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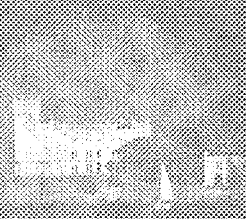

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Abstract

A theoretical investigation has been made of two new ultra-low-frequency electrostatic modes, namely, dust-cyclotron mode and dust-lower-hybrid mode, propagating perpendicular to the external magnetic field, in a self-gravitating magnetized two fluid dusty plasma system. It has been shown that the effect of the self-gravitational force, acting on both dust grains and ions, significantly modifies the dispersion properties of both of these two electrostatic modes. It is also found that under certain conditions, this self-gravitational effect can destabilize these ultra-low-frequency electrostatic modes. The implications of these results to some space and astrophysical dusty plasma systems, especially to planetary ring-systems and cometary tails, are briefly mentioned.

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Recently, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains), because of its vital role in the study of astrophysical and space environments, such as, asteroid zones, planetary atmospheres, interstellar media, circumstellar disks, dark molecular clouds, cometary tails, nebulae, earth's environment, etc. [1-7]. These dust grains are invariably immersed in the ambient plasma and radiative background. The interaction of these dust grains with the other plasma particles (viz. electrons and ions) is due to the charge carried by them. The dust grains are charged by a number of competing processes, depending upon the local conditions, such as, photo-electric emission stimulated by the ultra-violet radiation, collisional charging by electrons and ions, disruption and secondary emission due to the Maxwellian stress, etc. [8-12].

It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra [13-20]. Bliokh and Yaroshenko [13] studied electrostatic waves in dusty plasmas and applied their results in interpreting spoke-like structures in Saturn's rings (revealed by Voyager space mission [21]). Angelis *et al.* [14] investigated the propagation of ion-acoustic waves in a dusty plasma, in which a spatial inhomogeneity is created by a distribution of immobile dust particles [22]. They [14] applied their results in interpreting the low frequency noise enhancement observed by the *Vega and Giotto* space probes in the dusty regions of Halley's comet [23].

On the other hand, it has been shown both theoretically [24-33] and experimentally [34,35] that the dust charge dynamics introduces different new eigen modes, such as, dust-acoustic mode, dust-ion-acoustic mode, dust-drift mode, etc. [24-35]. These collective processes or wave phenomena in a dusty plasma (containing extremely massive dust grains) can be studied in either of the three possible regimes, namely, (i) the electromagnetic force is much greater than the gravitational force, (ii) the electromagnetic force is of the same order of the magnitude as the gravitational force, and (iii) the gravitational force is much greater than the electromagnetic force. Case (i) corresponds to usual laboratory plasma situations where Coulombic interaction is primary responsible for the plasma dielectric behavior. Case (ii) corresponds to planetary atmospheres and interstellar media [3, 36-38] where the thickness of the Jovian ring, spoke formation in Saturn's rings, etc. are thought to be due to the balance of these two forces. Case (iii) generally corresponds to astrophysical plasmas where the formation of large-scale structure is attributed to gravitational condensation [39].

Most of these studies [24-35] on these new modes (associated with extremely massive dust grains) are concerned with situation (i) but not with cases (ii) and (iii). Recently, a number of investigations [40-43] have been made on dust-acoustic waves in a self-gravitating dusty plasma system. Mahanta *et al.* [40] and Mamun [41] have studied the effect of the self-gravitational field on dust-acoustic waves by ignoring the ion dynamics, whereas Avinash & Shukla [42] and Verheest *et al.* [43] have investigated dust-acoustic waves in a self-gravitating unmagnetized dusty plasma, taking into account the dynamics of dust-grains and ions. The present work has considered a self-gravitating magnetized two fluid dusty plasma system and investigated two new ultra-low-frequency dust-electrostatic modes, namely, dust-cyclotron mode and dust-lower-hybrid mode, propagating perpendicular to the external magnetic field.

We consider a two-component, self-gravitating, warm, magnetized dusty plasma system consisting of negatively charged (extremely massive) dust and positively charged ion fluids. Thus, at equilibrium we have $Z_i \mathrm{n_{i0}} = \mathrm{Z_d n_{d0}}$, where $\mathrm{n_{i0}}$ ($\mathrm{n_{d0}}$) is the equilibrium ion (dust) number density and Z_d (Z_i) is the number of electrons (protons) residing on the dust grains (ions). This plasma system is assumed to be immersed in an external static magnetic field. It is also assumed here that the electron number density is highly depleted due to the attachment of almost all electrons to the surface of the extremely massive dust grains. This model is relevant to planetary ring-systems (e.g., Saturn's F-ring [3,26,31]) and laboratory experiments [34,35]. The macroscopic state of this self-gravitating, warm, magnetized dusty plasma system may be described by [27,31,43]:

$$\frac{\partial N_s}{\partial t} + \nabla \cdot (N_s \mathbf{U}_s) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{U}_s \cdot \nabla\right)\mathbf{U}_s = -\frac{q_s}{m_s}\nabla\Phi + \frac{q_s}{m_s c}(\mathbf{U}_s \times \mathbf{B}_0) - \nabla\Psi - \frac{1}{N_s m_s}\nabla P_s, \tag{2}$$

$$\nabla^2\Psi = 4\pi G \sum_s m_s N_s, \tag{3}$$

$$\nabla^2\Phi = 4\pi \sum_s q_s N_s, \tag{4}$$

where $\mathrm{m_s}$, $\mathrm{q_s}$ and $\mathrm{N_s}$ are, respectively, mass, charge and number density of the species s (dust grains and ions); $\mathbf{U}_s$ is the hydrodynamic velocity, $\mathrm{P_s} = \gamma_\mathrm{s} \mathrm{N_s k_B T_s}$ with $\mathrm{k_B T_s}$ being the thermal energy and γ_s being the adiabatic constant; Φ is the electrostatic wave potential; G is the universal gravitational constant; c is the speed of light in vacuum. We are interested in looking at different extremely low-frequency electrostatic modes (ω, k) propagating perpendicular to the external magnetic field $\mathbf{B}_0$ (we assume that $\mathbf{B}_0$ is along the x-axis, i.e., $\mathbf{B}_0 \parallel \hat{\mathrm{x}}$ and propagation vector $\mathbf{k}$ is along the y-axis, i.e., $\mathbf{k} \parallel \hat{\mathrm{y}}$). To study such electrostatic modes in a self-gravitating magnetized warm dusty plasma, we shall carry out a normal mode analysis. We first express our dependent variables N_s, U_s, Ψ and Φ in terms of their equilibrium and perturbed parts as

$$\left.\begin{aligned} N_s &= n_{s0} + n_s, \\ \mathbf{U}_s &= 0 + \mathbf{u}_s, \\ \Psi &= 0 + \psi, \\ \Phi &= 0 + \phi. \end{aligned}\right\} \tag{5}$$

Then, using these equations, we linearize our basic equations to a first order approximation and express them as

$$\frac{\partial n_s}{\partial t} + n_{s0}(\nabla \cdot \mathbf{u}_s) = 0, \tag{6}$$

$$\frac{\partial \mathbf{u}_s}{\partial t} = -\frac{q_s}{m_s}\nabla\phi + \frac{q_s}{m_s c}(\mathbf{u}_s \times \mathbf{B}_0) - \nabla\psi - \frac{v_{ts}^2}{n_{0s}}\nabla n_s, \tag{7}$$

$$\nabla^2\psi = 4\pi G \sum_s m_s n_s, \tag{8}$$

$$\nabla^2\phi = 4\pi \sum_s q_s n_s, \tag{9}$$

where $\mathrm{v_{ts}} = (\gamma_\mathrm{s} \mathrm{k_B T_s/m_s})^{1/2}$. Now, performing Fourier transformation of Eqs. (6)$-$(9) and using the first three of them, one can find n_d and n_i. The substitution of these n_d and n_i into the last one yields

$$\varepsilon\phi = 0, \tag{10}$$

where ε is the dielectric constant for the self-gravitating magnetized dusty plasma system, under consideration, and can be expressed as

$$\varepsilon = 1 - \frac{\omega_{pd}^2}{\beta\alpha_d\omega^2}\left[1 + \frac{1}{\alpha_i}\left(\frac{Z_i m_d}{Z_d m_i}\right)\frac{\omega_{Ji}^2}{\omega^2}\right] - \frac{\omega_{pi}^2}{\beta\alpha_i\omega^2}\left[1 + \frac{1}{\alpha_d}\left(\frac{Z_d m_i}{Z_i m_d}\right)\frac{\omega_{Jd}^2}{\omega^2}\right], \tag{11}$$

with

$$\begin{aligned}
\alpha_s &= 1 - \frac{\omega_{cs}^2}{\omega^2} + \frac{\omega_{Js}^2}{\omega^2} - \frac{k^2 v_{ts}^2}{\omega^2}, \\
\beta &= 1 - \frac{\omega_{Jd}^2\omega_{Ji}^2}{\alpha_d\alpha_i\omega^4}, \\
\omega_{ps} &= \sqrt{4\pi n_{s0} q_s^2/m_s}, \\
\omega_{Js} &= \sqrt{4\pi G m_s n_{s0}}. \\
\omega_{cs} &= \frac{|q_s|B_0}{m_s c}.
\end{aligned} \tag{12}$$

This equation $\varepsilon\phi = 0$ is compatible with non-zero ϕ only if $\varepsilon = 0$, i.e., ω and k are related by

$$\begin{aligned}
1 \; &- \; \frac{\omega_{pd}^2}{\omega^2 - \omega_{cd}^2 - k^2 v_{td}^2 + \omega_{Jd}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2 - k^2 v_{ti}^2 + \omega_{Ji}^2} \\
&- \; \frac{\omega_{pd}^2\omega_{Jd}^2 + \omega_{pi}^2\omega_{Ji}^2 + \omega_{Jd}^2\omega_{Ji}^2}{(\omega^2 - \omega_{cd}^2 - k^2 v_{td}^2 + \omega_{Jd}^2)(\omega^2 - \omega_{ci}^2 - k^2 v_{ti}^2 + \omega_{Ji}^2)} = 0.
\end{aligned} \tag{13}$$

This is the general dispersion relation for any electrostatic mode, propagating perpendicular to the external magnetic field, in a self-gravitating, magnetized, warm dusty plasma. However, our present interest is to study low-frequency electrostatic modes of two different new frequency limits, namely, $\omega \sim \omega_{cd}$ (where $\omega_\mathrm{Ji}, \omega_\mathrm{ci} < \mathrm{kv_{ti}}$ is valid) and $\omega_{cd} < \omega < \omega_{ci}$ (where $\omega_\mathrm{Ji}, \mathrm{kv_{ti}} < \omega_\mathrm{ci}$ is valid). The low-frequency electrostatic mode corresponding to the former case is termed as dust-cyclotron mode (analogous to ion-cyclotron mode) and corresponding to the latter case is termed as dust-lower-hybrid mode (analogous to ion-lower-hybrid mode).

A. Dust-cyclotron mode:

To study dust-cyclotron mode we use the approximations $\omega_{\mathrm{Ji}}, \omega_{\mathrm{ci}} << \mathrm{k}v_{\mathrm{ti}}$. This approximation reduces the general dispersion relation to a simple form:

$$\begin{aligned} \omega^2 = \omega_{cd}^2 + k^2 v_{td}^2 + \frac{k^2 C_d^2}{1+k^2\lambda_{Di}^2} & - \omega_{Jd}^2\left[1 + (\frac{Z_d m_i}{Z_i m_d})(\frac{1}{1+k^2\lambda_{Di}^2})\right] \\ & - \frac{\omega_{Ji}^2}{1+k^2\lambda_{Di}^2}\left[1 + G(\frac{m_i m_d}{Z_i Z_d e^2})\right], \end{aligned} \quad (14)$$

where $\mathrm{C_d} = (\gamma_\mathrm{i} \mathrm{Z_d k_B T_i / Z_i n_d})^{1/2}$ and $\lambda_{\mathrm{Di}} = (\gamma_\mathrm{i} \mathrm{k_B T_i} / 4\pi \mathrm{n_{i0} Z_i^2 e^2})^{1/2}$. This equation represents the dispersion relation for the dust-cyclotron mode, in which the effects of self-gravitational field (acting on both dust particles and ions), thermal pressures of dust and ion fluids, and ion dynamics are included. If we consider the unmagnetized case and neglect the effects of the self-gravitating field, ion dynamics and dust fluid temperature (i.e. $\omega_{cd} \to 0$, $\omega_{Jd,i} \to 0$, $\mathrm{k}^2\lambda_{\mathrm{Di}}^2 << 1$, and $\mathrm{v_{td}} \to 0$), this becomes the dispersion relation for the dust-acoustic mode studied by Rao *et al.* [24]. On the other hand, if we neglect effects of external magnetic field and dust fluid temperature, but not of the self-gravitational field and ion-dynamics, our dispersion relation reduces to that obtained by Verheest *et al.* [43].

It is shown from our dispersion relation for the dust-cyclotron mode that due to the effect of the self-gravitational force acting on dust grains and ions, this mode becomes unstable if

$$\left(\omega_{cd}^2 + k^2 v_{td}^2 + \delta k^2 C_d^2\right) < \left(\omega_{Jd}^2[1 + \delta(\frac{Z_d m_i}{Z_i m_d})] + \delta\omega_{Ji}^2[1 + G(\frac{m_i m_d}{Z_i Z_d e^2})]\right), \quad (15)$$

where $\delta = 1/(1 + \mathrm{k}^2\lambda_{\mathrm{Di}}^2)$. The criterion for this instability (known as gravitational instability), for $\mathrm{k}^2\lambda_{\mathrm{Di}}^2 << 1$ and $(\mathrm{Z_d m_i / Z_i m_d}) << 1$, can be simplified as

$$\left(\omega_{cd}^2 + k^2 v_{td}^2 + k^2 C_d^2\right) < \left(\omega_{Jd}^2 + \omega_{Ji}^2\right). \quad (16)$$

It is now obvious from this condition that the dust-cyclotron mode may become unstable due to the effect of the self-gravitational force acting on dust grains and ions. It is also shown that the effects of external magnetic field and thermal pressures of both dust and ion fluids try to stabilize this dust-cyclotron mode and counter the gravitational condensation of the dust grains.

B. Dust-lower-hybrid mode:

To examine the dust-lower-hybrid mode we use the approximations $\omega_{cd} < \omega < \omega_{ci}$ and ω_{Ji},

$kv_{ti} << \omega_{ci}$. These approximations reduce the general dispersion relation to a simple form:

$$\omega^2 = \omega_{cd}\omega_{ci}(1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2})^{-1} + k^2 v_{td}^2 \quad - \quad \omega_{Jd}^2 \left[1 + \frac{Z_d m_i}{Z_i m_d}(1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2})^{-1}\right] - \quad \omega_{Ji}^2 (1 + \frac{\omega_{cd}\omega_{ci}}{\omega_{pd}^2})^{-1} \left[1 + G(\frac{m_i m_d}{Z_i Z_d e^2})\right]. \quad (17)$$

This equation represents the dispersion relation for the dust-lower-hybrid mode where the effects of self-gravitational field (acting on both dust particles and ions), thermal pressures of dust and ion fluids are included.

It is shown from our dispersion relation for the dust-lower-hybrid mode that the effects of the self-gravitational field and dust fluid temperature modify this dust-lower-hybrid mode significantly, and that due to the effect of this self-gravitational field, this dust-lower-hybrid mode becomes unstable if

$$\left(\mu\omega_{cd}\omega_{ci} + k^2 v_{td}^2\right) < \left(\omega_{Jd}^2[1 + \mu(\frac{Z_d m_i}{Z_i m_d})] + \mu\omega_{Ji}^2[1 + G(\frac{m_i m_d}{Z_i Z_d e^2})]\right), \quad (18)$$

where $\mu = (1 + \omega_{cd}\omega_{ci}/\omega_{pd}^2)^{-1}$. The criterion for this instability (known as gravitational instability), for $\omega_{ci}\omega_{cd}/\omega_{pd}^2 << 1$ and $Z_d m_i/Z_i m_d << 1$, can be simplified as

$$\left(\omega_{cd}\omega_{ci} + k^2 v_{td}^2\right) < \left(\omega_{Jd}^2 + \omega_{Ji}^2\right). \quad (19)$$

It is now obvious from this condition that the effect of the self-gravitational force acting on dust grains and ions tries to destabilize the dust-lower-hybrid mode, whereas the effects of external magnetic field and thermal pressure of the dust fluid try to stabilize this mode and counter the gravitational condensation of the dust grains.

It is found that a dusty plasma system, containing negatively charged (extremely massive) dust grains and positively charged ions, may support two new ultra-low-frequency modes, namely, dust-cyclotron mode and dust-lower-hybrid mode, propagating perpendicular to the external magnetic field. If we compare this dust-cyclotron (dust-lower-hybrid) mode with ion-cyclotron (ion-lower-hybrid) mode [44], it can be shown from our dispersion relations that the phase velocity of the dust-cyclotron mode is approximately $Z_d m_i/Z_i m_d$ (whose value may range from 10^{-4} to 10^{-8}) times smaller than that of the ion-cyclotron mode, whereas the phase velocity of the dust-lower-hybrid mode is approximately $Z_d m_e/m_d$ (where m_e is the mass of an electron) times smaller than that of the ion-lower-hybrid mode.

It is observed that the effect of the gravitational force acting on both dust grains and ions tries to make all these low-frequency electrostatic modes (dust-acoustic mode, dust-cyclotron mode and dust-lower-hybrid mode) unstable, whereas effects of dust-temperature and external magnetic field play stabilizing role, i.e., try to make the mode stable and counter the gravitational condensation of the dust grains.

It may be stressed here that the results of the present investigation may be useful for understanding the electrostatic disturbances in a number of astrophysical dusty plasma systems, such as, planetary ring systems (viz. Saturn's rings [3,13,21]), cometary environments (viz. Halley's comet [14,23]), interstellar medium [3], etc., where negatively charged dust particulates and positively charged ions are the major plasma species.

It may be pointed out that this work should also play an important role, not only in the study of electrostatic waves in Saturn's rings and Halley's comet, proposed by Bliokh & Yaroshenko [13] and Angelis *et al.* [14], but also in understanding coagulation and gravitational condensation of the dust grains.

It may also be added here that the effects of inhomogeneities in plasma density and external magnetic field on these low-frequency electrostatic perturbation modes, and their instabilities are also problems of great importance, but beyond the scope of the present work.

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