THE INTERNAL CONVERSION AND e^+e^-- PAIRS CREATION FROM THE HEATED NUCLEI

S.N.Fedotkin, V.M.Kolomietz

For excitation energies less than neutron binding energy the nuclei decay predominantly by means of gamma radiation. Radiative transitions of such nuclei are studied in the different statistical models[1-5]. In this case the some averaged over the great number of excited states quantum transitions are considered instead of a individual radiative transitions between the nuclear levels. In parallel with radiative processes the processes of the internal conversion like e^+e^--pairs creation and atomic shell ionization take place. Our interest is in investigation of these processes for highly excited nuclei. In this situation as in a case of radiative transitions it makes sense to consider the averaged over the great number of the excited nuclear states the probability of the conversion transitions.

Let us consider the process of e^+e^--pairs creation from the heated nuclei. The differential probability of e^+e^--pair creation for conversion transition in the nucleus of multipolarity L from initial state i to the final state f can be writen as

$$\frac{dW_{e^+e^-}(\lambda L)}{d\epsilon_+ d\epsilon_-} = P_L^{(\lambda)}(\epsilon_+, \epsilon_-)\, W_{fi}(\lambda L)\, \delta(E_i - E_f - \epsilon_+ - \epsilon_-) \qquad (1)$$

Here $W_{fi}(\lambda L)$ is the probability of the radiative transition, $\lambda = 0(1)$ for magnetic (electric) transitions, E_i and E_f are the energies initial and final nuclear states, ϵ_+ and ϵ_- are the total energies of positron and electron. $P_L^{(\lambda)}(\epsilon_+, \epsilon_-)$ determines the differential conversion coefficient for internal pair creation $d\beta(\lambda L, \epsilon_+)$:

$$P_L^{(\lambda)}(\epsilon_+, \omega - \epsilon_+)\, d\epsilon_+ = d\beta(\lambda L, \epsilon_+), \qquad (2)$$

where $\omega = \epsilon_+ + \epsilon_-$ is total energy of the nuclear transition. The probability $W_{fi}(\lambda L, \omega)$ has the form [6]

$$W_{fi}(\lambda L, \omega) = f(\omega)\, B_{fi}(\lambda L), \qquad f(\omega) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2 \hbar} \frac{\omega^{2L+1}}{(c\hbar)^{2L+1}} \qquad (3)$$

where

$$B_{fi}(\lambda L) = \sum_{MM_f} |< f|Q_{LM}|i >|^2 \qquad (4)$$

is reduced probability of a transition, Q_{LM} is multipole operator.

Suppose that the nuclei have the sufficiently high excitation energy, so that the statistical approach for them can be used. Let us make in this case the averaging of the expression (1) over statistical ensemble of the initial and summation over all final nuclear states and obtain

$$\frac{dW_{e^+e^-}(\lambda L)}{d\epsilon_+ d\epsilon_-} = P_L^{(\lambda)}(\epsilon_+, \epsilon_-)\, f(\omega)\, S_L^{(\lambda)}(\omega, T) \qquad (5)$$

where function $S_L^{(\lambda)}(\omega, T)$ is determined as follows:

$$S_L^{(\lambda)}(\omega, T) = \sum_{if} exp\frac{\Omega - E_i}{T}\, B_{fi}(\lambda L)\, \delta(E_i - E_f - \omega) \qquad (6)$$

Here Ω and T are thermodynamics potential and temperature descriptive of the ensemble of nuclear states.

The function $S_L^{(\lambda)}(\omega, T)$ and averaged over statistical ensemble of the nuclear states gamma radiation probability $\bar{W}_\gamma(\lambda L, \omega)$ are connected as follows [3,5]:

$$\frac{d\bar{W}_\gamma(\lambda L, \omega)}{d\omega} = f(\omega)\, S_L^{(\lambda)}(\omega, T) \tag{7}$$

Using relation (7) we get the following expression for averaged probability of e^+e^- - pairs creation from the heated nucleus

$$\bar{W}_{e^+e^-}(\lambda L, T) = \int_m^{U-m} d\epsilon_+ \int_{\epsilon_+ + m}^{U} d\omega\, \frac{d\bar{W}_\gamma(\lambda L, \omega)}{d\omega}\, P_L^{(\lambda)}(\epsilon_+, \omega - \epsilon_+), \tag{8}$$

where U is the nuclear excitation energy, m is the electron rest mass. In this expression we carry out the integration over positron ϵ_+ and photon ω energies.

Let us consider now the process of atomic shell ionization for the highly excited nuclei. The differential atomic shell internal conversion probability can be writen as

$$\frac{dW_c(\lambda L)}{d\epsilon} = \alpha(\lambda L, \epsilon)\, W_{fi}(\lambda L)\, \delta(E_i - E_f - \epsilon),$$

where $\alpha(\lambda L, \epsilon)$ is the internal conversion coefficient, $\epsilon = E_i - E_f = \omega$ is the kinetic energy of conversion electron. Here we neglect the binding energy of the electron. We shall average this expression over statistical ensemble as well as in a case of e^+e^- -pairs creation and obtain

$$\bar{W}_c(\lambda L, T) = \int_0^U d\omega\, \frac{d\bar{W}_\gamma(\lambda L, \omega)}{d\omega}\, \alpha(\lambda L, \omega) \tag{9}$$

Therefore, if we can determine the averaged gamma radiation probability $\bar{W}_\gamma(\lambda L, \omega)$ we can calculate the averaged probabilities of e^+e^- -pairs creation $\bar{W}_{e^+e^-}(\lambda L, T)$ and atomic shell ionization $\bar{W}_c(\lambda L, T)$ from the expression (8) and (9).

The averaged γ-radiation probabilities described in different approaches[1-5]. In this paper we shall consider simple statistical Weisskopf model [1] descriptive of gamma decay of the heated nuclei. Gamma transition probability is given in this model by the ration of level density of final ρ_f and initial ρ_i nuclear states:

$$\frac{d\bar{W}_\gamma(\lambda L, \omega)}{d\omega} = W_w(\lambda L, \omega)\, \frac{\rho_f(U - \omega)}{\rho_i(U) D_o} \tag{10}$$

Here $W_w(\lambda L, \omega)$ is Weisskopf approximation for gamma transition probability, D_o is the characteristic single particle spacing.

Let us consider in this model the probability of e^+e^- -pairs creation from heated nuclei in the case of $E1$ -transitions. Function $P_1^{(1)}(\epsilon_+, \omega - \epsilon_+)$ in the Born approximation has the form ($\hbar = c = 1$) [7]

$$P_1^{(1)}(\epsilon_+, \omega - \epsilon_+) = \frac{\alpha}{\pi\omega^3}\, [(\epsilon_+^2 + \epsilon_-^2)\, ln \frac{(m^2 + \epsilon_+\epsilon_- + 2p_+p_-)}{m\omega} + 2p_+p_-], \tag{11}$$

where α is the fine-structure constant, $p_s = \sqrt{\epsilon_s^2 - m^2}$ are momenta of positron ($s = +$) or electron ($s = -$). Substituting the expression for $P_1^{(1)}(\epsilon_+, \omega - \epsilon_+)$ (11) in the formula (8) we obtain after integration over ω

$$W_{e^+e^-}(E1,T) = \frac{\alpha^2 R^2 m^3}{4\pi D_o} \int_m^{U-m} d\epsilon_+ \; J(\epsilon_+,T), \tag{12}$$

where R is the nuclear radius. The positron spectral distribution $J(\epsilon_+,T)$ approximately equal to

$$J(\epsilon_+,T) = \frac{T}{m}\, exp(-\frac{\epsilon_+}{T})\, (\frac{\epsilon_+^2}{m^2} - 1)^{\frac{1}{2}}\, [3K_1(\frac{m}{T}) + \frac{(\epsilon_+^2 - \epsilon_+ T + 4T^2)}{(mT + m\epsilon_+)}\, K_o(\frac{m}{T})] \tag{13}$$

Here $K_n(m/T)$ $(n = 0, 1)$ are Macdonald functions. Figure 1a shows the function $J(\epsilon_+,T)$ at different temperatures T. The position of maximum of positron spectral distribution is moved to the higher energies with increasing T and the distribution becomes broadly.

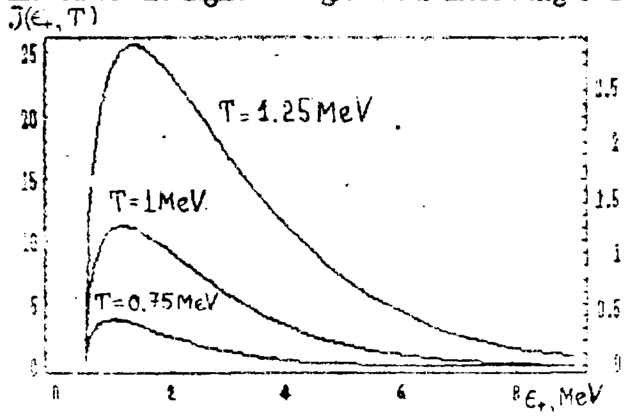

Fig.1a

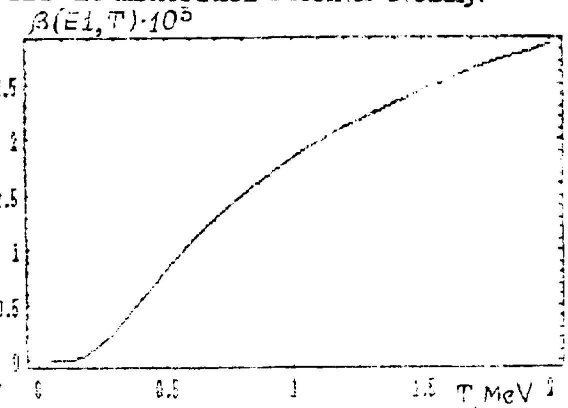

Fig.1b

We carry out an integration in (12) and obtain the net expression for the averaged e^+e^--pairs creation probability $W_{e^+e^-}(E1,T)$. The ratio of this probability to the averaged gamma radiation probability $\beta(E1,T) = W_{e^+e^-}(E1,T)/W_\gamma(E1,T)$ is given by

$$\beta(E1,T) = \frac{\alpha}{6\pi}\, x^2\, [3K_1^2(x) + K_o(x)(K_2(x) + \frac{1}{x}K_1(x))], \tag{14}$$

where $x = m/T$. Figure 1b shows the plot of the temperature dependent conversion coefficient for internal pair creation $\beta(E1,T)$ as a function of T.

Let us consider the K-shell ionization process within the framework of the statistical Weisskopf model. The internal conversion coefficient in the case of E1-transitions in the Born approximation has form [7]

$$\alpha_K(E1,\omega) = \alpha^4 Z^3\, (2 + \frac{m^2}{\omega^2})\, (1 + \frac{2m}{\omega})^{\frac{1}{2}}, \tag{15}$$

where Z is the nuclear charge. Substituting the expression (15) in the formula (9) we obtain

$$W_c(E1,T) = \frac{\alpha^5 Z^3 m^3 R^2}{4D_o} \int_0^U d\omega\; F(\omega,T), \tag{16}$$

where the conversion electron spectral distribution $F(\omega,T)$ has form

$$F(\omega,T) = exp(-\frac{\omega}{T})\, \frac{\omega^3}{m^3}\, (2 + \frac{m^2}{\omega^2})\, (1 + \frac{2m}{\omega})^{\frac{1}{2}}$$

The plot of function $F(\omega, T)$ at different temperatures T is given on Figure 2a. The width of spectral distribution $F(\omega, T)$ increases and the position of maximum is moved to the higher energies with increasing T.

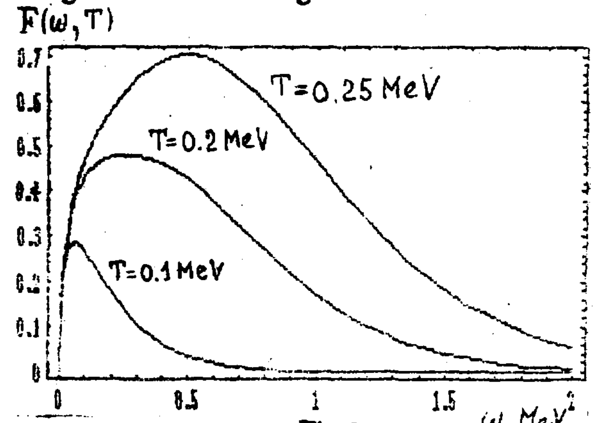

Fig.2a

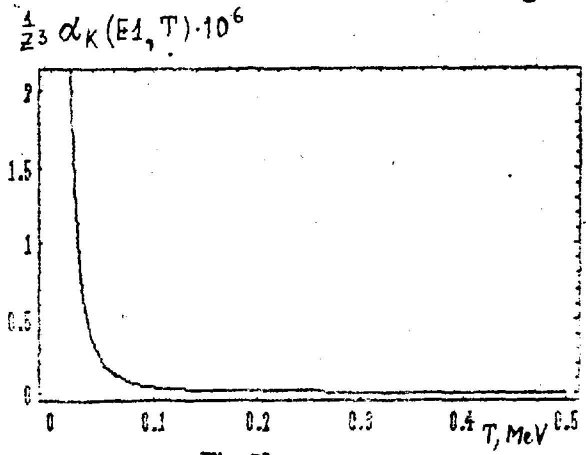

Fig.2b

Performing the integration in the expression (16) we obtain the temperature dependent conversion coefficient as the ratio $\alpha_K(E1,T) = \bar{W}_e(E1,T)/\bar{W}_\gamma(E1,T)$

$$\alpha_K(E1,T) = \frac{1}{3}\alpha^4 Z^3\, x^2\, exp(x)\, [K_2(x) + x(\frac{3}{2}K_1(x) - 2K_2(x) - \frac{dK_2(x)}{dx})] \qquad (17)$$

where $x = m/T$. The plot of $\alpha_K(E1,T)$ is shown in Figure 2b. For heavy nuclei with $Z = 100$ and at sufficiently small T the quantity $\alpha_K(E1,T)$ is equal to 10^{-2}. A related process for $E2$ -fluctuation electromagnetic field in framework of the thermodynamical approximation was investigated in paper [8].

It should be noted that because we calculated the ratio of internal conversion probability to the gamma radiation probability, some specific features of Weisskopf model disappear. Consequently, the expressions for the temperature dependent conversion coefficients $\beta(E1,T)$ and $\alpha_K(E1,T)$ contain more general information, than Weisskopf model. It is significant that the temperature dependent conversion coefficients contain the information about nuclear structure in contrast to usual conversion coefficients.

[1] Blatt J.M.and Weisskopf V.F.Theoretical nuclear physics(New York),1952,p.895

[2] Kuklin R.N.Sov.J.Nucl.Phys.1965,v.2,p.409

[3] Ignatjuk A.V.Bull.Acad.Sci.USSR(Phys.Ser.),1972,v.36,p.202

[4] Plujko V.A.Sov.J.Nucl.Phys.1990,v.52,p.1004

[5] Budnik A.P.and Svinyin I.R.Sov.J.Nucl.Phys.,1978,v.28,p.43

[6] Bohr A.and Mottelson B.Nuclear structure,v.1(N.Y.,Amsterdam,Benjamin),1969,p.456

[7] Akhieser A.J.and Berestecky W.B.Quantum Electrodynamics(M.,Fiz.Mat.),1959,p.650

[8] Grechukhin D.P.Sov.J.Nucl.Phys.1966,v.4,p.1134