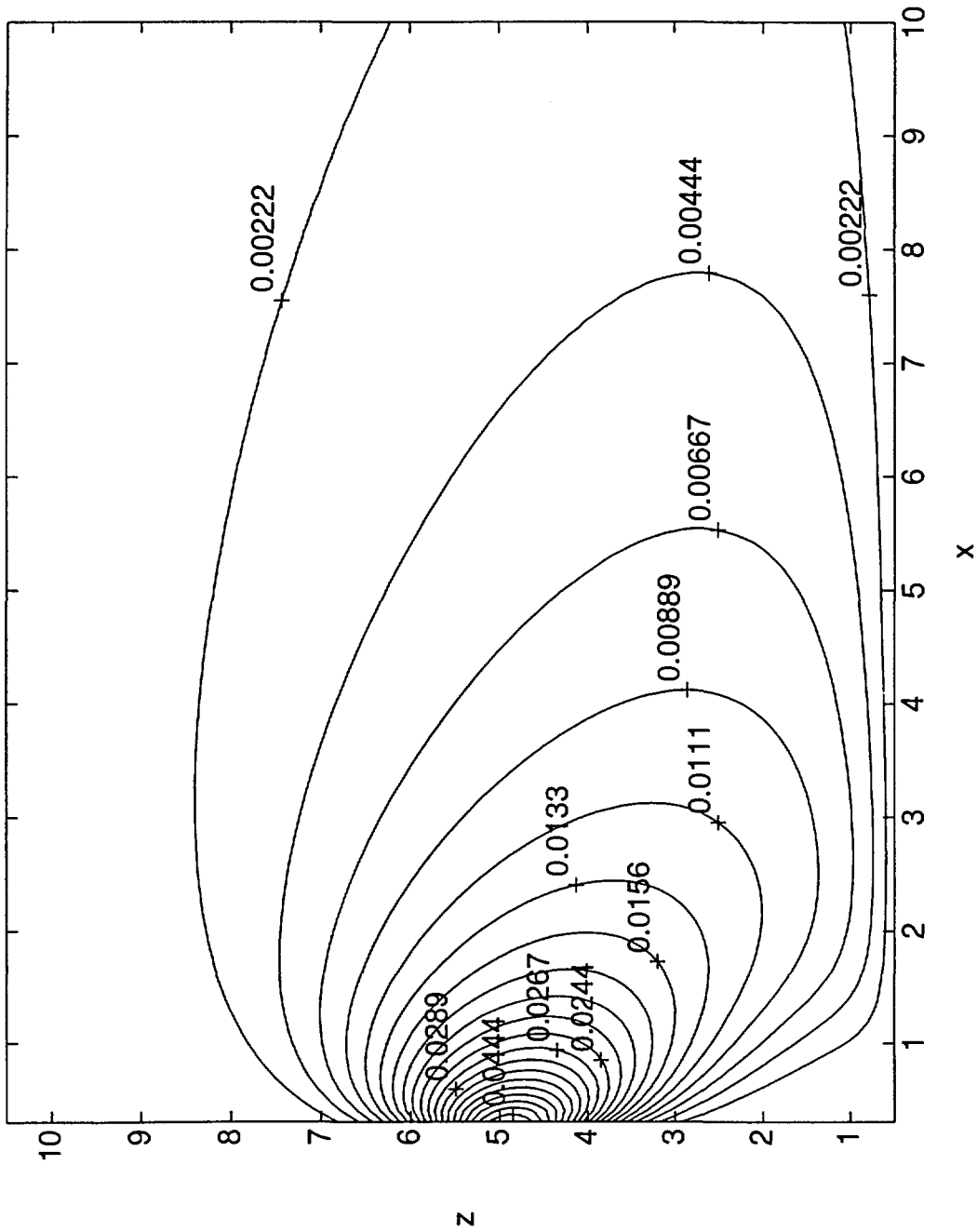


Figure 6c: $h = 5.0, z_0 = 0.5, \alpha = 1.0, b = 0.0, v = 0.5, \gamma = 1.0$

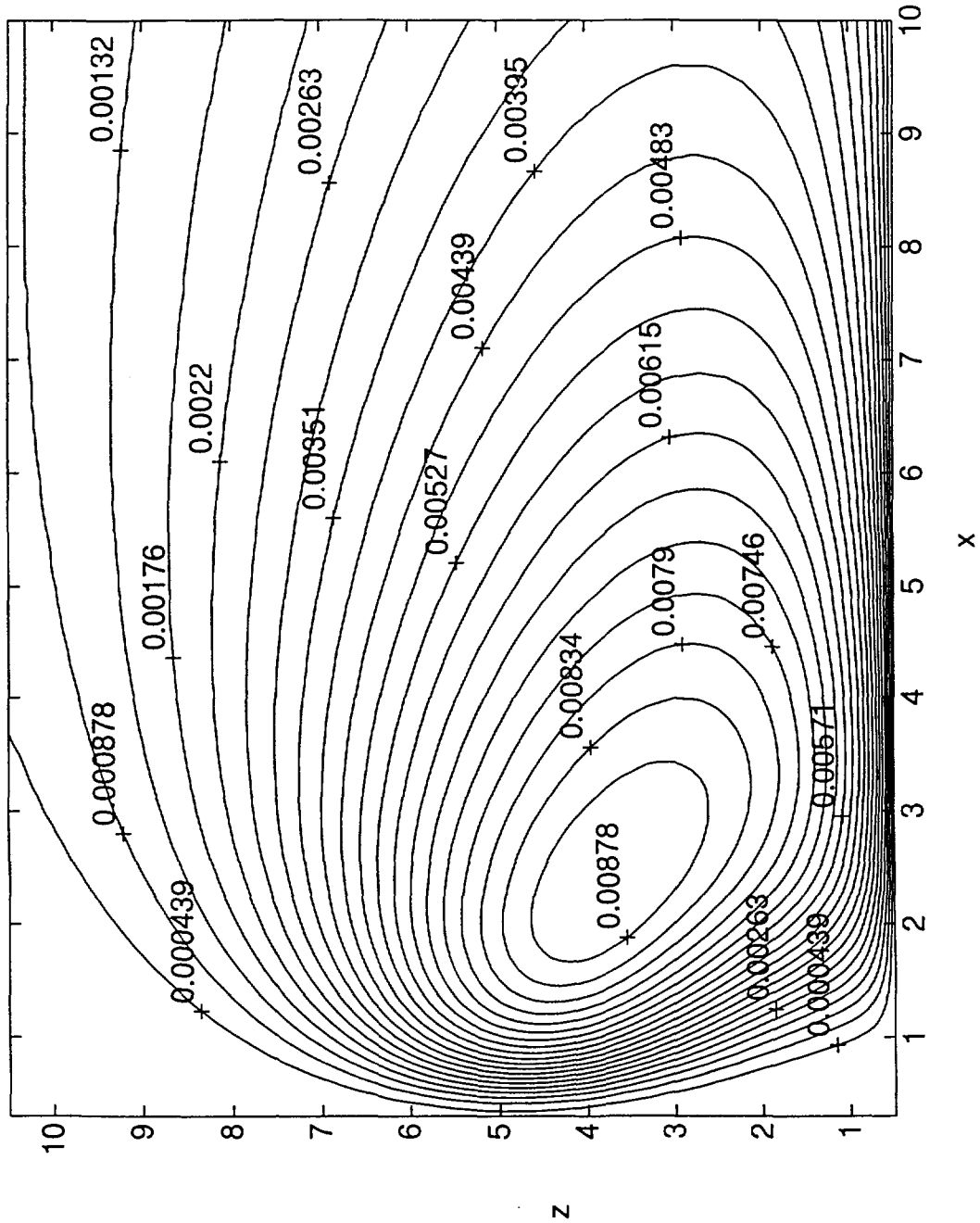


Figure 6d: $h = 5.0, z_0 = 0.5, a = 0.5, b = 0.0, v = 0.5, y = 2.0$

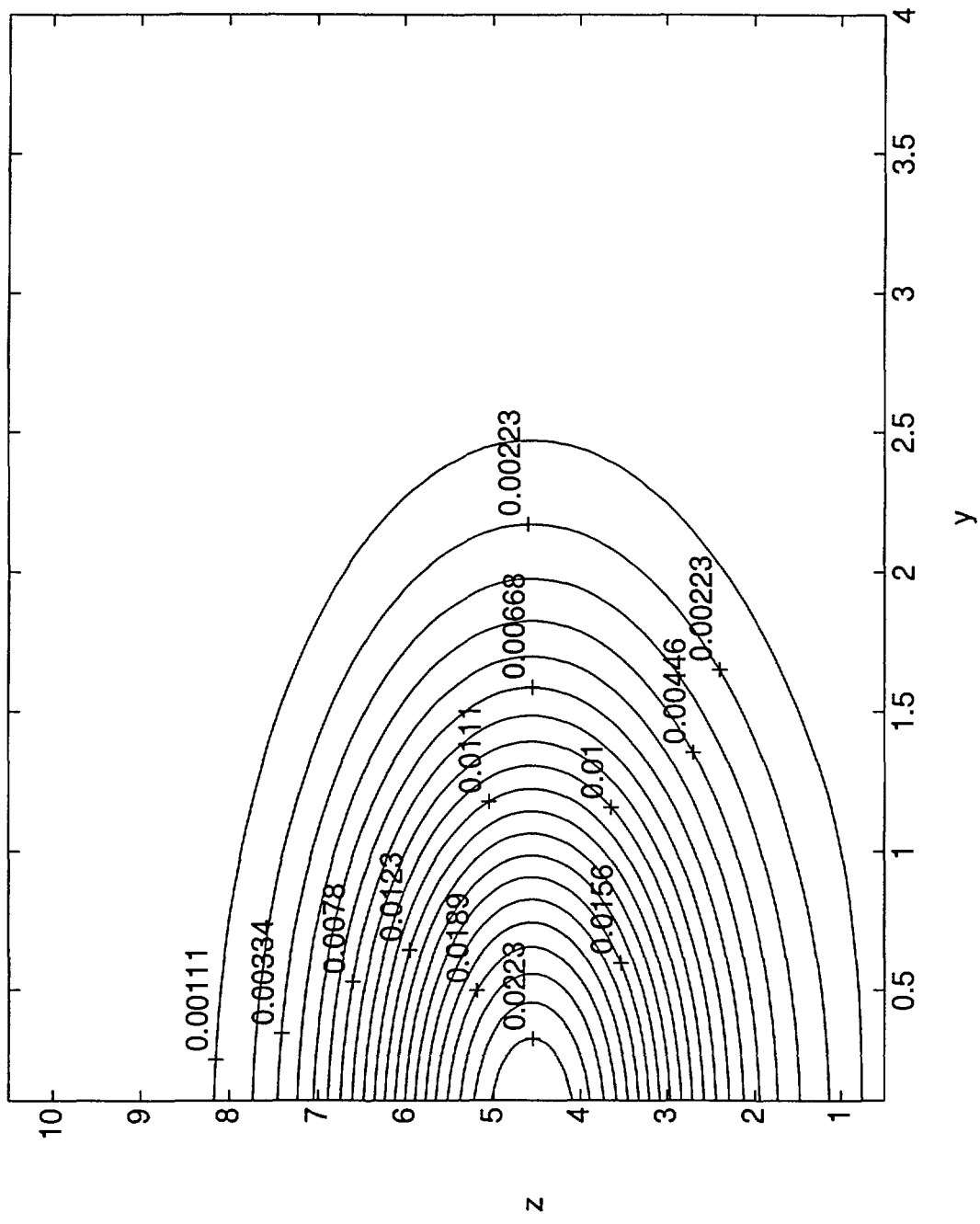


Figure 7a: $h = 5.0, z_0 = 0.5, a = 0.5, b = 0.0, v = 0.5, x = 1.0$

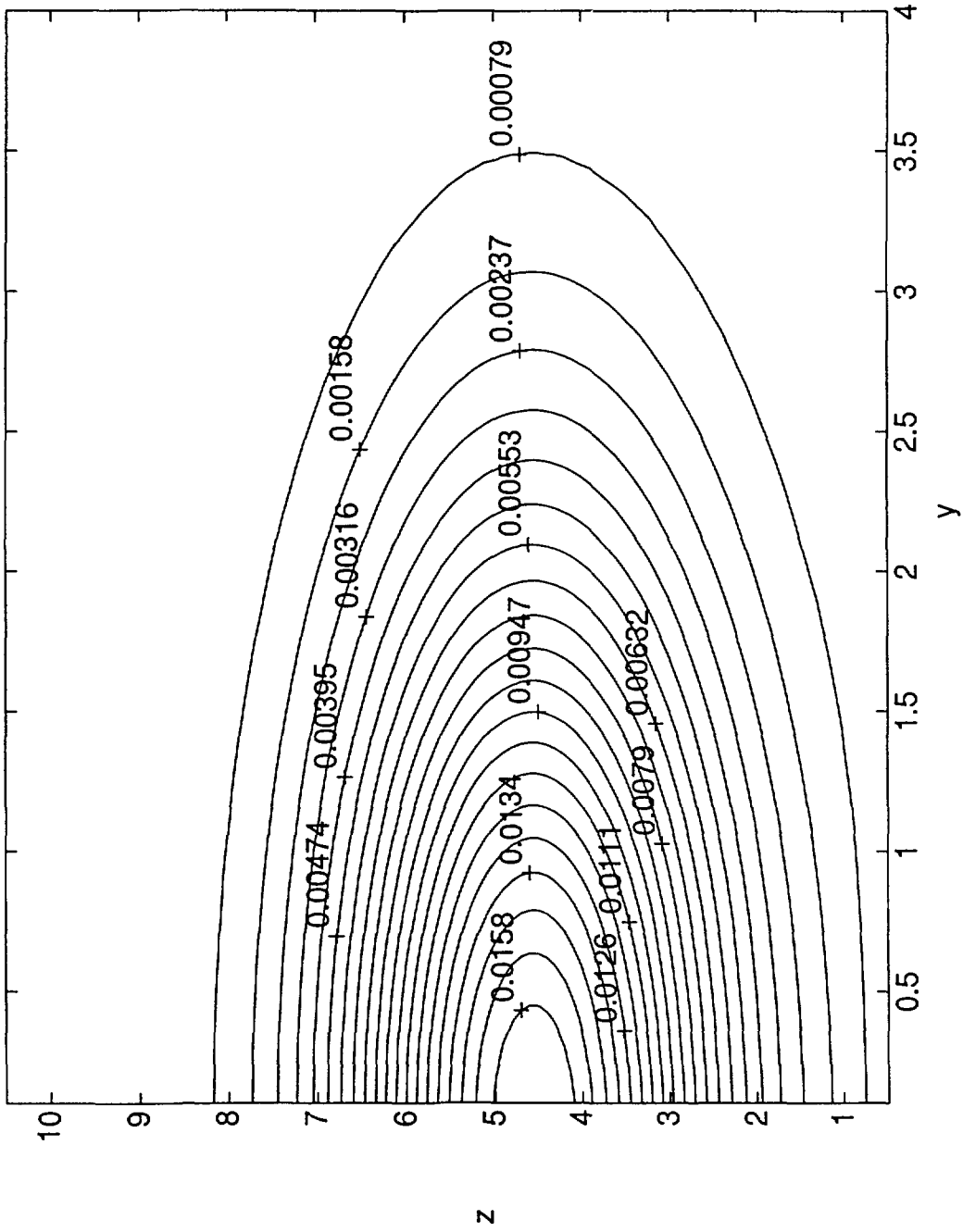


Figure 7b: $h = 5.0, z_0 = 0.5, a = 1.0, b = 0.0, v = 0.5, x = 1.0$

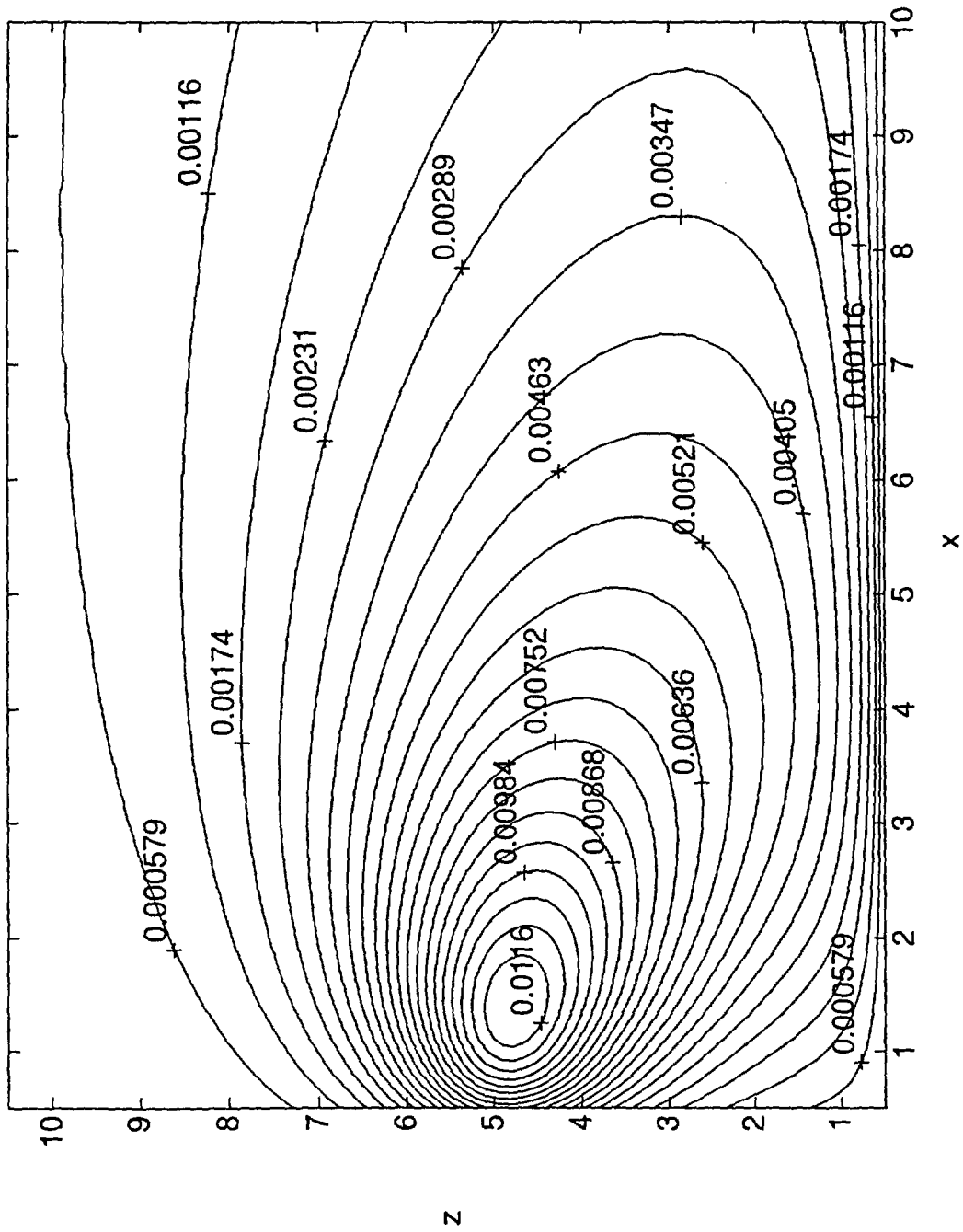


Figure 8a: $h = 5.0, z_0 = 0.5, a = 0.5, b = 1.0, v = 0.5, y = 1.0$

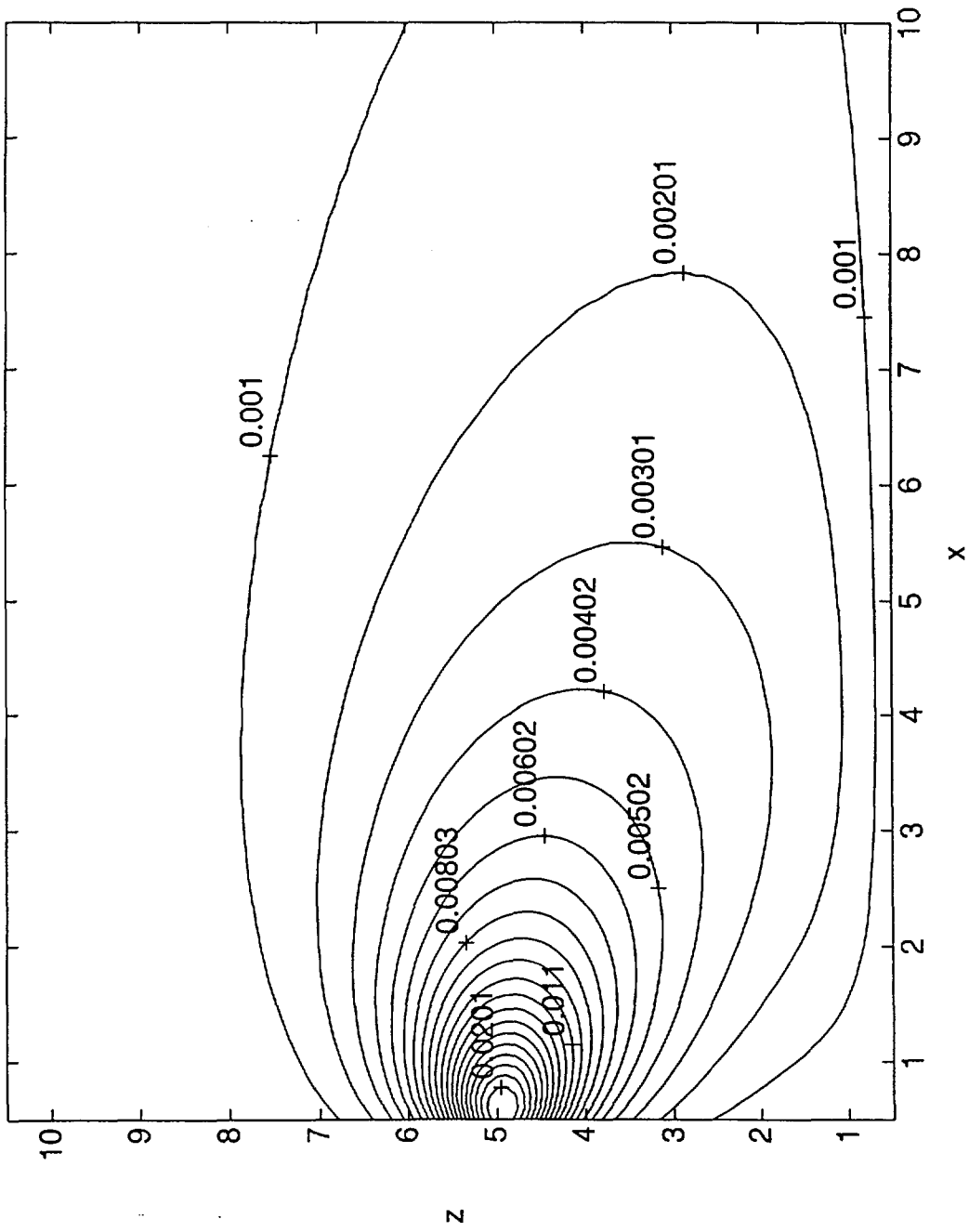


Figure 8b: $h = 5.0, z_0 = 0.5, a = 2.0, b = 1.0, y = 1.0$

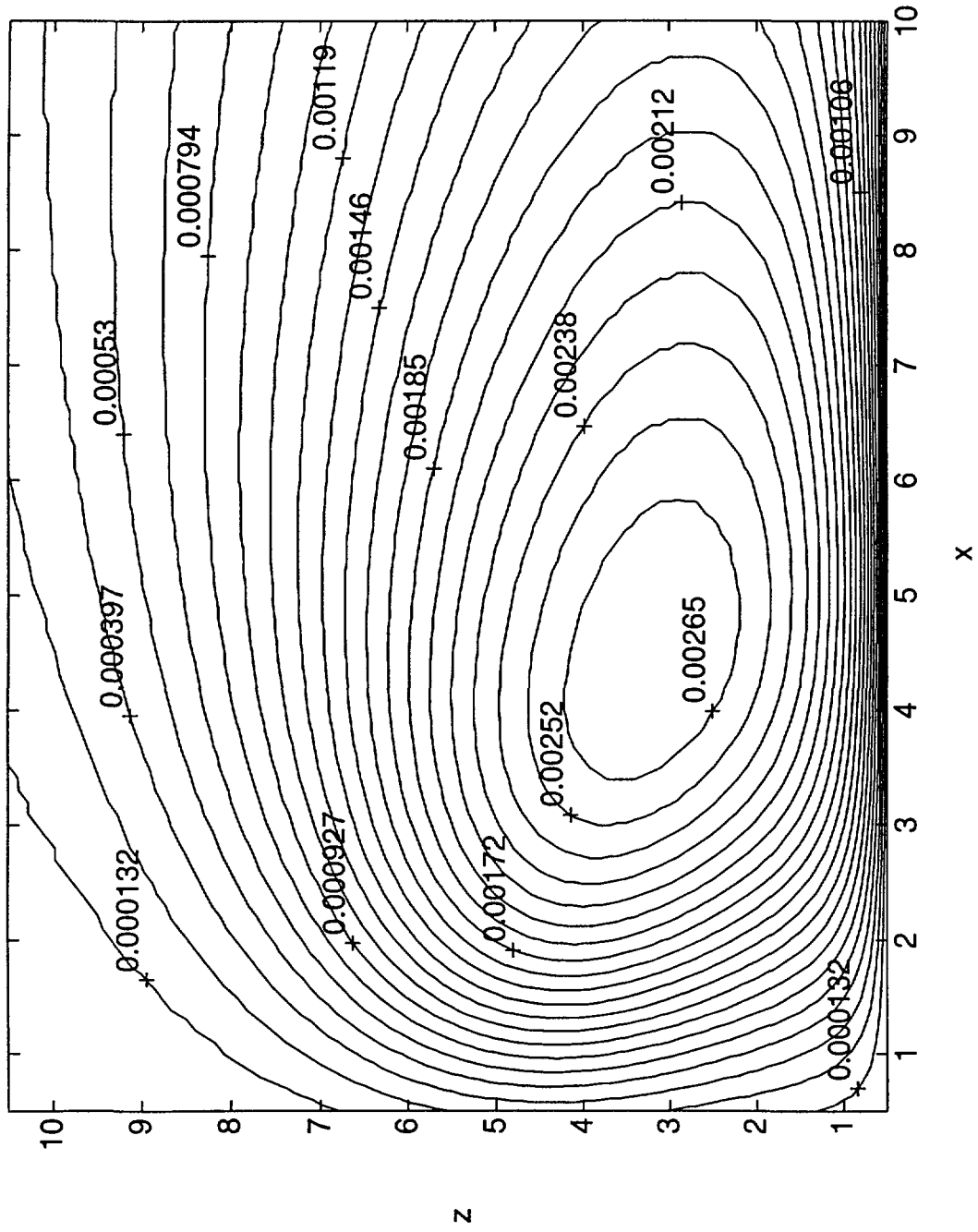


Figure 8c: $h = 5.0, z_0 = 0.5, a = 0.5, b = 0.5, y = 2.5$

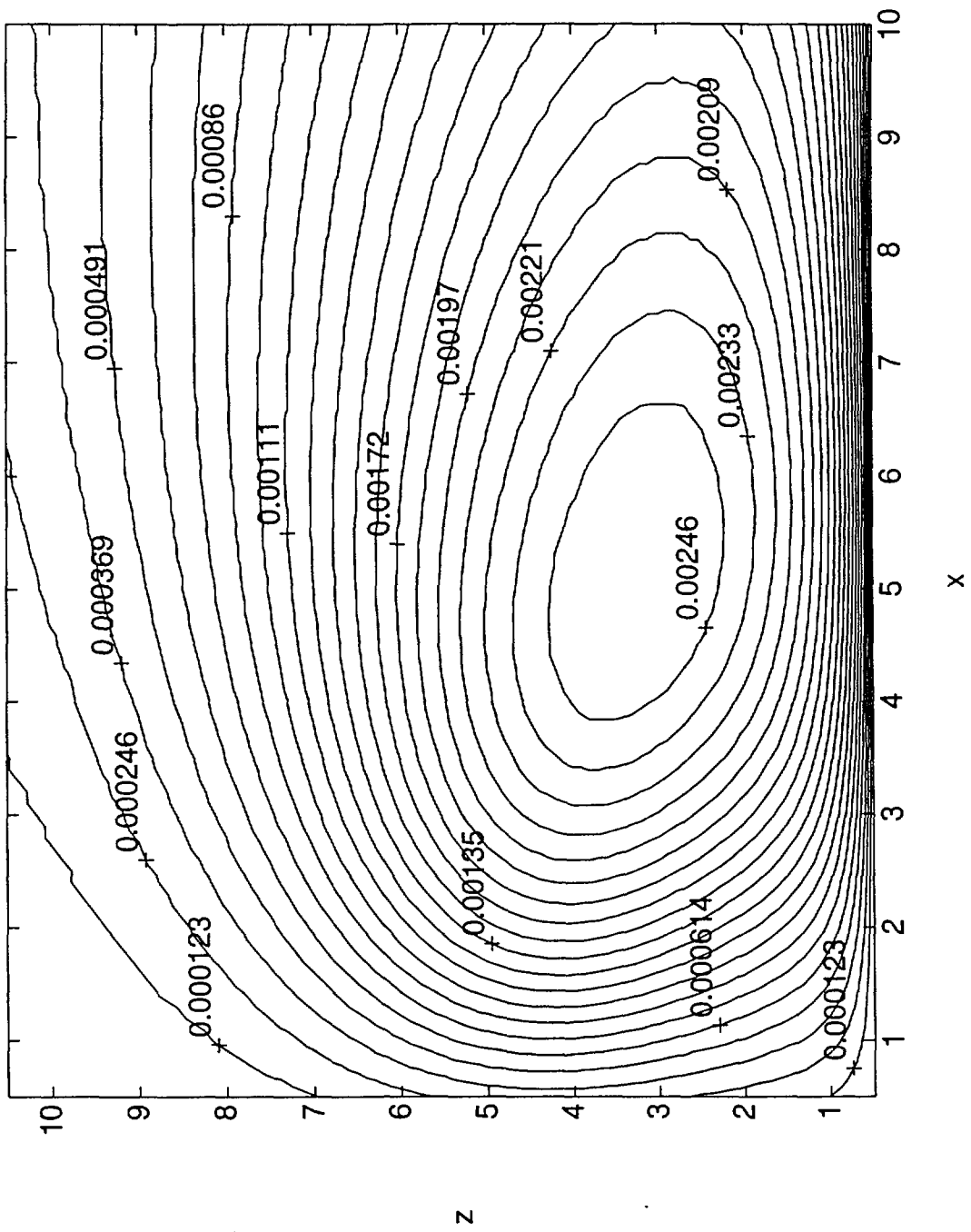


Figure 8d: $h = 5.0, z_0 = 0.5, a = 0.5, b = 1.0, y = 2.5$

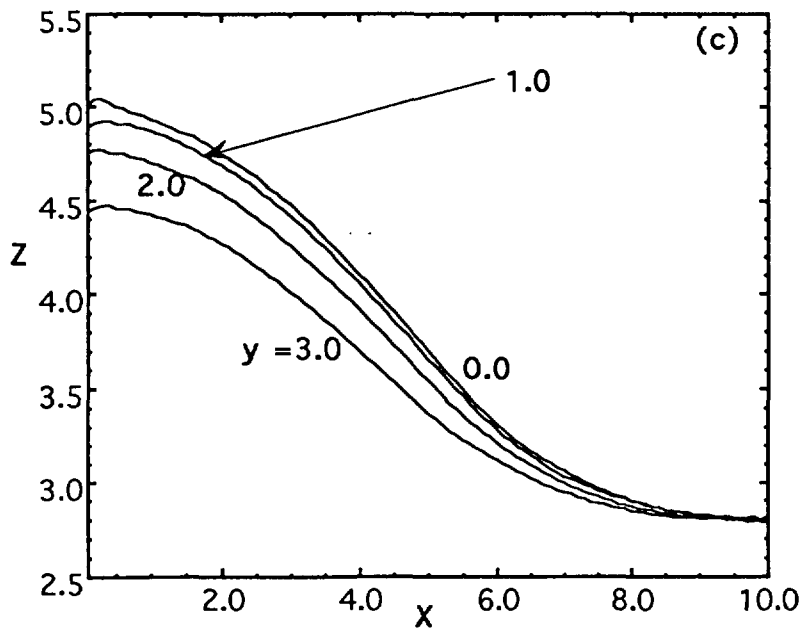
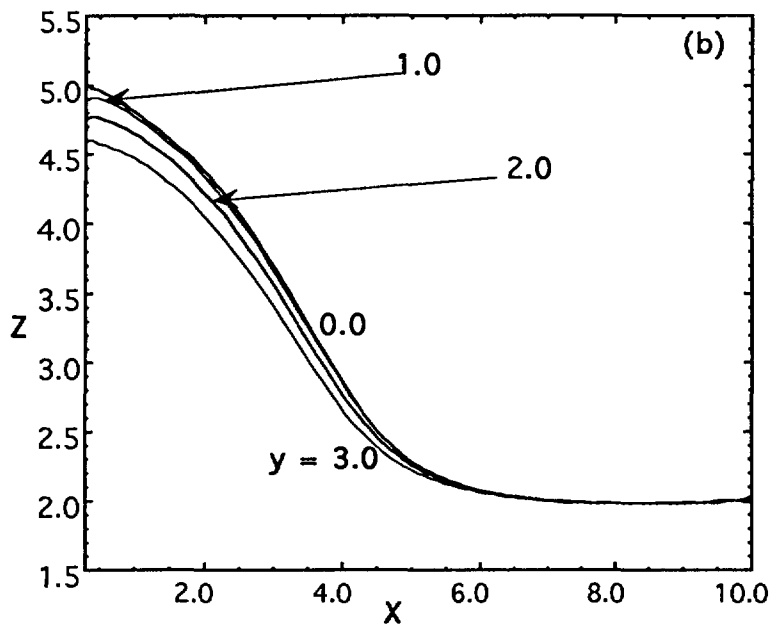
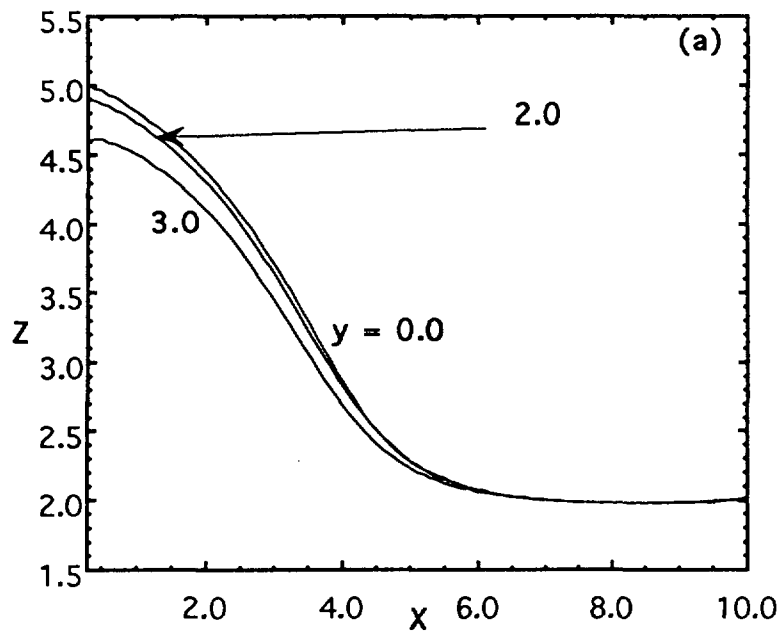


Figure 9