# Dirac Medal 1993

# Daniel Z. Freedman

Professor Daniel Z. Freedman is honoured:

"*for the discovery of supergravity theory and research in its subsequent development. Prior to his work in supergravity, he made significant contributions to the S matrix theory, discovering the so-called daughter trajectories. In the spring of 1976, in a seminal paper with Sergio Ferrara and Peter van Nieuwenhuizen, the first supergravity theory was proposed. This theory combines, in a non-trivial fashion, the spin 2 graviton with a spin 3/2 particle called the gravitino to elevate supersymmetry to a local gauge symmetry. This led to an explosion of interest in quantum gravity and it transformed the subject, playing a significant role in very important developments in string theory as well as Kaluza-Klein theory. Professor Freedman played a major role in the development of the subject, with his studies on the quantization of supergravity theories, coupling of supergravity to matter - in particular the introduction of Yang-Mills gauge coupling to supergravity. Currently any grand unified theory incorporating gravity is based on a supergravity theory coupled to matter in four dimensions. These theories emerge most naturally from the compactifications of the ten-dimensional heterotic string. Professor Freedman also made essential contributions to the understanding of the ultraviolet properties of supersymmetric sigma models in two dimensions, especially those on Kahler manifolds. Such models, which include the Calabi-Yau sigma models, currently play a central role in the study of superstring compactifications.*"

Professor Daniel Z. Freedman was born in 1939 at Hartford, Connecticut. He obtained his Ph.D. from the University of Wisconsin in 1964. Professor Freedman held postdoctoral positions at

Imperial College in London (1964-1965), the University of California at Berkeley (1965-1967) and the Institute for Advanced Study at Princeton (1967-1968). He was Assistant Professor (1968-1970), Associate Professor (1970-1975) and Professor (1968-1980) at the State University of New York at Stony Brook. He was Visiting Member at the Institute for Advanced Study at Princeton in 1973 and 1987. In 1974 and in 1985 he was Professeur d'Exchange at the Ecole Normale Supérieure in Paris. He is presently Professor of Applied Mathematics at MIT. Professor Freedman is a Fellow of the American Physical Society and Fellow of the American Academy of Arts and Sciences. He is the author of many scientific works on the theory of elementary particles and their interactions, particularly as related to the theory of gravitation.

# SOME BEAUTIFUL EQUATIONS OF MATHEMATICAL PHYSICS

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ABSTRACT. - *The basic ideas and the important role of gauge principles in modern elementary particle physics are outlined. There are three theoretically consistent gauge principles in quantum field theory: the spin-1 gauge principle of electromagnetism and the standard model, the spin-2 gauge principle of general relativity, and the spin-3/2 gauge principle of supergravity.*

Many quotations remind us of Dirac's ideas about the beauty of fundamental physical laws. For example, on a blackboard at the University of Moscow where visitors are asked to write a short statement for posterity, Dirac wrote: "A physical law must possess mathematical beauty." Elsewhere he wrote: "A great deal of my work is just playing with equations and seeing what they give.". And finally there is the famous statement: "It is more important for our equations to be beautiful than to have them fit experiment." This last statement is more extreme than I can accept. Nevertheless, as theoretical physicists we have been privileged to encounter in our education and in our research equations which

have simplicity and beauty and also the power to describe the real world. It is this privilege that makes scientific life worth living, and it is this and its close association with Dirac that suggested the title for this talk.

Yet it is a title which requires some qualification at the start. First, I deliberately chose to write "SOME beautiful equations ..." in full knowledge that it is only a small subset of such equations that I will discuss, chosen because of my own particular experiences. Other theorists could well choose an equally valid and interesting subset. In fact it is not a bad idea that every theorist "d'un certain âge" be required to give a lecture with the same title. This would be more creative and palatable than the alternative suggestion which is that every theorist be required to renew his/her professional license by retaking the Ph.D. qualifying exams.

Second, I do not wish to be held accountable for any precise definition of terms such as mathematical beauty, simplicity, naturalness, etc. I use these terms in a completely subjective way which is a product of the way I have looked at physics for the nearly 30 years of my professional life. I believe that equations speak louder than words, and that equations bring feelings for which the words above are roughly appropriate.

Finally, I want to dispel the notion that I have chosen a presentation for my own evil purposes. Some listeners probably anticipate that they will see equations from the work of Dirac, Einstein and other true giants. The equations of supergravity will then appear, and the audience will be left to draw its own conclusions. I assure you that I have no such delusions of grandeur. My career has been a mix of good years and bad years. If the good years teach good physics, then the bad years teach humility. Both are valuable.

The technical theme of this talk is that the ideas of spin, sym-

metry, and gauge symmetry, in particular determine the field equations of elementary particles. There are only three gauge principles which are theoretically consistent. The first of these is the spin-1 gauge principle which is part of Maxwell's equations and the heart and soul of the standard model. The second is the spin-2 gauge principle as embodied in general relativity. Both theories are confirmed by experiment. Between these is the now largely known theoretical structure of supersymmetry and the associated spin-3/2 gauge principle of supergravity. Does Nature know about this? Here, you can draw your own conclusions.

This viewpoint is what led me to work on supergravity in 1976. It is a view of the unification of forces before the unification program was profoundly affected by string theory. However, I confess that I myself think far less about unification now than I used to. Instead I think and worry about the survival of our profession and our quest to understand the laws of elementary particle physics. I hope that it is not a delusion to think that this presentation may contribute in a small positive way to the survival of that quest.

Let us start with the general idea that a particle is a unit of matter of definite mass $m$ and spin s. There are two classes of particles, the bosons with integer spin 0,1,2 ... and the fermions with half-integer spin 1/2, 3/2 .... We now know that whether a particle is "elementary" is not an absolute question. It depends on whether the experiments used to probe it can achieve a small enough spatial scale to detect an internal structure of smaller units. It is in this way that we have been led in the 20th century from atoms to nuclei to quarks. I will simplify that issue by saying that a particle is elementary if one can associate with it a wave equation and a local interaction Lagrangian and use these to account for experimental results within a certain range of scales. Those wave equations are restricted by Lorentz invariance and other symmetries. Underlying this is the beautiful mathematical structure which I will outline.

A spin-0 particle is described by a real scalar field. If massless it satisfies a very simple wave equation,

$$\Box\phi \equiv \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi\ (x,t) = 0 \tag{1}$$

which is the equation D'Alembert invented to describe acoustic waves in 1747. If it has a mass then there is another term, and one has the Klein-Gordon equation from the 1920's

$$(\Box + m^2)\phi = 0\ . \tag{2}$$

The particle physics of this equation is also very simple. The equation is second order in time. As initial data one must specify both $\phi(x,0)$ and $\partial_t\phi(x,0)$ at $t=0$. These two pieces of classical initial data correspond to a single quantum degree of freedom; for each possible momentum $p$, there is a one-particle state, usually denoted by the "ket" $|p>$, in a fertile notation we owe to Dirac.

Now we come to one of Dirac's major achievements, the wave equation he invented in 1927 to describe the spin-1/2 electron in a way consistent with the laws of special relativity. He postulated a first-order equation for a four-component complex field $\psi_\alpha(x,t)$. The equation requires a set of four matrices, now called $\gamma$ matrices, $\gamma^\mu$, satisfying the anti-commutation relations

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\ , \tag{3}$$

where $\eta^{\mu\nu} = (+,-,-,-)$ is the Minkowski metric of space-time. The Dirac equation can then be written in the massless and massive cases as

$$\begin{aligned} i\partial\!\!\!/\psi \equiv i\gamma^\mu \frac{\partial}{\partial x^\mu}\psi(x,t) &= 0 \\ (i\partial\!\!\!/ - m)\psi &= 0\ . \end{aligned} \tag{4}$$

It would require too long a digression to tell the full story of the physics contained in this equation, and I will just list a few things:

1. an accurate account of the spectrum of hydrogen;

2. prediction of the magnetic moment of the electron;

3. negative-energy states and anti-particles;

4. when applied to other spin-1/2 particles, namely the muon, proton and neutron, the Dirac equation and the system of $\gamma$-matrices provided the framework which established the form of the weak interactions in a very exciting chapter of 20th century physics;

5. the equation is one of the foundations of today's standard model of particle physics. It describes quarks, electrons, muons, and neutrinos, and their strong, electromagnetic, and weak interactions.

Despite this broad physical scope the basic particle physics of the Dirac equation is straightforward. It is a first-order equation so one must specify the four components of $\psi(x,0)$ as initial data. There are four quantum degrees of freedom, namely for each momentum $p$, a particle and antiparticle, each with two possible spins: $|p,\pm 1/2>$ and $|\bar{p},\pm 1/2>$. This is really the same ratio, namely 2/1, of independent classical data to particle states, because the four complex components of $\psi$ contain eight pieces of real information.

Following this approach one might think that a spin-1 particle should be described by a vector field $A_\mu(x,t)$ and the wave equation

$$\text{massless} \qquad \Box A_\mu = 0 \tag{5}$$

$$\text{massive} \quad (\Box - m^2)A_\mu = 0\ . \tag{6}$$

However trouble looms because there is a mismatch between the eight independent data for the classical initial value problem and the particle count required by Poincaré invariance, namely two

particle states of helicity $\pm$ 1 in the massless case and three states of helicity $\pm$ 1,0 in the massive case. Things get even worse because the extra components of the vector field give a quantum theory with negative probabilities, hence unacceptable.

It is here that the principle of gauge invariance comes to the rescue, with important consequences both for the linear wave equations of free field theory and the nonlinear equations which describe interactions. Gauge invariance is the idea that part of the information contained in the field $A_\mu$ is unphysical and unmeasurable, yet it is difficult and ill-advised to remove it entirely. It is a bit like writing a triangle on a piece of paper. The essential information about the triangle is contained in just three numbers, the side lengths, but for many purposes, such as to describe its relation to another figure on the paper it is useful to introduce a coordinate system and specify the coordinates $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ of the three vertices.

What is postulated is that the physical information in $A_\mu$ is specified by its "curl"

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \tag{7}$$

and this information is unchanged if $A_\mu$ is changed by the "gradient" of an arbitrary scalar function $\theta(x)$, viz.

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \theta \tag{8}$$

This is called a gauge transformation of $A_\mu$. The simplest wave equation which is invariant under this gauge transformation is

$$\begin{aligned} \partial^\mu F_{\mu\nu} &= 0 \\ \partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) &= 0 \\ \Box A_\mu - \partial_\mu \partial \cdot A &= 0 \ . \end{aligned} \tag{9}$$

These are all equivalent forms, and the last form shows that the new equation differs from the naïve one (4) by the fairly simple sec-

ond term. Yet this change is sufficient to solve previous problems, resulting in

1. a classical initial value problem with four independent initial data (this is usually shown using a gauge-fixing procedure not discussed here);

2. the quantum theory contains two polarization states $|p, \pm 1>$ of a massless spin-1 particle; and the gauge property can be used to show that these states transform properly under Lorentz transformations;

3. probabilities are positive.

We introduced gauge invariance to describe the photon, but there is a new and richer aspect, related to symmetry properties of the Dirac field. Let us look at the massive Dirac equation

$$i\partial\!\!\!/\psi - m\psi = 0 \ . \tag{10}$$

We make a transformation to a new spinor variable

$$\psi_\alpha(x) \rightarrow \psi'_\alpha(x) = e^{i\theta}\psi_\alpha(x) \ , \tag{11}$$

which is just a change of the complex phase of $\psi(x)$. It is obvious that

$$(i\partial\!\!\!/ - m)\psi'(x) = e^{i\theta}(i\partial\!\!\!/ - m)\psi(x) = 0 \ , \tag{12}$$

so $\psi'(x)$ satisfies the Dirac equation if $\psi(x)$ does.

This is a symmetry – a transformation of a set of fields which takes one solution of the field equations into another. The phase angle $\theta$ is called the symmetry parameter. In this case we have a global or rigid symmetry because $\theta$ is a constant, independent of $\vec{x}$.

However, we are reminded, if only for alphabetic reasons, of our description of the electromagnetic field. There we saw that

the gauge transformation (8) with an arbitrary function $\theta(x)$ is a symmetry. What happens if we try to generalize the previous phase symmetry to

$$\psi_\alpha(x) \to \psi'_\alpha(x) = e^{i\theta(x)}\psi_\alpha(x) \ ? \tag{13}$$

We must again test whether $\psi'(x)$ satisfies the same field equation, and we find

$$(i\partial\!\!\!/ - m)\psi'(x) = e^{i\theta(x)}(i\partial\!\!\!/ - \gamma^\mu\partial_\mu\theta - m)\ \psi(x) = -e^{i\theta}\gamma^\mu\partial_\mu\theta\psi \ . \tag{14}$$

So symmetry fails unless $\partial_\mu\theta(x) = 0$, and we are back to a global symmetry.

Now comes the powerful step. Suppose that we introduce a new interaction between $A_\mu(x)$ and $\psi(x)$, using the covariant derivative

$$D_\mu\psi \equiv (\partial_\mu - ieA_\mu)\psi(x) \tag{15}$$

and the modified Dirac equation

$$i\gamma^\mu D_\mu\psi - m\psi = 0 \ . \tag{16}$$

It is easy to see that this equation is invariant under the simultaneous transformation

$$\begin{aligned} \psi(x) \to \psi'(x) &= e^{i\theta(x)}\psi(x) \\ A_\mu(x) \to A'_\mu &= A_\mu(x) + i\partial_\mu\theta(x) \ . \end{aligned} \tag{17}$$

So we now have a nonlinear field equation with local symmetry.

The final step is to require that the combined Dirac and Maxwell equations be obtained from a gauge invariant Lagrangian, which turns out to be

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}(\gamma^\mu D_\mu - m)\psi \tag{18}$$

The $\delta\bar{\psi}$ variation of $\mathcal{L}$ produces the gauge-invariant Dirac equation, while the $\delta A_\nu$ variation produces the modified Maxwell equation

$$\partial^\mu(\partial_\mu A_\nu - \partial_\nu A_\mu) = e\bar{\psi}\gamma_\nu\psi \tag{19}$$

in which the "current" $J_\nu = e\bar{\psi}\gamma_\nu\psi$ is the source. If we take the divergence of both sides of the equation, then the left side vanishes identically because of $\mu\nu$ antisymmetry, so $J_\nu$ must satisfy the equation of continuity

$$\partial^\nu J_\nu = \frac{\partial}{\partial t} J_0 - \vec{\nabla}\cdot\vec{J} = 0 \ . \tag{20}$$

In turn, one can verify that this current conservation equation is satisfied because of (16) and its complex conjugate. So gauge invariance produces a system of field equations linked by subtle consistency conditions. Of course one must not forget to mention that what we have obtained in this way are the field equations of quantum electrodynamics, which have been verified experimentally with high precision. Indeed it is this theory and its coupling constant $e^2/4\pi\hbar c = 1/137$ that controls, in Dirac's words, "all of chemistry and much of physics."

It is worth summarizing what we have done because the same strategy has worked at least twice more in this century:

1. we promoted the rigid phase symmetry of $\psi(x)$ to a local symmetry by coupling to the gauge field $A_\mu(x)$ using covariant derivatives;

2. in the resulting gauge invariant theory, the conserved current of the matter field $\psi$ becomes a source of the gauge field;

3. a fundamental force of Nature is described in this way.

Let us introduce an aesthetic subtheme in this talk, namely the occurrence of the equations of physics in public art and design. A millenium ago, 1964 to be exact, I was a postdoctoral

fellow at Imperial College in London. I noticed then, and on subsequent visits, the frieze over the main door of the physics building, where some important equations and facts are carved in black marble. I was lucky enough to get (with the considerable help of Dr. K. Stelle of Imperial College), some transparencies showing this frieze. There is a full view showing four blocks of mathematical material interspersed with graphics. And there is an enlargement of the mathematical blocks. The third block is devoted to electromagnetism, with Maxwell's equations in full 19th century form very prominent. In the first block there is quantum mechanics with the Dirac equation in the upper-right corner. The second block is a mix of special relativity, Newtonian gravity (why not general relativity?) and thermodynamics. I call your attention only to the numerical relation

$$\frac{e^2}{Gm_e m_P} = 2.27 \times 10^{39} \tag{21}$$

which gives the ratio of strength of electric and gravitational forces between the electron and proton. From this one can easily compute that if there were no electromagnetism, the Bohr radius of gravitationally bound hydrogen would be $10^{32}$ cm $\sim 10^{15}$ light years. Reciprocally one can see that it is only on an energy scale of $10^{19}$ GeV that quantum gravitational effects among elementary particles become important.

Because of its importance in the modern picture of particle interactions, I must describe the non-Abelian generalization of gauge theory obtained by Yang and Mills in 1954. The mathematical background is a Lie group $G$ of dimension $N$, with the matrices $T^a_{ij}$ of an $n$-dimensional irreducible representation, structure constants $f^{abc}$, and commutators

$$[T^a, T^a] = if^{abc}T^c \ . \tag{22}$$

At the global level one has $N$ symmetry parameters, $\theta^a$, a set of

$n$ fermion fields $\psi_{i\alpha}(x)$ and infinitesimal transformation rule

$$\delta\psi_i = i\theta^a T^a{}_{ij}\psi_j \ . \tag{23}$$

To achieve local invariance one needs a set of $N$ gauge potentials $A^a_\mu(x)$. It is then straightforward to "covariantize" equations for $\psi_i$ using the non-Abelian covariant derivative

$$D_\mu\psi_i \equiv \partial_\mu\psi_i - igA^a_\mu T^a{}_{ij}\psi_j \ . \tag{24}$$

The new feature here is that the gauge field is in part its own source. This is reflected in its transformation rule

$$\delta A^a_\mu = \partial_\mu\theta^a + g\ f^{abc}A^b_\mu\theta^a \tag{25}$$

in which there is both a gradient term similar to the electromagnetic case (8) plus a "rotation" term which survives for constant $\theta^a$. The non-Abelian field strength is nonlinear,

$$F^a_{\mu\nu} = \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + g\ f^{abc}A^b_\mu A^c_\nu \ , \tag{26}$$

and so is the Yang-Mills field equation

$$D^\mu F^a_{\mu\nu} \equiv \partial^\mu F^a_{\mu\nu} + g\ f^{abc}A^b_\mu F^c_{\mu\nu} = g\bar{\psi}_i\gamma_\mu T^a{}_{ij}\psi_j \ . \tag{27}$$

One can show that $F^a_{\mu\nu}$ and $D^\mu F^a_{\mu\nu}$ transform homogeneously, e.g.,

$$\delta F^a_{\mu\nu} = g\ f^{abc}F^b_{\mu\nu}\theta^c \tag{28}$$

which means that they are covariant under non-Abelian gauge transformations. Non-Abelian gauge invariance is the fundamental principle underlying the standard model of elementary particles, and there is strong experimental evidence that this model, with gauge group $SU(3) \times SU(2) \times U(1)$, describes the strong, electromagnetic, and weak forces.

Our profession is a difficult one. To find the right field equations is only part of the job. It is far more difficult to solve those

equations in the context of quantum dynamics where each field variable is an operator in Hilbert space. Our knowledge of gauge field dynamics comes from a combination of experiment and theoretical insight. It is fortunate in many ways that there is a weak coupling regime in which perturbation theory is valid, and precise calculations using Feynman diagrams can be performed.

The only aspect of this dynamics that I will discuss here is the question of spontaneous symmetry breaking. This is the phenomon that when field equations are invariant under a large transformation group $G$, only a subgroup, $H \subset G$, need be realized directly in the mass spectrum and scattering amplitudes which would be observed experimentally. For example, realization of the full symmetry group $G$ means that all observed particles can be organized in multiplets which are representations of $G$ with the same mass for all particles in a given multiplet. If the symmetry is broken, then only a subgroup $H$ is realized in this way, but there are other observable signals of the larger group $G$. The situation for broken global symmetry is covered by the Goldstone theorem, which states that if G has dimension $N$, and $H$ has dimension $M$, then there must be $N-M$ massless scalar particles whose scattering amplitudes have characteristic properties at low energies. For broken gauge symmetry, one has instead the Higgs mechanism. The gauge fields reorganize into $M$ massless fields of the subgroup $H$, plus $N-M$ fields which appear as massive spin-1 particles. It is quantum dynamics that must tell us whether symmetry is broken or not. This depends on whether wave functions invariant under $G$ or $H$ have lower energy.

It is time for another aesthetic interlude, this time from Washington D.C. Near the National Academy of Sciences building, and completely accessible to the public, is a full size statue of Albert Einstein. He holds a tablet on which the enduring part of his life's

work is summarized in these three equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R &= \kappa T_{\mu\nu} \\ eV &= h\nu - A \\ E &= mc^2 \end{aligned} \tag{29}$$

from general relativity, the photo electric effect, and special relativity. Underneath the equations is his signature. This is a powerful artistic statement, which makes one proud to be a physicist. (I thank my MIT colleague Prof. A. Toomre for obtaining slides of this statue for me.)

I want to discuss general relativity very briefly from the viewpoint of gauging spacetime symmetry. The theoretical principle of special relativity is that physical field equations should be invariant under translations and Lorentz transformations of space-time. These are transformations between two coordinate systems $x^\mu$ and $x'^\mu$ related by

$$x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu \ . \tag{30}$$

In a special relativity, this is a global symmetry. There are four translation parameters $a^\mu$, while $\Lambda^\mu_\nu$ is a matrix of the group 0(3,1) containing six parameters which describe the relative angular orientation and velocities of the two coordinate systems. There is a great deal to say about the often counterintuitive effects of the mixing of space and time in special relativity, but for the purposes of today's talk, I speak only about two formal consequences:

1. particles are classified by their mass m and spin s; technically these numbers specify a representation of the group;

2. there is a conserved symmetric stress tensor $T^{\mu\nu}$ whose integrals $P^\mu = \int d^3x T^{0\mu}$ are the energy and momentum of a system of fields or particles.

For the electromagnetic field this stress tensor is

$$T^{\mu\nu} = F^{\mu\rho}F^{\nu}_{\rho} - \frac{1}{4}\,\eta^{\mu\nu}(F_{\rho\sigma})^2 \ .$$

If you look carefully you can find the conservation equation on the Imperial College frieze.

The gauging of this space-time symmetry is a fairly complicated process, but the elements are similar to those of the spin-1 gauge principle. I must oversimplify and state that one seeks a set of equations which are invariant under general coordinate transformations, in which two sets of space-time coordinates $x^\mu$ and $x'^\mu$ are related in a completely arbitrary way:

$$\begin{aligned} x'^\mu &= a^\mu(x^\nu) \\ &\approx x^\mu + \xi^\mu(x) \ . \end{aligned} \tag{31}$$

where the last form holds for infinitesimal transformations. The gauge parameter is the vector $\xi^\mu(x)$, and the gauge field is a symmetric tensor $g_{\mu\nu}(x)$ with the transformation rule

$$\begin{aligned} \delta g_{\mu\nu}(x) &= \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho} - \xi^\rho \partial_\rho g_{\mu\nu} \\ &= D_\mu \xi_\nu + D_\mu \xi_\nu \ . \end{aligned} \tag{32}$$

In the first line one sees a mix of gradient terms plus a translation term, indicating that the resulting theory is self-sourced. This is a funny way to say that the gravitational field itself carries energy and momentum. In the second line I just want to indicate that things can be organized into covariant derivatives which also simplify the coupling of $g_{\mu\nu}$ to matter fields.

Finally the analogue of the field strength $F_{\mu\nu}$ is the curvature tensor $R^\lambda_{\mu\rho\nu}$, from which one forms the Ricci tensor $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$ and the Riemann scalar $R = g^{\mu\nu}R_{\mu\nu}$. . These are the elements of the Einstein field equation

$$R^{\mu\nu} - \frac{1}{2}\,g^{\mu\nu}R = \kappa\, T^{\mu\nu} \tag{33}$$

in which the source of the curvature is the energy-momentum tensor of the matter fields in the system.

In a single general lecture one can give neither an adequate technical account of general relativity nor an adequate discussion of the ideas it embodies as a theory of gravity. I will make three brief comments.

1. On the formal side, gauge invariance guarantees that the particle content of the field $g_{\mu\nu}$ is the massless spin-2 graviton with two helicity states $|\vec{k}, \pm 2>$, with positive probabilities and interactions which maintain these properties.

2. On the side of ideas is the remarkable fact that $g_{\mu\nu}(x)$ is the metric tensor of space-time. So the theory of gravity is a theory of space-time geometry, a fact that has captivated many physicists.

3. Best of all is the experimental side. The theory is right in the classical domain. Several subtle effects which distinguish Einstein gravity from alternative theories (e.g., Newton's) have been observed. A fairly recent example is the accurate measurement of the decay of the orbit of the binary pulsar at the rate expected from quadrupole gravitational radiation.

It is a straightforward matter to take the standard model and couple it to gravity by the procedure I have hinted at above. This is completely described in several textbooks. But one learns little because the direct quantum effects of gravity are negligible at the energy of any conceivable particle accelerator. So for practical purposes one can drop the gravitational terms and concentrate on the dynamics of particle physics. Here there are many interesting unsolved problems, and for the last three years I have been working on some of them.

However, most physicists agree that one must eventually understand gravity at the quantum level perhaps only as an intellectual question (but perhaps more). One can be fairly certain that there is "new physics" at the quantum gravity or Planck scale of $10^{19}$ GeV, because theories obtained by the straightforward coupling of matter contain uncontrollable infinities. They are non-renormalizable in roughly the same way that the effective Fermi theory of the weak interactions is unrenormalizable.

The Weinberg-Salam-Glashow model of the electroweak interactions was put forward in 1968. It contained new ideas and was renormalizable. It predicted weak neutral currents which were found in 1972 at the scale of accelerator experiments realizable at that time. The W and Z bosons which were the key to the modification of the Fermi theory were found a decade later with masses just below 100 GeV which was close to the weak scale of 300 GeV at which the Fermi theory necessarily broke down. Analogously one can hope that new ideas about quantum gravity could have somewhat indirect consequences well below $10^{19}$ GeV and perhaps answer some of the questions left open by the standard model. These could include the following. Does the group of the standard model appear as a subgroup H (unbroken at the weak scale) of a larger unification group G? Are there some restrictions among the free parameters of the model, most of them from the poorly understood sector of non-gauge fermion couplings? This is the pragmatic component of the motivation for supersymmetry and supergravity and also string theory. There is also an aesthetic motivation, namely the search for beauty and symmetry in physical laws, which I think would have pleased Dirac.

Supersymmetry is a symmetry of relativistic field theories connecting fields of different spin. There are transformation rules containing a spinor parameter which rotate a bosonic field into a fermionic superpartner and vice versa. That such a symmetry is theoretically consistent was a surprise because earlier work, es-

pecially the Coleman-Mandula theorem, had indicated that the invariance groups permitted in quantum field theory were limited to the Poincaré group of space-time symmetries and a Lie group $G$ for internal symmetries is described above in connection with non-Abelian gauge theories. Neither contains spin-changing symmetry operators. However, in 1971, Gol'fand and Likhtman [1] sought to go beyond the limitations of the Coleman-Mandula theorem. They wrote down the algebraic relations of an extension of the Poincaré algebra containing spinor generators, and an interacting field theory which is invariant. The mathematical structure is that of a Lie superalgebra, which was not considered in earlier work. In 1972, Volkov and Akulov [2] obtained another invariant field theory with a different and pretty structure; it described a spontaneously broken form of supersymmetry. Finally in 1973 Wess and Zumino [3] discovered supersymmetry in four-dimensional field theories by generalizing a structure found in early work on superstring theory. Their paper contained the basic supersymmetric theories, with their off-shell multiplet structure, and systematic rules for constructing invariant interacting Lagrangians. The paper of Wess and Zumino was the springboard for the work of many physicists who contributed to the formal and phenomenological development of the subject.

Let us look at the example of supersymmetric Yang Mills theory, first obtained by Ferrara and Zumino [4] and Salam and Strathdee [5], which is the simplest interacting theory where you can see both that supersymmetry works and that it has some depth. It is important to look at an interacting theory because there are many possible symmetries of a free theory which are spurious, because one cannot introduce interactions. So I will present the full theory, and a guide to the manipulations needed to show that it is invariant.

The fields of the theory are the $N$ gauge bosons $A^a_\mu(x)$ and their superpartners, a set of $N$-gauginos $\chi^a(x)$ which are Majo-

rana spinors. A Majorana spinor satisfies a linear condition which means that only four independent real functions are required as initial data. It describes a spin-1/2 particle which is its own anti-particle. The minimal Lagrangian which is gauge invariant, namely

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{i}{2}\,\bar{\chi}^a\gamma^\mu\,(D_\mu\chi)^a \tag{34}$$

also possesses global supersymmetry. It is invariant under the following transformations which mix bosons and fermions

$$\begin{aligned} \delta A^a_\mu &= i\bar{\epsilon}\,\gamma_\mu\chi^a \\ \delta\chi^a &= \sigma^{\mu\nu}F^a_{\mu\nu}\epsilon \end{aligned} \tag{35}$$

where

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{4}\,[\gamma^\mu,\gamma^\nu] \\ D_\mu\chi^a &= \partial_\mu\chi^a + g\,f^{abc}A^b_\mu\chi^c \\ F_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,f^{abc}A^b_\mu A^c_\nu\ . \end{aligned} \tag{36}$$

To show supersymmetry in a simplified way, let us establish the invariance of the free equations of motion. We need to show that if $\chi$ and $A_\mu$ satisfy the Dirac and Maxwell equations

$$\begin{aligned} i\gamma^\mu\partial_\mu\chi &= 0 \\ \partial^\mu F_{\mu\nu} &= 0 \end{aligned} \tag{37}$$

then so do their variations $\delta\chi$ and $\delta A_\mu$. For the Maxwell equation, we need

$$\begin{aligned} \delta F_{\mu\nu} &= \partial_\mu\delta A_\nu - \partial_\nu\delta A_\mu \\ &= i\bar{\epsilon}(\gamma_\mu\partial_\nu - \gamma_\nu\partial_\mu)\chi\ . \end{aligned} \tag{38}$$

Then

$$\partial^\mu\delta F_{\mu\nu} = i\bar{\epsilon}(\not{\partial}\partial_\nu\chi - \gamma_\nu\,\Box\chi)\ . \tag{39}$$

Both terms vanish separately if $\partial\!\!\!/\chi = 0$. The Dirac equation is a little more involved and more instructive

$$\gamma^\mu \partial_\mu \delta\chi = \gamma^\mu \sigma^{\lambda\rho} \partial_\mu F_{\lambda\rho} \epsilon \ . \tag{40}$$

We substitute the standard Dirac matrix identity

$$\begin{aligned} \gamma^\mu \sigma^{\lambda\rho} &= \frac{1}{2}\,[\gamma^\mu, \sigma^{\lambda\rho}] + \frac{1}{2}\,\{\gamma^\mu, \sigma^{\lambda\rho}\} \\ &= \frac{1}{2}(\eta^{\mu\lambda}\gamma^\rho - \eta^{\mu\rho}\gamma^\lambda) + \frac{1}{2}\, i\epsilon^{\mu\lambda\rho\sigma}\gamma_5\gamma_\sigma \ . \end{aligned} \tag{41}$$

in (40) and find

$$\gamma^\mu \partial_\mu \delta\chi = \frac{1}{2}\{(\eta^{\mu\lambda}\gamma^\rho - \eta^{\mu\rho}\gamma^\lambda)\partial_\mu F_{\lambda\rho} + i\gamma_5\gamma_\sigma\epsilon^{\mu\lambda\rho\sigma}\partial_\mu F_{\lambda\rho}\}\epsilon \ . \tag{42}$$

The first two terms vanish by the Maxwell equation above, and the last vanishes if one substitutes $F_{\lambda\rho} = \partial_\lambda A_\rho - \partial_\rho A_\lambda$ and uses the fact that $\epsilon^{\mu\lambda\rho\sigma}$ is totally anti symmetric.

In the interacting non-Abelian theory things are a little more complicated. The Ricci and Bianchi identities of Yang-Mills theory are required

$$\begin{aligned} [D_\mu, D_\nu]\chi^a &= g\ f^{abc} F^b_{\mu\nu} \chi^c \\ \epsilon^{\lambda\mu\nu\rho} D_\mu F^a_{\nu\rho} &= 0 \end{aligned} \tag{43}$$

and one then finds that the term (in $\delta\mathcal{L}$)

$$g\ f^{abc}\ \bar{\epsilon}\gamma^\mu \chi^a \bar{\chi}^b \gamma_\mu \chi^c \tag{44}$$

must vanish as the final test of invariance. It can be shown to vanish as a consequence of the Fierz rearrangement identity for the $\gamma$ matrices and the crucial fact that the spinor quantities $\chi^a(x)$ and $\epsilon$ must anticommute because of the Pauli exclusion principle.

It is worthwhile to summarize the ingredients of the proof:

a. the Ricci and Bianchi identities which are fundamental to the non-Abelian gauge invariance;

b. properties of the $\gamma$ matrix algebra used in the relativistic treatment of spin;

c. anti-commutativity of fermionic quantities required by the connection of particle spin and statistics.

If the discussion above does not convince you that supersymmetry is a principle of great depth, then let me describe one more fact. This is the relation of supersymmetry to the space-time transformations of the Poincaré group. For any physical field $\Phi(x)$ of a supersymmetric theory one can make repeated supersymmetry variations, with spinor parameters, $\epsilon_1$ and $\epsilon_2$. The commutator of two transformations is

$$(\delta_{\epsilon_1}\delta_{\epsilon_2} - \delta_{\epsilon_2}\delta_{\epsilon_1})\Phi(x) = i\bar{\epsilon}_1\gamma^\mu\epsilon_2 \;\; \partial_\mu\Phi(x) \; . \tag{45}$$

Thus the commutator is an infinitesimal translation in space-time with displacement parameter $\delta a^\mu = i\bar{\epsilon}_1\gamma^\mu\epsilon_2$. So supersymmetry is a "square root" of translations in much the same way that the Dirac equation is said to be the "square root" of the scalar wave equation. This is already enough to see that the local form of supersymmetry must involve gravity, and we will return to this shortly.

In a theory with a non-Abelian gauge group $G$, we have seen that the fields are organized in representation of $G$. In a supersymmetric theory there is the analogous Poincaré super-algebra, including translations, Lorentz and SUSY transformations. Fields are organized in multiplets of this algebra, the basic ones contain fields of spins

| | |
|---|---|
| chiral multiplet | $(1/2, 0^+, 0^-)$ |
| gauge multiplet | $(1, 1/2)$ |

$$
\begin{array}{rll}
 & (3/2,1) & N \geq 2 \text{ supergravity} \\
N = 1 \text{ supergravity} & (2,3/2) & \\
 & (5/2,2) & \\
 & (s, s-1/2) &
\end{array}
\tag{46}
$$

A field theory with local supersymmetry is called a supergravity theory. Let us see what is required for such a theory by applying what we have learned about the spin-1 and spin-2 gauge principles. We want invariance with respect to transformations with an arbitrary spinor function $\epsilon_\alpha(x)$, so we should expect to require a gauge field with an additional vector index, a vector-spinor field $\psi_{\mu\alpha}(x)$. A free field theory for $\psi_{\mu\alpha}(x)$ had been formulated in 1941 by Rarita and Schwinger, describing a spin-3/2 particle. For a Majorana field, their Lagrangian is

$$
\mathcal{L} = -\frac{1}{2}\,\epsilon^{\lambda\rho\mu\nu}\,\bar{\psi}_\lambda \gamma_5 \gamma_\mu\,\partial_\nu \psi_\rho\ . \tag{47}
$$

One can easily see that it is invariant under the gauge transformation $\delta\psi_\rho = \partial_\rho \epsilon$. So the spin-3/2 field is the natural candidate for the gravitino, the superpartner of the graviton, and we should expect that the particle content of the basic supergravity theory should be given by the (2, 3/2) supermultiplet above.

However it was not clear that the theory could be mathematically consistent because of the infamous history of attempts to add interactions to the Rarita-Schwinger theory. All such attempts had led to inconsistencies. For example if $\psi_\rho$ is coupled to an electromagnetic field using the covariant derivative, $D_\nu \psi_\rho = (\partial_\nu - ieA_\nu)\psi_\rho$, the resulting theory, although formally relativistic, has propagation of signals at velocities faster than light. We now know that such problems arise when the interactions fail to incorporate the gauge invariance of the free theory.

Our approach [6] to the construction of supergravity was to start with the minimal elements required in a gravitational La-

grangian with fermions. These were:

$$\begin{aligned}
&\text{vierbein} \quad e_\mu^a \\
&\text{spin connection} \quad \overset{\circ}{\omega}_{\mu ab} = \frac{1}{2}\,[e_a^\nu(\partial_\mu e_{b\nu} - \partial_\nu e_{b\mu}) \\
&\qquad\qquad + e_a^\rho e_b^\sigma(\partial_\sigma e_{c\rho})e_\mu^c - (a \leftrightarrow b)] \\
&\text{curvature tensor} \quad R_{\mu\nu ab} = \partial_\mu \overset{\circ}{\omega}_{\nu ab} + \overset{\circ}{\omega}_{\mu a}{}^{c}\,\overset{\circ}{\omega}_{\nu cb} \\
&\qquad\qquad - (\mu \leftrightarrow \nu) \\
&\text{Lorentz covariant derivative} \quad D_\nu \psi_\rho = (\partial_\nu + \frac{1}{2}\,\overset{\circ}{\omega}_{\nu ab}\,\sigma^{ab})\psi_\rho \,. \quad (48)
\end{aligned}$$

From these we formed the Lagrangian

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_2 + \mathcal{L}_{3/2} \\
&\equiv -\frac{\det e}{4\kappa^2}\, e^{a\mu} e^{b\nu} R_{\mu\nu ab} - \frac{1}{2}\,\epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda \gamma_5\, e_\mu^a \gamma_a D_\nu \psi_\rho \quad (49)
\end{aligned}$$

where $\kappa$ is the gravitational coupling constant. The first term is the standard pure gravity action in vierbein form, and the second the Rarita-Schwinger Lagrangian with minimal gravitational coupling. This Lagrangian is acceptable from the viewpoint of the spin-2 gauge principle, and the next question is whether it is locally supersymmetric.

For this one needs transformation rules. It was natural to postulate

$$\delta\psi_\rho = \frac{1}{\kappa}\, D_\rho \epsilon = \frac{1}{\kappa}\,(\partial_\rho \epsilon + \frac{1}{2}\,\overset{\circ}{\omega}_{\rho ab}\,\sigma^{ab}\epsilon) \qquad (50)$$

because this is both gravitationally covariant and contains the expected mix of gradient plus Bose-Fermi mixing terms. The vierbein variation

$$\delta\, e_\mu^a = -i\,\kappa \bar{\epsilon}\gamma^a \psi_\mu \qquad (51)$$

is almost uniquely determined by invariance arguments.

I will present the first steps in the proof of local supersymmetry which shows that terms of order $\kappa^{-1}\bar{\epsilon}\psi$ vanish in the variation of

the action. In conventional vierbein gravity the variation of $\mathcal{L}_2$ for any $\delta e^a_\mu$ is

$$\Delta S_2 = \frac{1}{2\kappa^2}\int d^4x \det e(R^{a\mu} - \frac{1}{2}e^{a\mu}R)\delta e_{a\mu} \ . \tag{52}$$

It is the Einstein tensor in frame form that multiplies $\delta e^a_\mu$. We now compute the $\delta\psi$ variation of $\mathcal{L}_{3/2}$, getting a factor of two by varying $\bar{\psi}_\lambda$ and $\psi_\rho$ according to the rules for Majorana spinors,

$$\begin{aligned}\Delta_1 S_{3/2} &= -\frac{1}{\kappa}\int d^4x\ \epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu D_\nu D_\rho\epsilon \\ &= -\frac{1}{4\kappa}\int d^4x\ \epsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu R_{\nu\rho bc}\sigma^{bc}\epsilon \end{aligned}\tag{53}$$

where we have used the gravitational Ricci identity in the last line. We now use (41) in the form

$$\gamma_\mu\sigma^{bc} = \frac{1}{2}(e^b_\mu\gamma^c - e^c_\mu\gamma^b) + \frac{1}{2}ie^a_\mu\epsilon^{abcd}\gamma_5\gamma_d \tag{54}$$

When this is inserted in (53) the first two terms give contractions

$$\epsilon^{\lambda\rho\mu\nu}R_{\nu\rho\mu b} = 0 \tag{55}$$

which vanish due to the first Bianchi identity for the curvature tensor.

After use of $\bar{\psi}_\lambda\gamma_d\epsilon = -\bar{\epsilon}\gamma_d\psi_\lambda$, which holds for anti-commuting Majorana spinors, we are left with

$$\Delta_1 S_{3/2} = \frac{1}{8\kappa}i\int d^4x\epsilon^{\mu\lambda\rho\nu}\epsilon^{abcd}e^a_\mu R_{\nu\rho bc}\bar{\epsilon}\gamma_d\psi_\lambda \ . \tag{56}$$

It is less fun, but straightforward, to compute the contraction of the $\epsilon$ tensors

$$\begin{aligned}\epsilon^{\mu\lambda\rho\nu}\epsilon^{abcd}e^a{}_\mu R_{\nu\rho bc} &= 2\det e(e^{b\lambda}e^{c\rho}e^{dr} + e^{c\lambda}e^{d\rho}e^{b\nu} \\ &\quad + e^{d\lambda}e^{b\rho}e^{c\lambda})R_{\nu\rho bc} \\ &= 4\det e(R^{d\lambda} - \frac{1}{2}e^{d\lambda}R) \ . \end{aligned}\tag{57}$$

When this is inserted in (56) and (51) is inserted in (52) we find an exact cancellation!

The situation is similar to that of supersymmetric gauge theories. The cancellation is due to the combined effects of the gravitational Ricci and Bianchi identities and the Dirac $\gamma$-matrix algebra. This lowest order cancellation showed that we were on the right track, but there was more work to be done because there is a non-vanishing variation $\Delta_3 S_{3/2}$ of order $\kappa\bar{\epsilon}\psi^3$.

Here we struggled for many weeks because it was hard to perceive a pattern in quantities with so many indices. Finally we devised a systematic approach involving:

a) a general ansatz for a modified gravitino transformation of order $\delta'\psi = \kappa\bar{\epsilon}\psi^2$;

b) an analogous general ansatz for a contact Lagrangian of order $\mathcal{L}_4 = \kappa^2(\bar{\psi}\psi)^2$.

The $\delta'\psi$ variation of $\mathcal{L}_{3/2}$ and the $\delta\psi$ variation of $\mathcal{L}_4$ give additional order $\kappa\epsilon\psi^3$ terms and we were able to find unique choices for $\delta'\psi$ and $\mathcal{L}_4$ to make the total variation vanish. Fierz rearrangement was required here.

Unfortunately new and complicated terms of order $\kappa^3\bar{\epsilon}\psi^5$ are generated by $\delta e$ and $\delta'\psi$ variations of $\mathcal{L}_4$. One could show easily that no further modification of the framework could be made, and these terms had to vanish or the theory failed. We were able to show that they vanished by a computer calculation in FORTRAN language with explicit input of the $\gamma$-matrices and a program to implement the anti-symmetrization implicit for fermionic variables.

Soon thereafter an important simplification of the resulting theory was obtained by Deser and Zumino [7], with a further simpli-

fying step [8] somewhat later. This involved the idea that the gravitino modifies the space-time geometry by including torsion. The net result is that the Riemannian spin connection $\overset{\circ}{\omega}_{\mu ab}$ is replaced by

$$\omega_{\mu ab} = \overset{\circ}{\omega}_{\mu ab} + \frac{1}{2}\, i\kappa^2\, (\bar{\psi}_\mu \gamma_a \psi_b - \bar{\psi}_\mu \gamma_b \psi_a - \bar{\psi}_a \gamma_\mu \psi_b) \qquad (58)$$

in the Lagrangian (49) and transformation rule (50). This grouping of terms gives a complete and succint definition of the theory, and a simpler proof of invariance.

The subsequent development of supergravity included:

1. coupling supergravity to the chiral and gauge multiplets of global supersymmetry; these couplings involve conserved supercurrents and super-covariant derivatives; there are now relatively simple tensor methods to obtain the most general form of these theories;

2. developing extended supergravity with $N \leq 8$ gravitinos; the maximal $N = 8$ theory was once thought to be the best candidate for a unified field theory.

3. Higher dimensional supergravity culminating in the 10 and 11 dimensional theories. The 10 dimensional version is important both historically and practically for superstrings.

Earlier we said that the only theoretically consistent gauge principles are those of spin-1, spin-2, and spin-3/2. This information comes from a set of theorems, due to Coleman and Mandula and Haag and Lopuszanski, which limit the symmetries permitted in an interacting theory. These theorems hold under certain assumptions which must be examined critically. But it appears that they are essentially correct. For example, one can write free field theories for a spin-5/2 field, but attempts to include interactions have all failed.

There is time to describe only very briefly what now appears to be the most plausible scenario for experimental verification of these ideas. This is the global supersymmetric extension of the standard model with fields grouped in chiral and gauge multiplets. The known quark, lepton, gauge, and Higgs fields all have superpartners. One then couples this large set of matter fields to supergravity. Observed supersymmetry requires that a particle and its superpartner must have the same mass. This is decidedly false, so one must expect supersymmetry breaking, and superpartners are predicted with masses between 100 GeV and 1 TeV.

Without the supergravity couplings explicit mechanisms for the symmetry breaking have not been found. There could be a subtle dynamical breaking mechanism, but in any case the spontaneous global supersymmetry breaking would give a Goldstino, a massless spin-1/2 particle that is excluded experimentally. So the role of supergravity in these models is to break supersymmetry such that the gravitino becomes massive by a super-Higgs mechanism, without generating a cosmological constant.

The first model which correctly described this super-Higgs mechanism was obtained by Polonyi [9]. General studies of the conditions for the super-Higgs effect by Cremmer, Julia, Scherk, Ferrara, Girardello and van Nieuwenhuizen [10] and by Cremmer, Ferrara, Girardello and Van Proeyen [11] also contain the most general $N = 1$ supergravity actions. It is this work which has been widely applied to supersymmetric extensions of the standard model. A very early discussion [12] of the super-Higgs effect for $N$ spin-3/2 fields and $N$ Goldstone fermions is incorrect both for general $N$ and in the special case $N = 1$.

Discovery of the superpartners is the key requirement to confirm the picture of broken supersymmetry, but there is also a less direct set of predictions related to the unification scale of gauge coupling of the standard model, the rate of proton decay and the

masses expected for the top quark and Higgs bosons. There is now favorable experimental evidence on the unification scale and the observed lower limit on the proton lifetime. These facts appear quite naturally in the supergravity models but not in the simplest forms of theories without supersymmetry.

In this lecture I have not done justice to string theory and the beautiful ideas it contains. Therefore I must clearly state that the $N = 1$ supersymmetry/supergravity framework I have discussed cannot give a complete theory. There are non-renormalizable infinities which require new physics at the Planck scale. A more fundamental superstring theory could well be correct and there are well studied scenarios by which such a theory can lead, for energies less than $10^{19}$ GeV, to an effective $N = 1$ supergravity theory.

I have not had the time to discuss some of the pragmatic features of supersymmetry/ supergravity theories which make them attactive as a candidate for physics beyond the standard model. There are review articles to consult about this very active subject of research. Instead what I have tried to say is that these theories are based on the only theoretically consistent symmetry principle not so far confirmed in Nature. This suggests that it is historically inevitable for supersymmetry to play a role. Of course this could be as dangerous as the prediction that "capitalism contains within itself the seeds of its own destruction." Experiment is the ultimate test of theoretical speculation. New experiments are needed together with theorists who are willing to devote a good part of their effort to support the experimental enterprise.

### Note

We can include explicit reference only to a few of the original papers on supersymmetry and supergravity. Many important papers are omitted, and it is fortunate that they are reprinted and reviewed in the following collections which are also a very good way to learn the subject.

a. "Supersymmetry and Supergravity", ed. M. Jacob, North Holland, Amsterdam (1986), a collection of Physics Reports by J.Ellis, P. Fayet and S. Ferrara, H. Haber and G. Kane, C. Llewellyn Smith, D. Nanopoulos, P. van Nieuwenhuizen, H. Nilles, A. Savoy-Navarro and M. Sohnius.

b. "Supersymmetry", 2 Vols., ed. S. Ferrara, North Holland, Amsterdam (1987).

c. "Supergravities in Diverse Dimensions", 2 Vols., eds. A. Salam and E. Sezgin, North Holland/World Scientific (1989).

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