Open Problems in Non-Equilibrium Physics

Dimitri KUSNEZOV

Yale University

Open Problems in Non-Equilibrium Physics

The current understanding of non-equilibrium physics is largely limited to expansions around equilibrium. From the statistical mechanics point of view we would like to know how to approach general non-equilibrium physics.

For instance, we can ask the following:

- ❐ **What can we say about dynamical systems when we are far from equilibrium?**
- ❐ **Can we define a non-equilibrium ensemble and use that to compute properties of the system?**
- ❐ **What role does the chaotic nature of the dynamics play a role in transport?**

Outline:

I. Approaches to Non-Equilibrium Statistical Mechanics;

II. Classical and Quantum Processes in Chaotic Environments;

III. Classical Fields in Non-Equilibrium Situations: Real time dynamics at finite temperature;

IV. Phase Transitions in Non-Equilibrium conditions;

V. Conclusions

Non-Equilibrium distributions

Is there a measure for non-equilibrium systems?

- **We consider a dynamical evolution:** (S_t, A, μ)
  Birkhoff proved ('30s) the closest thing to what Boltzmann wanted:

$$\int F(x)d\mu = \int \overline{F(x)}d\mu$$

- **More recently we have the *SRB theorem* (Sinai-Ruelle-Bowen) which tells us that for certain types of dynamical evolution, we can relate time averages to a measure:**

$$\lim_{t\to\infty} \frac{1}{t} \int dt' g(S_t \vec{x}) = \int g \, d\mu / \int d\mu$$

μ is called the *SRB measure.*

This measure does not have to be the canonical measure when you are in 'equilibrium', and it is difficult to compute even in simple systems.

We now find that the measures that are determined in the NESS are *fractal*. and of measure zero in the full dynamical space.

Real time quantum dynamics with temperature: *(DK, Bulgac, DoDang...)*

It is possible to develop a real time, finite temperature theory using chaos.

Start with a Hamiltonian and identify the few important degrees of freedom. If the remaining terms are chaotic, then the interactions can be described by a suitably designed random matrix theory. The influence functional can be analytically computed.

We can be prove that:

☆ Quantum 'chaotic' interactions lead to true thermodynamic behavior in only a singular limit;

☆ The examination of the second cumulant alone is not sufficient to conclude that the dynamics is thermal;

☆ A real time density matrix can evolve to a stationary state described by non-thermal distributions.

III. Fields Theory in Non-Equilibrium Environments

There are many approaches that one can take to developing a finite temperature formalism:

- **Influence functionals - integrate out hard modes** ***(Son [gauge theory], Muller, Greiner [ϕ^4],...)***

- **Thermo-field dynamics** ***(Umezawa,...);*** **Local equilibrium** ***(Zubarev...);***

- **Fokker-Planck equations** ***(Gyulassy,...)***

⋮

And so forth. The main distinction is how the thermal fluctuations are introduced. However these are not well suited for general situations, such as temperature or pressure gradients which are large.

It is worth examining what might happen to a field theory when it is strongly out of equilibrium.

IV. Non-Equilibrium Phase Transitions

Using such techniques, we are exploring what happens to phase boundaries in the NESS.

For example, by placing a phase transition in thermal boundary conditions, we can explore the steady states phase boundary determined by the local temperature T(x).

It is simple to extend the formalism to *SU(N)* to treat gauge theories *(DK '93)*.

One can add dynamic cooling, such as the simulation of thermal expansion:

$$T_{hot}(t) = T_{hot,0}/(t + t_0)$$

One can then ask how the hot regions cool and what happens to the phase boundaries. Do pockets develop, or is it rapid evaporation?

V. Conclusions

The only thing we really know about transport is the weak field limit. For general features of non-equilibrium problems, we are guided by simple models and simulations. From these we find:

Phase space distributions can become fractal;

There can be large departures from Green-Kubo predictions;

Non-equilibrium steady states have many unusual properties which do not arise as simple extensions of equilibrium ideas; an entirely new approach seems needed;

We have applied these ideas to classical and quantum systems - but now we need to extend these to field theory.

In the non-equilibrium steady state, it seems that entropy is not defined, even in the weak field limit. Its value diverges. Is there a consistent definition we can use here?

Does any of this survive in the continuum limit?