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ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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PARALLEL-PIPELINED INTERPRETATION OF NEW ALGORITHM OF SECOND KIND FREDGOLM's INTEGRAL EQUATION WITH TOEPLITZ TYPE KERNEL NUMERICAL SOLUTION ON SYSTOLIC ARRAY.

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Նախատիպ ԵՖԻ-1405(16)-93

Ս. Հ. ՄԽԻԹԱՐՅԱՆ

ՖՐԵԴՀՈԼՄԻ ԵՐԿՐՈՐԴ ԿԱՐԳԻ ՏԵՂԱՓՈԽՈՒԹՅԱՆ ՏԻՊԻ ԿՈՐԻԶՈՎ ԻՆՏԵԳՐԱԼ ՀԱՎԱՍԱՐՈՒՄԻ ԹՎԱՅԻՆ ԼՈՒԾՄԱՆ ՆՈՐ ԱԼԳՈՐԻԹՄԻ ԶՈՒԳԱՀԵՌ-ՀՈՍՔԱՅԻՆ ՍԵԳՆԱՐԱՆՈՒՄԸ ՍԻՍՏՈԼԻԿ ՑԱՆՑԱՎԱԾԻ ՕԳՆՈՒԹՅԱՄԲ

Առաջարկված է [illegible] ձևափոխված Ֆրեդհոլմի երկրորդ կարգի Տեղափոխության տիպի կորիզով ինտեգրալ հավասարումների թվային լուծման նոր ալգորիթմի իրագործման [illegible]: Ցուցաբերված կառուցվածքի համար որոշված է [illegible] գործակիցը, որը թույլ է տալիս գնահատել [illegible] բազմապրոցեսորային [illegible] կիրառման տարբերակները:

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ԵՐԵՎԱՆ - ...

The fast algorithms of numerical solution second kind Fredgolm's integral equations

$$y(x)= \int_0^1 K(x,t)y(t)dt+f(x), \qquad x \in [0,1]. \qquad (1)$$

with Toeplitz type kernels are very important because many applied problems, in particular, physical, come to. The new algorithms for the analogical purpose have been proposed in [1]. Since the realization time of the algorithms is long on scalar computer because of their large operational complications ($O(N^2)$, NL8), the parallel-pipelined interpretations is very actual on VLSI (Very Large Scale Integration) base. The parallel-pipelined interpretation one of them is presented.

The Toeplitz type kernel is satisfied the condition

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} \right) K(x,t)= \sum_{k=1}^{m} p_k(x)q_k(t). \qquad (2)$$

Let F(i,n) equal F(ih,nh):

$$F(i,n) \overset{def}{=} F(ih,nh),$$

where $i, n \in Z$, $F(i,n)$ is any of the mentioned below functions. The discretization process may be considered on the net with nodes, having integer co-ordinates.

The structure of one of this algorithms solution is following:

```
/input data/
for 0≤i≤N do
begin
Φ(i,o)=Φ_1(i,o)=K(O,i);
Ψ(i,O)=Ψ_1(i,O)=K(i,O);
y(i,o)=f(i);
for 1≤k≤m do (m < N)
begin
P_k(i,O)=p_k(i);
Q_k(i,O)=q_k(i)
end
end

/internal calculations/
 for 0≤n≤N-1 do
 for 0≤i≤N    do
 begin
 if n<N-1 then
 begin
 Φ(i,n+1)=Φ(i,n)+nΦ(n,n)Φ_1(i,n);                    (3)
 Ψ(i,n+1)=Ψ(i,n)+hΨ(n,n)Ψ_1(i,n);                    (4)
 for 1≤k≤m do
 begin
 P_k(i,n+1)=P_k(i,n)[illegible]_k(n,n)Ψ_1(i,n);      (5)
 Q_k(i,n+1)=Q_k(i,n[illegible]_k(n,n)Φ_1(i,n)        (6)
```

```
end
end
else
begin
Φ(n+1,n+1)=Φ(n+1,n)+hΨ(n,n)Φ_1(n+1,n);
Ψ(n+1,n+1)=Ψ(n+1,n)+hΦ(n,n)Ψ_1(n+1,n)
end
else
begin
Φ_1(i,n+1)=Φ_1(i-1,n)+hΨ(n,n)Φ(i-1,n)+ Σ_{k=1}^{m} hP_k(n,n)Q_k(i-1,n);    (7)

Ψ_1(i,n+1)=Ψ_1(i-1,n)+hΦ(n,n)Ψ(i-1,n)+ Σ_{k=1}^{m} hQ_k(n,n)P_k(i-1,n)     (8)

end;

y(i,n+1)=y(i,n)+hy(n,n)Ψ_1(i,n)     (9)

end;

/output calculations/
for 1≤n≤N-1 do
for 0≤i≤N
begin
α^(1)=y(i,n);
α^(2)=Φ_1(i,n+1);
α^(3)=Ψ_1(i,n+1)
end
```

The quantities $\Phi_1(i,n+1)$ and $\Psi_1(i,n+1)$ provide the second kind integral operators factorization. We introduce the following designations $\beta_{nk}=hP_k(n,n)$; $\gamma_n=h\psi(n,n)$; $\rho_{nk}=hQ_k(n,n)$; $\alpha_n=h\Phi(n,n)$; $w_n=hy(n,n)$. The recurrent relations (3) – (9) are transformed so:

$$\Phi(i,n+1)=\Phi(i,n)+\alpha_n\Phi_1(i,n); \qquad (10)$$

$$\Psi(i,n+1)=\Psi(i,n)+\gamma_n\Psi_1(i,n+1); \qquad (11)$$

$$P_k(i,n+1)=P_k(i,n)+\beta_{nk}\Psi_1(i,n); \qquad (12)$$

$$Q_k(i,n+1)=Q_k(i,n)+\rho_{nk}\Phi_1(i,n); \qquad (13)$$

$$\Phi_1(i,n+1)=\Phi_1(i-1,n)+\gamma_n\Phi(i-1,n)+\sum_{k=1}^{m}\beta_{nk}Q_k(i-1,n); \qquad (14)$$

$$\Psi_1(i,n+1)=\Psi_1(i-1,n)+\alpha_n\Psi(i-1,n)+\sum_{k=1}^{m}\rho_{nk}P_k(i-1,n); \qquad (15)$$

$$y(i,n+1)=y(i,n)+w_n\Psi_1(i,n) \qquad (16)$$

The operational complication of this algorithm is $\Omega=(4m+5)N^2+O(N)$. The temporal step is defined as the time of accumulator operation $(s\leftarrow s+ab)$ implementation. The temporal complication (the realization time on the scalar computer) T_1 is equal Ω. This algorithm may be implemented by parallel-pipelined way on the computational network, consisting of N+1 processors. The work of the every computer becomes complicated by calculations of quantities $\Phi(i,n+1)$, $Q_k(i,n+1)$; $P_k(i,n+1)$ $(1\le k\le m)$: $\Phi_1(i,n+1)$; $\Psi_1(i,n+1)$; $y(i,n+1)$ on the every stage.

We will analyze the structure of recurrent relations. Quantities $\Phi(i,n+1)$; $Q_k(i,n+1)$ depend on $\Phi_1(i,n)$ and other $\Psi(i,n+1)$; $P_k(i,n+1)$; $y(i,n)$ depend on $\Psi_1(i,n+1)$. The calculation $\Phi_1(i,n+1)$ $(\Psi_1(i,n+1))$ is required m+1 accumulator operations. (m+1) analogical operations are implemented for summary calculations of quantities $\Phi(i,n+1)$; $Q_k(i,n+1)$ $(\Psi(i,n+1)$; $P_k(i,n+1))$ $(1\le k\le m)$.

It's advisable to use the cylindrical computational network, consisting of 4(N+1) processors (fig. 1). The cylindrical form provides the one-directed of horizontal communications.

Four chains are united in the computational network, but every one of them is the autonomic computational environment, in which the necessary operands are received from other chains corresponding functional devices.

The every chain's processors are intended for following quantities calculations:

1 chain's – $\Phi(i,n+1)$; $Q_k(i,n+1)$ $(1\le k\le m)$;

2 chain's – $\Phi_1(i,n+1)$ $(i>0)$;

3 chain's – $\Psi_1(i,n+1)$ $(i>0)$;

4 chain's – $\Psi(i,n+1)$; $P_k(i,n+1)$ $(1\le k\le m)$.

We describe the work of (i,j)–computer, where j –chain's number (j=1, 2, 3, 4); i – number of the j-chain's processor. Boolean variables $\alpha_k^{(0)}$, $\alpha_k^{(1)}$, ϵ; $\delta_1^{(0)}$, $\delta_2^{(0)}$; $\delta_3^{(0)}$ (constant signals); X'_k , X_k, r' , r (internal registers). The constant signals equal 0, when they present, and equal 1, otherwise.

The (0,1)-processor (fig. 2a) functioning program is following:

if $\varepsilon=1$ then

begin

1: if $\alpha_0^{(0)}=0$ then begin $X'_0=b_{in}$; $X_0=hb_{in}$; $a_{out}=b_{out}=X_0$; $X_0=0$; go to 2 end;

2: for $1\le k\le m$ do

if $\alpha_k^{(0)}=0$ then begin $X'_k=b_{in}$; $X_k=hb_{in}$; $a_{out}=b_{out}=X_k$; $X_k=0$ end;

if $\alpha_m^{(0)}=0$ then begin $X'_m=b_{in}$; $X_m=hb_{in}$; $a_{out}=b_{out}=X_m$; go to 3 end;

3: if $\alpha_0^{(0)}=1$ then begin $X_0=X'_0+a_{in}^{(1)}X'_0$; $b_{out}=c_{out}=a_{in}^{(2)}$; go to 4 end;

4: if $\alpha_1^{(0)}=1$ then begin $X_1=X'_1+a_{in}X'_0$; $b_{out}=X'_0$; $X'_0=X_0$; $X'_0=0$; go to 5 end;

5: if $\alpha_2^{(0)}=1$ then begin $X_2=X'_2+a_{in}X_0$; $b_{out}=X'_1$; $X'_1=X_1$; $X_1=0$; go to 6 end;

6: if $\alpha_3^{(0)}=1$ then begin $X_3=X'_3+a_{in}X'_0$; $r=c_{in}$; $b_{out}=X'_2$; $X'_2=X_2$; $X_2=0$; go to 7 end;

7: for $4\le k\le m-1$ do

begin

if $\alpha_k^{(0)}=1$ then begin $X_k=X'_k+a_{in}X'_0$; $b_{out}=X'_{k-1}$; $X'_{k-1}=X_{k-1}$; $X_{k-1}=0$; go to 8 end

end;

8: if $\alpha_m^{(0)}=1$ then begin $X_m=X'_m+a_{in}X'_0$; $b_{out}=X'_{m-1}$; $X'_{m-1}=X_{m-1}$; $X_{m-1}=0$; go to 9 end;

9: if $\delta_1^{(0)}=1$ then begin $b_{out}=X'_m$; $X'_m=X_m$; $X'_m=0$; go to 10 end;

10: if $\alpha_0^{(1)}=0$ then begin $X_0=X_0'+a_{in}^{(1)}r$; $a_{out}^{(1)}=a_{in}^{(1)}$;
$b_{out}=c_{out}=a_{in}^{(2)}$; go to 11 end;

11: if $\alpha_1^{(1)}=0$ then begin $a_{out}=a_{in}$; $X_1=X_1'+a_{in}^{(1)}r$; $b_{out}=X_0'$; $X_0'=X_0$;
$b_{out}^{(2)}=r$; $X_0=0$; go to 9 end;

12: if $\alpha_2^{(1)}=0$ then begin $a_{out}=a_{in}$; $X_2=X_2'+a_{in}r$; $b_{out}=X_1'$; $X_1'=X_1$;
$X_1=0$; go to 13 end;

13: if $\alpha_3^{(1)}=0$ then begin $a_{out}=a_{in}$; $X_3=X_3'+a_{in}r$; $r'=c_{in}$; $X_2'=X_2$;
$b_{out}=X_2$; $X_2=0$; go to 14 end;

14: for $4\le k\le m-1$ do
if $\alpha_k^{(1)}=1$ then begin $X_k=X_k'+a_{in}r$; $b_{out}=X_{k-1}'$; $X_{k-1}'=X_{k-1}$;
$X_{k-1}=0$; $a_{out}=a_{in}$ end;
if $\alpha_m^{(1)}=1$ then begin $X_m=X_m'+a_{in}r$; $b_{out}=X_{m-1}'$; $X_{m-1}'=X_{m-1}$;
$X_{m-1}=0$; $a_{out}=a_{in}$ end
end
else
$a_{in}=c_{out}$;

The sequence of values $\alpha_k^{(0)}=0$ $(0\le k\le m)$; $\delta_1^{(0)}=1$ $(\varepsilon=1)$ operates the calculations on the first stage, the other $\alpha_k^{(0)}=1$ $(0\le k\le m)$, $\delta_2^{(0)}=1$ works at rest of data processing time; the value $\varepsilon=0$ ends the computational course in (0,1)-processor. The external shower of 0 and 1, is directed to data computer in the plane, perpendicular to the surface of the systolic structure.

The functioning scheme of the first chain's other processors (fig. 2 b,c, d) may be presented with some modifications. The count of (N,1)-computer's internal registers is equal half the analogical quantity of the first chain's other processors, because operands aren't translated in the vertica direction (down).

(i,2)-processor calculates the quantity $\Phi_1(i,m+1)$ on the every stage. that the computational process is simplified (the count of states don't depend on m).

The following boolean variables organize the data processing management: α', β', γ': X (the internal register). The shower of values is directed to the second chain in plane, perpendicular to the surface of the systolic array during calculating time. The shower for first stage is pictured on fig. 3.

So, the program of (i,2)-computer's (fig. 2e) work is following:

if $\alpha'=\beta'=\gamma'=w'=0$ then $b_{out}=b_{in}$;

if $\alpha'=\gamma'=w'=0$ and $\beta'=1$ then begin $b_{out}=b_{in}$; $a_{out}=c_{in}$ end;

if $\alpha'=1$ and $\beta'=\gamma'=w'=0$ then $a_{out}=c_{in}$;

if $\gamma'=1$ and $\alpha'=\beta'=w'=0$ then begin $c_{out}=c_{in}$; $X=a_{in}b_{in}+b_{in}$ end;

if $\alpha'=\beta'=\gamma'=0$ and $w'=1$ then $X=X+a_{in}b_{in}$;

if $\alpha'=\beta'=1$ and $\gamma'=w'=0$ then begin $X=X+a_{in}b_{in}$; $d_{out}=X$ end;

if $\alpha'=w'=0$ and $\gamma'=\beta'=1$ then begin $a_{out}=a_{in}$; $X=b_{in}^{(1)}+b_{in}^{(2)}a_{in}$; [illegible] $=c_{in}$;

[illegible] $=\beta'=0$ and $\gamma'=w'=1$ then begin $a_{out}=a_{in}$; $X=X+a_{in}b_{in}$ end;

[illegible] $=w'=1$ and $\beta'=\gamma'=0$ then begin $X=X+a_{in}b_{in}$; $d_{out}=X$; [illegible] end;

if [illegible] $=1$ and $\alpha'=\gamma'=0$ then begin $a_{out}=a_{in}$; $X=a_{in}b_{in}+b_{in}$ end;

if $\alpha'=\gamma'=1$ and $\beta'=w'=0$ then $X=b_{in}^{(1)}+a_{in}b_{in}^{(2)}$;

if $\alpha'=\beta'=\gamma'=1$ and $w'=0$ then begin $a_{out}=a_{in}$; $X=b_{in}^{(1)}+a_{in}b_{in}^{(2)}$ end;

Functions of 3 and 4 chains processors (fig. 2 a'–e')may be described, if b_{in} is replaced by c_{in}, b_{out} by c_{out}, c_{in} by b_{in}, c_{out} by b_{out}, d_{in} by d'_{in}, d_{out} by d'_{out} in afore-cited programs. (1,4)-processor (fig. 2e') calculates $y(1,n)$ in addition. The last circumstance corrects the work of (1,4)-processor slightly.

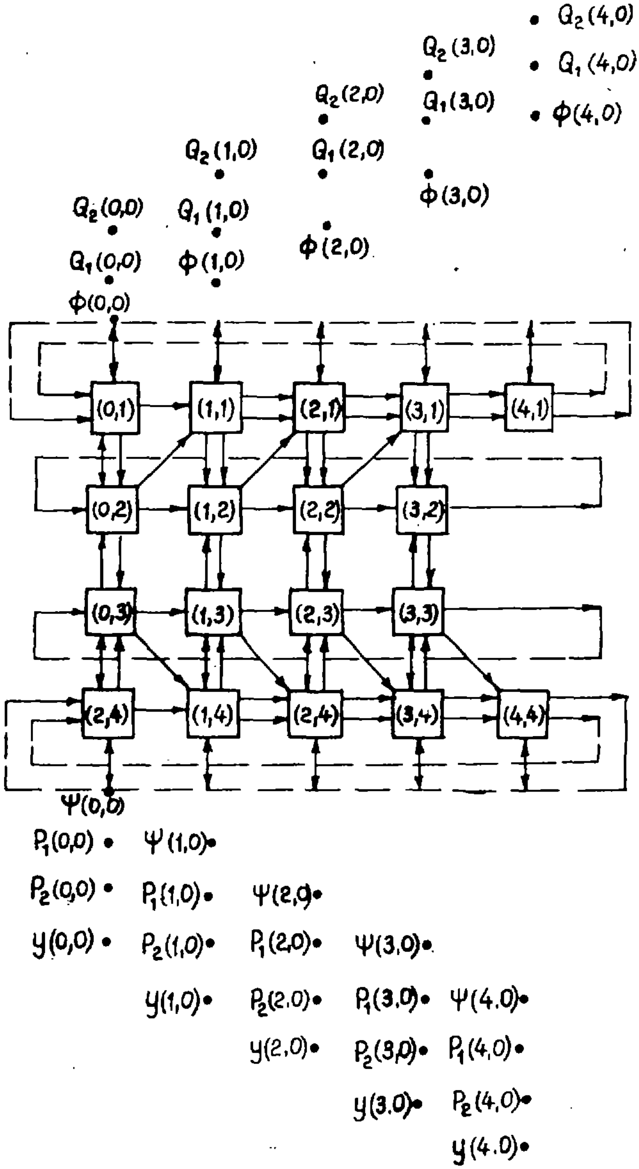
The computational structure works symmetrically: quantities $\Phi(1,n+1)$ and $\Psi(1,n+1)$, $Q_k(1,n+1)$ and $P_k(1,n+1)$ $(1 \leq k \leq m)$, $\Phi_1(1,n+1)$ and $\Psi_1(1,n+1)$ are calculated simultaneously.

The $\Phi(1,n+1)$ $(i>n)$ is calculated on the $t_{\Phi(i,n+1)}=2+(m+2)n+i$ temporal step, $\Phi(i,n+1)$ $(i \leq n)$ on $t_{\Phi(i,n+1)}=3+(m+2)n+N+i$; Q_k $(1 \leq k \leq m)$ on $t_{Q_k(i,n+1)}=t_{\Phi(i,n+1)}+k$; $y(1,n+1)$ on $t_{y(1,n+1)}=$ $=t_{\Phi(1,n+1)}+m+1$; $\Phi_1(1,n+1)$ $(0 \leq n \leq N)$ on $t_{\Phi_1(1,n+1)}=t_{\Phi(1,n+1)}+2$; $\Phi_1(1,n+1)$ on $t_{\Phi_1}(1,N)=2+(m+3)N+1$.

The computational process is ended with the output $\Phi_1(N,N)$ $(\Psi_1(N,N))$ from (0,1)-processor (fig. 4). So the time of this algorithm parallel-pipeline realization is $T_{4N+2}=(m+4)N+3$.

The efficiency coefficient is

$$K_{ef} = \frac{m+\frac{5}{4}}{m+4} .$$



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Q2(3,0)
Q1(4,0)
Q2(2,0)
Q1(3,0)
φ(4,0)
Q2(1,0)
Q1(2,0)
φ(3,0)
Q2(0,0)
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Ψ(0,0)
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P1(1,0)
Ψ(2,0)
y(0,0)
P2(1,0)
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Ψ(3,0)
y(1,0)
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P1(3,0)
Ψ(4,0)
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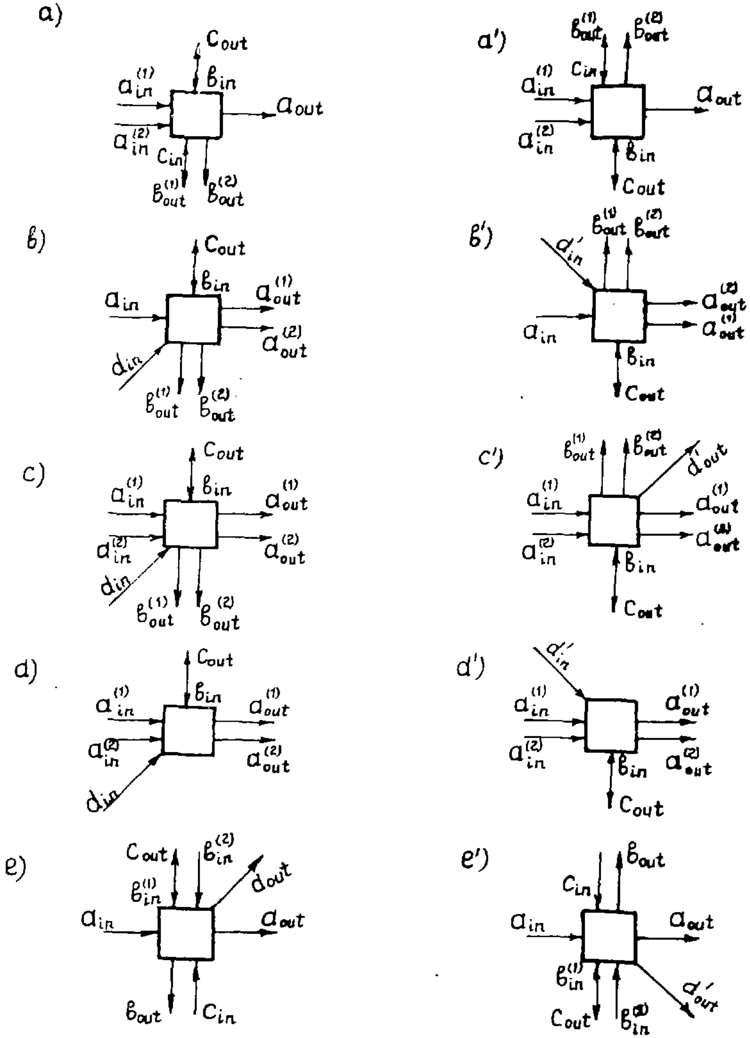
P2(3,0)
P1(4,0)
y(3,0)
P2(4,0)
y(4,0)

Fig.1



a)
Cout
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ain(2)
Cin
bout(1)
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a')
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bout(2)
Cin
ain(1)
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ain(2)
bin
Cout
b)
Cout
bin
aout(1)
ain
aout(2)
din
bout(1)
bout(2)
b')
din'
bout(1)
bout(2)
aout(2)
ain
aout(1)
bin
Cout
c)
Cout
ain(1)
bin
aout(1)
ain(2)
aout(2)
din
bout(1)
bout(2)
c')
bout(1)
bout(2)
dout'
ain(1)
aout(1)
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bin
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d)
Cout
ain(1)
bin
aout(1)
ain(2)
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d')
din'
ain(1)
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e)
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Cout
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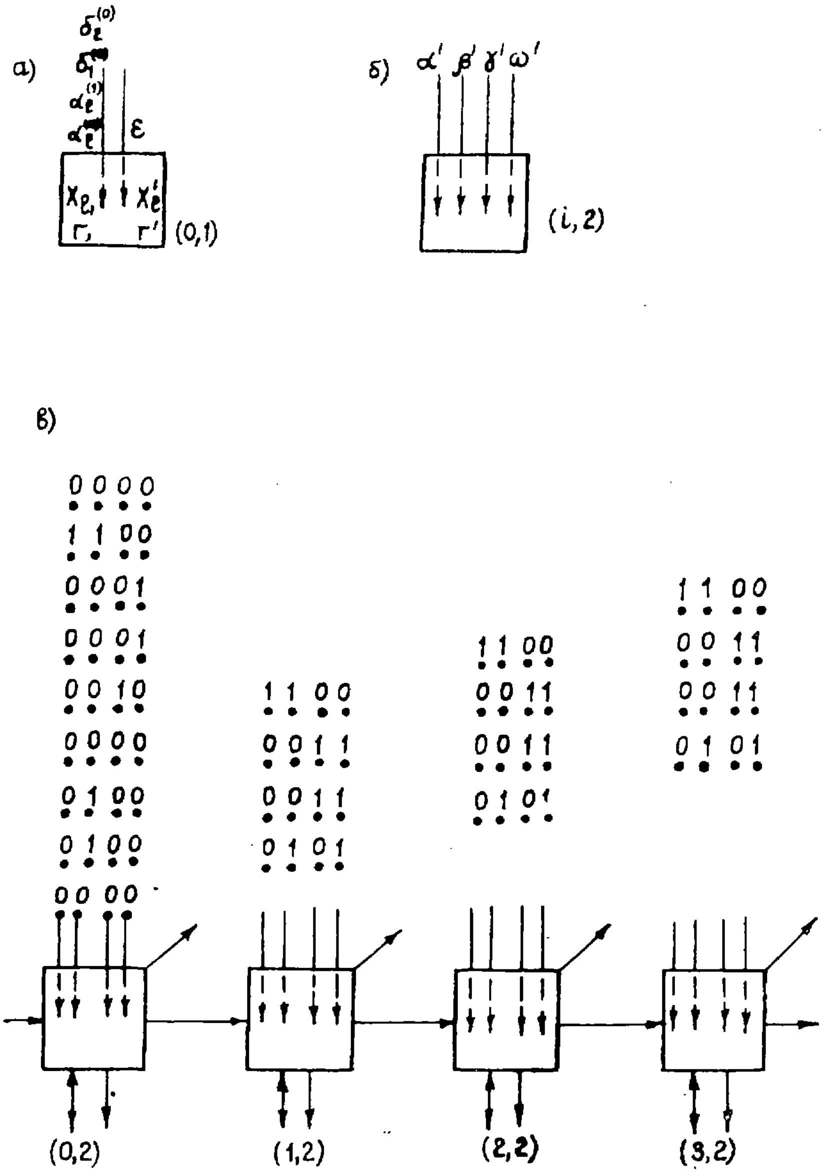
Fig.2


а)
$\delta_\ell^{(0)}$
$\alpha_\ell^{(1)}$
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X'_ℓ Г'
(0,1)
б)
α' β' γ' ω'
(i,2)
в)
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0011
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0101
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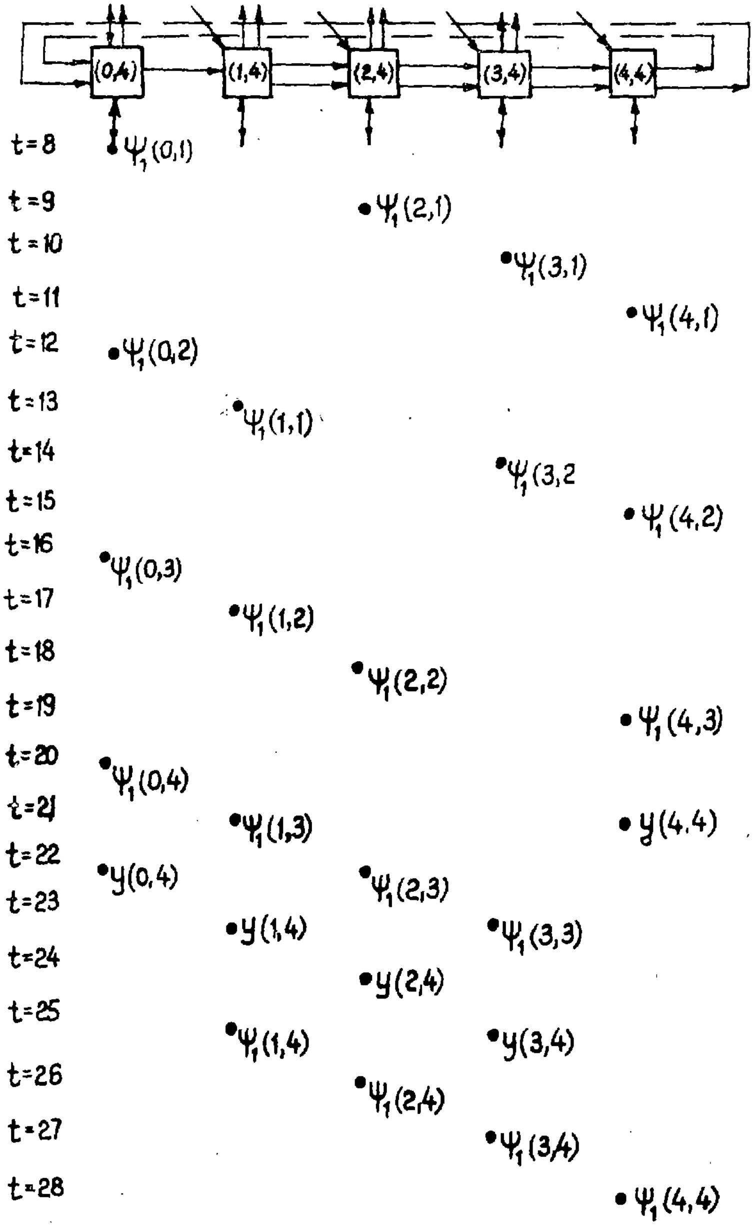
(2,2)
(3,2)

Fig.3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t=22
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t=24
t=25
t=26
t=27
t=28
ψ1(0,1)
ψ1(2,1)
ψ1(3,1)
ψ1(4,1)
ψ1(0,2)
ψ1(1,1)
ψ1(3,2
ψ1(4,2)
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ψ1(1,2)
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ψ1(4,3)
ψ1(0,4)
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y(4.4)
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y(2,4)
ψ1(1,4)
y(3,4)
ψ1(2,4)
ψ1(3,4)
ψ1(4,4)

Fig.4

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ИНДЕКС 3649

Препринт ЕФИ-1405(16)-93

М. А. МХИТАРЯН

ПАРАЛЛЕЛЬНО-ПОТОЧНАЯ ИНТЕРПРЕТАЦИЯ НОВОГО АЛГОРИТМА ЧИСЛЕННОГО РЕШЕНИЯ ИНТЕГРАЛЬНОГО УРАВНЕНИЯ ФРЕДГОЛЬМА ВТОРОГО РОДА С ЯДРОМ ТЕПЛИЦЕВА ТИПА.

В данной работе предложен систолический массив для реализации нового алгоритма численного решения интегрального уравнения Фредгольма с ядром теплицева типа. Для каждой структуры определяется коэффициент эффективности вычислений, позволяющий оценить степень использования оборудования (процессорных элементов).

ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
ЕРЕВАН-1993

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M. A. [illegible]

PARALLEL-PIPE[illegible] SECOND KIND FREDHOLM[illegible] INTEGRAL [illegible] KERNEL [illegible]

[illegible] systolic array [illegible] Fredholm [illegible] coefficient is determined for [illegible] to estimate the efficiency of process[illegible]

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