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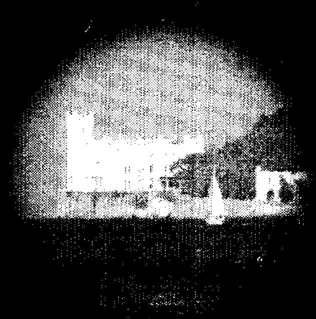

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GENERAL RELATIVISTIC GALVANO-GRAVITOMAGNETIC EFFECT IN CURRENT CARRYING CONDUCTORS

B.J. Ahmedov[1]
Institute of Nuclear Physics,
Ulughbek, Tashkent 702132, Uzbekistan
and
The Abdus Salam International Centre for Theoretical Physics,
Trieste, Italy.

Abstract

The analogy between general relativity and electromagnetism suggests that there is a galvano-gravitomagnetic effect, which is the gravitational analogue of the Hall effect. This new effect takes place when a current carrying conductor is placed in a gravitomagnetic field and the conduction electrons moving inside the conductor are deflected transversally with respect to the current flow. In connection with this galvano-gravitomagnetic effect, we explore the possibility of using current carrying conductors for detecting the gravitomagnetic field of the Earth.

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[1]Fax: +7-3712-642590; E-mail: ahmedov@suninp.tashkent.su

In general relativity, the equation of motion for a test particle with mass m and velocity $\mathbf{v}$ in the weak-field and slow-motion limit has form [1]

$$m\frac{d\mathbf{v}}{dt} \cong m(\mathbf{E}_g + \frac{1}{c}\mathbf{v} \times \mathbf{B_g}). \tag{1}$$

Equation (1) shows that, there is an interesting and useful analogy between weak gravitational fields and electromagnetic fields according to which the gravitational field in the weak limit can be decomposed into a "gravitoelectric" part

$$\mathbf{E}_g = -\frac{GM}{r^2}\mathbf{r}$$

and a "gravitomagnetic" one

$$\mathbf{B}_g = \frac{2G}{c}[\frac{(\mathbf{J} - 3(\mathbf{J}\cdot\mathbf{r})\mathbf{r})}{r^3}],$$

with $\mathbf{E}_g$ and $\mathbf{B}_g$ being the analogues of electric $\mathbf{E}$ and magnetic $\mathbf{B}$ fields. M and $\mathbf{J}$ are the mass and angular momentum of gravitational object, $\mathbf{r}$ is the position of test particle. This analogy brings up the question: are there gravitational analogues of the effects known in electrodynamics?

Indeed, in the case of low velocities and weak fields, the formal analogy between gravity and electromagnetism gives rise to a number of similar phenomena collected under the name gravitoelectromagnetism (see, for review, [1,2]). Due to the weakness of the gravitomagnetic effects, however, their existence has yet to be verified. For slowly rotating masses, such as the Earth or the Sun, the gravitomagnetic field is expected to be extremely weak. Even near the surface of the Earth, the gravitomagnetic contribution is about 10^6 times smaller than the gravitoelectric (monopolar) one. However, this prediction of general relativity has been recently tested [3] by means of data obtained from laser-ranging observations of LAGEOS satellites and is expected to be verified again in the near future via the Gravity Probe B experiment (which measures the mechanical precession of superconducting gyroscopes carried by drag-free satellite in a polar orbit around the Earth [4]).

The use of highly sensitive SQUIDs for detecting general relativistic effects can significantly reduce the use of mechanical measuring devices such as gyroscopes and favour the use of equivalent but more precise electromagnetic devices. This circumstance motivates the investigation of gravitoelectric and gravitomagnetic effects on electromagnetic processes in (super-)conductors.

The electric field induced by the gravitoelectric field in conductors has been widely discussed starting with Shiff and Barnhill (SB) who observed [5] that electrons inside a metal would sag under gravity, until a constant gravitoelectric field is balanced by the electrostatic force of compression. This would create an electric field $E = \frac{mg}{e} = -5.6 \times 10^{-11} V/m$ inside the metal that would exactly compensate the acceleration due to gravity on the electron. Moreover, Dessler et al (DMRT) also observed [6] that not only the electrons but also the ions should sag under gravitoelectric field to produce an effect $10^3 - 10^4$ times greater and of oposite sign with $E \approx +10^{-6} V/m$. The conclusion is that both effects described by SB and DMRT occur, but under certain conditions there is a competition between them [7-9].

Similarly, the influence of the gravitomagnetic field on the magnetic properties of (super-)conductors has been investigated by a number of authors starting with De Witt [10] and Papini [11] both from an experimental and theoretical point of view. For example, proposals have been made to detect the magnetic London moment generated by the gravitomagnetic field in superconductors. Unfortunately the present technology is still unsufficient to detect such weak magnetic fields.

However, we expect that the interplay between the gravitomagnetic field and the electric current can amplify the general relativistic effects and the use of it might therefore improve the experimental accuracy. In this respect, conductors are more suitable for research than superconductors, since electric currents vanish inside the superconducting media. To the best of our knowledge no attempt has been made to investigate such effects and our goal here is to study the gravitomagnetic effects on the electric current flowing in a conductor with the aim to find new general relativistic effects.

It is well-known that the macroscopic consequence of the Lorentz force on the conduction current is the Hall effect [12], according to which an electric field

$$\mathbf{E}_H = R_H \mathbf{j} \times \mathbf{B} \tag{2}$$

appears across a conductor immersed in a magnetic field $\mathbf{B}$ and crossed by a current $\mathbf{j}$. Here R_H is the Hall constant. Driven by the gravitomagnetic analogies we might ask whether the gravitomagnetic field can act on the moving conduction electrons in the media as does the magnetic one, i.e. is there gravitomagnetic analogue of the Hall effect?

In order to get an answer to this question we first formulate Ohm's law for a conductor embedded in the weak external gravitational field derived starting from the equation of motion of conduction electrons inside a conductor subjected to an electromagnetic and gravitational fields

$$m\frac{d\mathbf{v}}{dt} \cong m(\mathbf{E}_g + \frac{1}{c}\mathbf{v} \times \mathbf{B}_g) - e(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}) + \frac{ne^2}{\lambda}\mathbf{v}. \tag{3}$$

The last term in (3) is due to the resistance force acting on conduction electrons from continuous medium with conductivity λ, n is the electron's concentration.

In a steady state, with $\frac{d\mathbf{v}}{dt} = 0$ everywhere, the equation (3) gives Ohm's law

$$\mathbf{E} = \frac{1}{\lambda}\mathbf{j} - R_H \mathbf{j} \times \mathbf{B} + \frac{A}{c^2}\mathbf{E}_g + \frac{R_{gg}}{2c}\mathbf{j} \times \mathbf{B}_g, \tag{4}$$

where $\mathbf{j} = ne\mathbf{v}$ is conduction current density. Here $A = \frac{(m-\gamma M_a)c^2}{e}$ and $R_{gg} = \frac{2m}{ne^2}$ are the coefficients of proportionality, the value of a parameter γ depends on model and typically is of order 0.1 for metals [13]. In the formula (4) we add the term arising from the change of the electrochemical potential μ in the weak gravitational field and being proportional to the atomic mass M_a and gravitoelectric field E_g (see, for example, [13, 14, 15]).

The first two terms on the right hand side of equation (4) are standard classical terms. The third contribution is due to the gravitoelectric field. The last term in the equation (4) is new and never discussed before. It has pure general relativistic nature without Newtonian analogue, and is caused by the effect of gravitomagnetic force on the conduction current.

According to expression (4), an azimuthal voltage

$$V = \frac{R_{gg}}{2cd} i_r B_g \tag{5}$$

will be induced across a conductor carrying radial current i_r even if an external magnetic field is assumed to be absent (d is the thickness of the conductor). Conduction electrons moving in radial direction would be deflected in azimuthal direction until the gravitomagnetic force is compensated by the electric field arising from charge separation on the lateral sides of the conductor. This new general relativistic effect arising from newly incorporated term in Ohm's law is of purely gravitomagnetic origin and so can be referred to as galvano-gravitomagnetic one (galvano-gyroscopic effect, which is the rotational analogue of the Hall effect, is discussed in [19]).

Alternatively, the equation (4) can be obtained directly from the general relativistic constitutive equation [16]

$$F_{\alpha\beta}u^\beta = \frac{1}{\lambda}\jmath_\alpha + R_H(F_{\nu\alpha} + u_\alpha u^\sigma F_{\nu\sigma})\jmath^\nu + Aw_\alpha - b\jmath^\beta A_{\alpha\beta} \tag{6}$$

for conductors embedded in the stationary gravitational field. Here $F_{\alpha\beta}$ is the electromagnetic field tensor, $E_\alpha = F_{\alpha\beta}u^\beta$ and $B_\alpha = -\frac{1}{2}e_{\alpha\beta\mu\nu}u^\beta F^{\mu\nu}$ are the electric and magnetic fields as measured by an observer at rest with respect to the conductor, $A_{\beta\alpha} = u_{[\alpha,\beta]} + u_{[\beta}w_{\alpha]}$ is the relativistic rate of rotation, $w_\alpha = u_{\alpha;\beta}u^\beta$ is the absolute acceleration, $b = \frac{2mc}{ne^2}$ is the parameter for the conductor with the four velocity u^α, $[\cdots]$ denotes the antisymmetrization. The last term in the right hand side of equation (6) is the one caused by the gravitomagnetic effect on conduction current.

Ohm's law for conduction current has been generalized to include effects of gravity and inertia in the recent papers [7, 13, 15], but the effect of gravitomagnetic force upon an electric current has been taken into account only in [16]. Recently, Khanna [17] has derived the general relativistic Ohm's law for a two-component plasma and concluded that it has no new terms as compared with special relativity in the limit of quasi-neutral plasma. The gravitomagnetic terms did not appear in Ohm's law because of the magnetohydrodynamic approximation used in [17] for the fully ionized plasma, but would appear in the case of a weakly ionized uncharged plasma.

Suppose the conductor is at rest in the spacetime of the slow rotating gravitational body of mass M and angular momentum J [18][2]:

$$ds^2 = -\Big(1 - \frac{\alpha}{r}\Big)(dx^0)^2 + \Big(1 - \frac{\alpha}{r}\Big)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 - \frac{2a\alpha \sin^2\theta}{cr} dx^0 d\varphi \qquad (7)$$

with $a = J/cM$ and $\alpha = 2GM/c^2$. Then with the absolute acceleration of conductor

$$w_\alpha\{0, -\frac{\alpha/2r^2}{1-\alpha/r}, 0, 0\}$$

and with the nonvanishing components of the relativistic rate of rotation

$$A_{13} = \frac{a\alpha/2r^2}{c(1-\alpha/r)^{3/2}} \sin^2\theta; \quad A_{23} = -\frac{a\alpha/r}{c(1-\alpha/r)^{1/2}} \sin\theta\cos\theta,$$

the the general-relativistic Ohm's law (6) takes the form (4).

However, metric (7) can be transformed to the reference frame of a satellite orbiting at a radius r_0 with angular velocity $\Omega = \epsilon\sqrt{\frac{\alpha c^2}{2r_0^3}}$:

$$ds^2 = -\Big(1 - \frac{\alpha}{r}\Big)(dx^0)^2 + \Big(1 - \frac{\alpha}{r}\Big)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2 + 2\Big(\frac{\epsilon\Omega r^2}{c} - \frac{a\alpha}{cr}\Big) \sin^2\theta dx^0 d\varphi, \qquad (8)$$

assuming + and − signs of parameter $\epsilon = \pm$ apply to direct and retrograte orbits, respectively.

Ohm's law (6) in the metric (8) takes form

$$\mathbf{E} = \frac{1}{\lambda}\mathbf{j} - R_H \mathbf{j} \times \mathbf{B} + \frac{A}{c^2}\mathbf{E}_g + \frac{R_{gg}}{2c}\mathbf{j} \times \mathbf{B}_g - R_{gg}\mathbf{j} \times \mathbf{\Omega}. \qquad (9)$$

[2]For the Earth, the dimensionless gravitomagnetic contribution $GJ/c^3r^2 < 4 \times 10^{-16}$ is much less than the gravitoelectric one $GM/c^2r < 7 \times 10^{-10}$. Hence, in what follows the angular momentum will be taken into account only to first order.

Restricting ourselves to an equatoril orbit, for the sake of simplicity, we calculate from (9) the voltage across a conductor carrying radial current i_r

$$V = V_{Sch} + V_{LT} = \epsilon \frac{R_{gg}}{d} i_r \Omega + \frac{R_{gg}}{2cd} i_r B_g. \tag{10}$$

The first term in the right hand side of (10) results from the rotational motion of the conductor through the curved space-time around the Earth and is proportional to the square root of the Schwarzschild parameter α. We are here interested in the second term which is due to rotation of central body and corresponds to the gravitomagnetic effect. For experimental purposes, it is clearly better to eliminate the lower order terms in expression (10). In order to do this, we could send two conductors in geosynchronous and anti-geosynchronous orbits and compare their data when they pass overhead. The gravitomagnetic term will have the same sign for both satellites but Schwarzschild term changes the sign from one station to another. So the simple addition of the data from the counter-rotating stations will isolate the pure gravitomafnetic term, and therefore, it is possible to separate gravitomagnetic effect from the dominant Schwarzschild effect.

The gravitomagnetic effect may be regarded, among other things, as a consequence of gravitational analogue of Larmor's theorem [20] according to which the effect of the gravitomagnetic force on an electron is the same as that of the Lorentz force $\mathbf{F} = -\frac{e}{c}(\mathbf{v} \times \mathbf{B})$ due to magnetic field $\mathbf{B}$, if $\mathbf{B}$ satisfies

$$\mathbf{B} = -(2mc/e)\mathbf{\Omega}_{LT}, \tag{11}$$

where the gravitomagnetic field $\mathbf{B}_g$ is identified with the Lense-Thirring frequency $-2c\mathbf{\Omega}_{LT}$ of the local inertial frames with respect to observers at infinity. Since the macroscopic consequence of the Lorentz forces on the conduction electrons is the Hall effect (2), the result (5) follows, in fact, from the gravitomagnetic Larmor's theorem (11).

The value of the potential difference in (5) can be adjusted by the magnitude of the electric current and this dependence may give one more possibility to detect the gravitomagnetism by measuring the difference in a voltage generated across two current carrying conductors orbiting in pro- and retrograte directions around the Earth. Of course, ground based laboratory experiment for measurement of the galvano-gravitomagnetic effect could be also derived. But on Earth, the angular velocity of the conductor with respect to a local inertial frame $\mathbf{\Omega}_L$ is given by [1]:

$$\mathbf{\Omega}_L = \mathbf{\Omega}_{VLBI} - \mathbf{\Omega}_{Th} - \mathbf{\Omega}_S - \mathbf{\Omega}_{LT},$$

where $\mathbf{\Omega}_{VLBI}$ is the angular velocity of the laboratory with respect to an asymptotic inertial frame, $\mathbf{\Omega}_{Th}$ and $\mathbf{\Omega}_S$ are, respectively, the contributions of the Thomas precession arising from non-gravitational forces and of the de Sitter or geodetic precession. As a result, in order to detect $\mathbf{\Omega}_{LT}$ one should measure $\mathbf{\Omega}_L$ and then substract from it the independently measured value of $\mathbf{\Omega}_{VLBI}$ with VLBI (Very Long Baseline Interferometry, see, for example, [21]) and the contributions due to the Thomas and de Sitter precessions. A number of disturbing effects such as seismic accelerations, local gravitational noise, atmospheric turbulence have long made ground-based experiment difficult. Satellite based experiments can offer the possibility of reduction in these disturbances, although they will also raise other additional difficulties.

The valuable galvano-gravitomagnetic coefficient $R_{gg} \approx 0.8 \times 10^{-22}s$ is estimated for the typical semiconductor. Then the difference in voltage developed in two counter-rotating semiconductors is $2V_{LT} = 2(R_{gg}/d)i_r\Omega_{LT} \approx 0.6 \times 10^{-19}V$ if we put $a_\oplus = \frac{2}{5}\Omega_\oplus R_\oplus^2$, $\alpha_\oplus = 2 \times G\frac{M_\oplus}{c^2} = 2 \times 0.44cm$, $R_\oplus \approx 6.37 \times 10^8 cm$, $\Omega_\oplus = 7.27 \times 10^{-5} rad/s$, $i_r = 10^3 A$ and $d = 10^{-2}cm$.

In principle, it is possible to measure a voltage of $10^{-19}V$ with today's SQUID tecnology with precise voltage accuracies [22] of 1 part in $10^{22}V$. In a possible experiment a superconducting loop of SQUID can be connected across carrying radial current semiconductor which will form normal layer of Josephson junction with source of constant voltage V_{LT}. The nonvanishing potential difference V_{LT} would lead to a time varying magnetic flux through the loop. The change in magnetic flux Φ_b inside the circuit during the time interval $[0, t]$ is

$$\Delta\Phi_b = \Delta n\Phi_0 + c\int_0^t V_{LT}dt \tag{12}$$

where $\Phi_0 = \pi\hbar c/e = 2\times 10^{-7} Gauss\cdot cm^2$ is quantum of the magnetic flux. As long as $\Delta\Phi_b < \Phi_0$, n will remain constant and $\Delta\Phi_b$ will increase linearly with time until $\Delta\Phi_b = \Phi_0$, then the order of the step n will change as flux quantum enters the loop. Thus this particular loop is sensitive to the V_{LT} and in this connection to the Lense-Thirring frequency. Moreover, it should be underlined that the proposed scheme of flux measurement (12) is cumulative and the long time measurement can be used for accumulation of data towards better detection of the gravitomagnetic effect.

The experiment is quite difficult since the measured effect is small compared to potential disturbances and careful attention must be paid to possible systematic experimental errors. While we do not intend to present here a detailed proposal but to point out the possibility of new electromagnetic test of general relativity.

We can foresee two major experimental difficulties: (i) ensuring that the deviation of the semiconductor's orbit from idealized one is in the limit of the required accuracy, and (ii) shielding the Earth's magnetic fields which can induce much bigger voltage due to the Hall effect and superconducting shells provide a natural means of shielding of apparatus from external magnetic field. Any experimental project would need to take these problems into serious consideration.

In this letter, we have shown that the effect of the gravitomagnetic force on the conduction current is to induce a galvano-gravitomagnetic potential difference just as the action of magnetic field is to develop a Hall voltage across current carrying conductor. This is the general relativistic analogue of the Hall effect and when current carrying semiconductor is subject to general-relativistic gravitational field of Earth, a potential difference around $10^{-19}V$ will be developed across it. The general relativistic galvano-gravitomagnetic effect seems to be experimentally verifiable with the present technology. Because of the great importance of gravitomagnetic fields for astrophysics and fundamental physics, such experiment would also constitute an important and direct measurement of the general relativistic Lense-Thirring effect.

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