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Abstract

In our previous work (hep-th/9812044) we have proposed the sigma-model action, conjectured to be the NSR analogue of superstring theory on $AdS_5 \times S^5$. This sigma-model is the NSR superstring action with potential term corresponding to the exotic 5-form vertex operator (branelike state). This 5-form potential plays the role of cosmological term, effectively curving the flat space-time geometry to that of $AdS_5 \times S^5$. In this paper we study this ansatz in more detail and provide the derivation of the correlators of the four-dimensional super Yang-Mills theory from the above mentioned sigma-model. In particular, we show that the correlation function of two dilaton vertex operators in such a model reproduces the well-known result for the two-point function in $N = 4$ four-dimensional super Yang-Mills theory: $< F^2(x)F^2(y) > \sim \frac{N^2}{|x-y|^8}$.

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Introduction

One of the most profound questions with regard to critical superstring theory in ten dimensions is its relation to our four-dimensional world. Superstring theory on $AdS_5 \times S^5$, whose low energy limit is the anti de Sitter supergravity, is presumed to be related to $D = 4$ $N = 4$ super Yang-Mills theory due to the holography principle and to contain crucial information about the large N gauge dynamics in four dimensions [1,2,3,4]. Construction of the action for this theory in the GS formalism has been considered in [5,6,7,8,9]. The GS superstring action in the $AdS_5 \times S^5$ metric is essentially non-linear and its quantization in a straightforward way and computation of string scattering amplitudes seem to be problematic. In the previous work [10] we attempted to formulate the NSR version of superstring theory on AdS. Namely, we have proposed the σ-model action, which we claimede to be the NSR analogue of the superstring action on $AdS_5 \times S^5$ (refs-Tseytlin) in the GS formalism. Up to picture-changing transformation, the action for this sigma-model is given by:

$$S_{NSR} = \int d^2z[\frac{1}{2}\partial X^m \bar{\partial} X_m + \frac{1}{2}(\psi^m \bar{\partial}\psi_m + \bar{\psi}^m \partial \bar{\psi}_m) \\ +\lambda(X)\epsilon^{p_1p_2p_3p_4}[e^{\phi}\psi_{p_1}\psi_{p_2}\psi_{p_3}\psi_{p_4}\psi_t\bar{\partial}y^t + \partial(e^{\phi}\psi_{p_1}\psi_{p_2}\psi_{p_3})\bar{\partial}\tilde{x}_{p_4}] + ghosts] \tag{1}$$

where $X^m \equiv (x^p, y^t)$ are ten-dimensional space-time coordinates, split in the $''4+6''$ way, $p = 0,...,3; t = 4,...,9$; ψ are worldsheet NSR fermions, ϕ is bosonized superconformal ghost field. In other words, the action (1) consists of a kinetic part which is just the standard NSR free superstring action and the sigma-model type potential, determined by the massless 5-form vertex operator, whose zero-momentum part is given by

$$V_5 = \lambda\epsilon^{p_1...p_4}\psi_{p_1}...\psi_{p_4}\psi_t\bar{\partial}X^t + ghosts \tag{2}$$

where λ is constant. We will see, from analyzing the BRST invariance condition that λ must infact be constant in the space-time, i.e. the potential term in the sigma-model is cosmological-like. The BRST-invariant 5-form massless vertex operator (2) does not seem to correspond to any known physical particle in perturbative string spectrum. On the contrary, it is related to the brane dynamics and space-time geometry in some very intriguing way. In this paper we will discuss it in much more details. The related potential part in the action (1) is presumed to play the role of cosmological term, effectively curving the flat ten-dimensional space-time to that of $AdS_5 \times S^5$; the position of the four-dimensional boundary of AdS_5 in ten-dimensional Minkowski space-time is related to polarization of the 5-form vertex operator (2). This note is organized as follows. We begin with reviewing the arguments that relate the string theory on $AdS_5 \times S^5$ to the sigma-model with the 5-form state (2). Then we analyze the BRST invariance of the five-form vertex operator. Firstly, we find that the BRST invariance condition for the vertex operator (2) restricts the dynamics of the scalar field to four dimensions, longitudinal with respect to the underlying three-brane; then the condition of the worldsheet conformal invariance further restricts λ, requiring it to be constant (which is, in fact, related to N). Then we compute two-point dilaton correlation function in the sigma-model (1) (up to the order of λ^2) (which, according to the AdS/CFT correspondents should be related with the correlators of gauge-invariant F^2 operators in the $D = 4$ $N = 4$ SYM). We find that this two-point correlator reproduces the $< F^2F^2 >$ correlator in the four-dimensional Yang-Mills, thus producing the important evidence of the connection between the brane-like sigma-model (1) and the large N limit of gauge theory.

NSR Strings on AdS and the brane-like sigma-model

Let us briefly recall the arguments of the previous paper that led us to relate the sigma-model (1) to the AdS superstrings. We start with the $SO(1,3) \times SO(6)$-invariant $AdS_5 \times S^5$ action in the GS formalism with fixed kappa-symmetry. The action is given by [6]:

$$S = -\frac{1}{2}\int d^2z\{\sqrt{-g}g^{ij}(y^2(\partial_i x^p - 2i\bar\theta\Gamma^p\partial_i\theta)(\partial_j x^p - 2i\bar\theta\Gamma^p\partial_j\theta) + \frac{1}{y^2}\partial_i y^t\partial_j y^t) + 4\epsilon^{ij}\partial_i y^t\bar\theta\Gamma^t\partial_j\theta\} \quad (3)$$

On the other hand, as the $AdS_5 \times S^5$ space is the near-horizon limit of the D3-brane solution of the type IIB supergravity in ten dimensions, one may alternatively view the string theory on $AdS_5 \times S^5$ as a theory of closed strings propagating in the *flat* space-time in the presence of N parallel D3-branes or, equivalently, open Dirichlet strings. The stress-energy tensor of a GS superstring in the flat background is given by:

$$\begin{aligned} T_{ij} &= \Pi_i^m \Pi_{mj} \\ \Pi_i^m &= \partial_i X^m + i\bar\theta^\alpha\Gamma^m_{\alpha\beta}\partial_i\theta^\beta \end{aligned} \quad (4)$$

Because of the D3-brane presence the holomorphic and anti-holomorphic spinors θ and $\bar\theta$ are no longer independent; they are related as

$$\bar\theta_\alpha = \lambda(N)(\Gamma^0...\Gamma^3)_{\alpha\beta}\theta_\beta \quad (5)$$

where $\lambda(N)$ is some constant depending on the number N of parallel D3-branes. Now in order to find the NSR analogue of the superstring action in the presence of N parallel D3-branes, one has to use the standard relation between GS space-time spinor θ and the NSR matter + ghost spin operator of zero conformal dimension:

$$\theta_\alpha(z) = e^{\frac{\phi}{2}}\Sigma_\alpha(z) + ghosts \tag{6}$$

with the additional constraint (5). In the absence of the constraint (5) substituting (6) into the GS expression (4) for the stress-energy tensor would have been given , up to picture-changing, simply by the stress-energy tensor of a free NSR superstring.The constraint (5), however, modifies the result of the internal normal reordering that one has to perform in the Ramond-Ramond terms of the form $\bar\theta\partial\theta\bar\partial X$ in the GS stress-energy tensor (4). Namely, using the relations (5), (6) and the O.P.E. formula between spin operators: : $\Sigma_\alpha(z)\Sigma_\beta(w) :\sim \frac{\epsilon_{\alpha\beta}}{(z-w)^{\frac{5}{4}}} + \sum_p \frac{\Gamma_{\alpha\beta}^{m_1...m_p}\psi_{m_1}...\psi_{m_5}(w)}{(z-w)^{\frac{5}{4}-p}}$ we find that

$$:\bar\theta\Gamma^m\partial\theta\bar\partial X_m := \epsilon^{p_1...p_4}\partial e^{\phi}\psi_{p_1}...\psi_{p_3}\partial X_{p_4} + \epsilon^{p_1...p_4}e^{\phi}\psi_{p_1}...\psi_{p_4}\psi_t\partial X^t + ghosts \tag{7}$$

Other terms in the stress-energy tensor (4) reproduce, up to picture changing, the standard matter+ghost stress-energy tensor of free NSR theory. The action corresponding to the stress tensor (7) is then the one given by the sigma-model (1). The above arguments led us to suggest that the $AdS_5 \times S^5$ structure of the the space-time may in fact be "depicted" by the potential term in the sigma-model (1) corresponding to the exotic 5-form branelike vertex (2). In this paper we will test this conjecture by computing the correlators in the sigma-model (1) and showing them to correspond to the four-dimensional correlators in the large N gauge theory. First of all we would like to point out at some important properties of the vertex operator V_5. At zero momentum its form is given by (2). At a given momentum k the straightforward generalization would be $V_5(k) = \lambda(k)e^{\phi}\psi_{p_1}...psi_{p_4}\psi_t(\partial X^t+i(k\psi)\psi^t)e^{ikX}$; however the BRST invariance imposes constraints on possible values of k. In fact, it restricts k to the longitudinal directions, effectively reducing the dynamics of the field $\lambda(k)$ to four dimensions ! The easiest way to see it is to consider the internal singularities inside the vertex $V_5(k)$. Recall that if one takes, for instance, the vertex operator of a vector boson of the form $e_m(k)(\partial X^m + ...)e^{ikX}$, there is the internal singularity due to the coupling between ∂X and the exponent e^{ikX}: $\partial X^m(z)e^{ikX}(w) \sim \frac{k^m}{z-w}e^{ikX}$. To remove this internal singularity or, equivalently, to insure the BRST invariance of the vector vertex operator, one has to require the transversality of the polarization vector at non-zero values of the momentum: $k_m e^m(k) = 0$. In case of the vertex operator V_5 the 5-form state is BRST invariant and the internal singularities are absent if the transverse field $\bar{\partial} X^t$ does not couple to the exponent at all, i.e. the exponent depends on the longitudinal components of the momentum only since the O.P.E. $\bar{\partial} X^t(z) e^{ik^{||}X}(w)$ is non-singular ($k^{||}X \equiv k_p X^p; p = 0,1,2,3$).The same condition can be derived in a straightforward way, by computing the commmutator of $V_5(k)$ with the BRST charge. Therefore the BRST-invariant 5-form vertex operator is purely longitudinal and the scalar field λ is confined to four dimensions:

$$V_5(k^{||}) = \lambda(k^{||})\epsilon^{p_1 \dots p_4} e^{\phi} \psi_{p_1} \dots psi_{p_4} \psi_t (\bar{\partial} X^t + i(k^{||}\bar{\psi})\bar{\psi}^t) e^{ik^{||}X} \tag{8}$$

However, this is not yet the end of the story. Apart from the BRST invariance of the V_5 vertex, we also need to insure that the 5-form term in the sigma-model action (1) does not violate the worldsheet conformal invariance. Consider the stress tensor:

$$T(z) = \frac{1}{2}(\partial X^m \partial X_m + \psi^m \partial \psi_m) + \lambda(x^{||}) e^{\phi} \psi_{p_1} \dots \psi_{p_4} \psi_t \partial X^t(z) \tag{9}$$

The two-point correlation function is given by:

$$< T(z)T(w) >= \frac{1}{2|z-w|^4}(15 + 2 < \lambda(x^{||}(z,\bar{z}))\lambda(x^{||}(w,\bar{w})) >) \tag{10}$$

(note that ∂X^t doesn't interact with λ. It follows that, in order to preserve the conformally invariant form of the O.P.E. for two stress-energy tensors in the two-dimensional CFT (that is, there are no extra singularities coming from the O.P.E. $\lambda(x^{||}(z))\lambda(x^{||}(w))$) we have to require that $< \lambda(x^{||}(z,\bar{z}))\lambda(x^{||}(w,\bar{w})) >$ is independent on $(z,\bar{z})$ and $(w,\bar{w})$ i.e. it is given by a constant. This is achieved for two choices of $\lambda(k)$: the trivial one, when $\lambda(k)$ is taken to be a four-dimensional delta-function (then λ is constant in the longitudinal four-dimensional subspace) or if one takes $\lambda(k)$ to be be inversely proportional to the fourth power of $|k^{||}|$:

$$\lambda(k) \sim \frac{\lambda_0^{(+1)}}{k^{||4}} \tag{11}$$

where $\lambda_0^{(+1)}$ is $k^{||}$-independent and the label $(+1)$ refers to the ghost number. Indeed, in the latter case the correlation function is given by $< \lambda(x^{||}(z,\bar{z}))\lambda(x^{||}(w,\bar{w})) >\sim {\lambda_0^{(+1)}}^2 \int \frac{d^4k^{||}}{k^{||4}} e^{-k^{||2} ln|z-w|}$ and the integral is taken over the entire four-dimensional momentum space. The dependence on $|z-w|$ is then eliminated by rescaling ${k^{||}}^2 ln|z-w| \to \tilde{k^{||}}$.

Correlation functions in the brane-like sigma-model

In our calculation of the two-point dilaton correlation in the sigma-model with the 5-form state we would like to use the slightly modified version of (1).Namely, in order to maintain the correct ghost number balance in correlation functions on the sphere it is convenient to choose the linear combination of V_5 in the +1-picture with its picture -3 version

$$V^{(-3)} = \frac{\lambda_0^{(-3)}}{|k^{||}|^4}\epsilon^{p_1...p_4}e^{-3\phi}\psi_{p_1}...\psi_{p_4}\psi_t(\bar{\partial}X^t + i(k^{||}\bar{\psi})\bar{\psi}^t)e^{ik^{||}x^{||}} \quad (12)$$

where $\lambda^{(-1)}$ and $\lambda^{(-3)}$ are some k-independent constants to be specified later(in fact both are related to the parameter N of the gauge theory) The generating functional for the dilaton amplitude is then given by:

$$Z(\varphi,\lambda) = \int D[X]D[\Psi][ghostsf(\Gamma)]exp\{\int d^2z\frac{1}{2}(\partial X_m\bar{\partial}X^m + \psi_m\bar{\partial}\psi^m)$$
$$+\int\frac{d^4k^{||}}{k^{||^4}}\epsilon^{p_1...p_4}(\lambda_0^{(-3)}e^{-3\phi} + \lambda_0^{(+1)}e^{\phi})\psi_{p_1}...\psi_{p_4}\psi_t(\bar{\partial}X^t + i(k^{||}\psi)\psi^t)e^{ik^{||}x} + cc \quad (13)$$
$$+\int d^{10}kV_\varphi(k)\varphi(k)\}$$

where $V(k)$ is the dilaton vertex operator at momentum k and $f(\Gamma)$ is certain function of picture-changing operator Γ, necessary to insure the correct ghost number balance in correlation functions.The function $f(\Gamma)$ will be specified later. Of course, it is possible to rewrite the functional (13) in an equivalent form so that it would only contain the picture +1 five-form in the potential, but with the different measure deformation $f(\Gamma)$. We have dropped the term with the full derivative of the three-form, which does not contribute to correlators in our calculation; $V_\varphi(k) = \partial X^m\bar{\partial}X^n(\eta_{mn} - k_m\bar{k}_n - k_n\bar{k}_m)e^{ikX}$ is the dilaton vertex operator; in this paper we will ignore the longitudinal $(k,\bar{k})$ part of the dilaton vertex since it is not important for the correlation functions.

The two-point dilaton correlation function is given by:

$$< V_\varphi(p_1)V_\varphi(p_2) > = \frac{\delta Z(\varphi,\lambda)}{\delta\varphi(p_1)\delta\varphi(p_2)}|_{\varphi=0} \quad (14)$$

The first non-trivial (of order λ^2) contribution to this correlation function is given by

$$A =< V_\varphi(p_1)V_\varphi(p_2) >\sim \lambda^{(+1)}\lambda^{(-3)}\int\frac{d^4k_1^{||}}{k_1^{||^4}}\int\frac{d^4k_2^{||}}{k_2^{||^4}} < V_\varphi(p_1)V_\varphi(p_2)V_5(k_1^{||})V_5(k_2^{||}) > \quad (15)$$

We shall see that the term of order λ^2 in the expansion corresponds to the contribution of the s-wave of the dilaton in the AdS picture. As for higher order terms in the expansion in λ, we shall argue that they correspond to higher partial waves of the dilaton field on the AdS_5. Due to the momentum conservation, we have $p_2 = -p_1 - k_1^{||} - k_2^{||}$, therefore the λ^2 part of dilaton amplitude is given by:

$$\begin{aligned} A(p_1) = \lambda^{(+1)}\lambda^{(-3)} \int \frac{d^4 k_1^{||}}{k_1^{||4}} \int \frac{d^4 k_2^{||}}{k_2^{||4}} < V_\varphi(p_1) V_\varphi(-p_1 - k_1^{||} - k_2^{||}) >= \\ \int \frac{d^4 k_1^{||}}{k_1^{||4}} \int \frac{d^4 k_2^{||}}{k_2^{||4}} \int d^2 z_1 \int d^2 z_2 \int d^2 w_1 \int d^2 w_2 \{ \\ < e^{-3\phi} : \psi^5_{t_1} : \bar\partial X^{t_1} e^{ik_1^{||}X}(z_1, \bar z_1) e^{\phi} : \psi^5_{t_2} : \bar\partial X^{t_2} e^{ik_2^{||}X}(z_2, \bar z_2) \\ \times e^{-\bar\phi}(\partial X_{m_1} + i(p_1\psi)\psi_{m_1})\bar\psi^{m_1} e^{ip_1 X}(w_1, \bar w_1) \\ \times e^{-\bar\phi}(\partial X_{m_1} + i(p_2\psi)\psi_{m_1})\bar\psi^{m_1} e^{ip_2 X}(w_2, \bar w_2) >\} \end{aligned} \tag{16}$$

where we have introduced the notation $\psi^5_t = \epsilon^{p_1 p_2 p_3 p_4} : \psi_{p_1}\psi_{p_2}\psi_{p_3}\psi_{p_4}\psi_t :$ and the correlators are now to be evaluated in the free theory. Dividing by the $SL(2, C)$ volume one has:

$$\begin{aligned} A(p_1) = \lambda^{(+1)}\lambda^{(-3)} \int \frac{d^4 k_1^{||}}{k_1^{||4}} \int \frac{d^4 k_2^{||}}{k_2^{||4}} \int d^2 w_1 |z_1 - w_2|^2 |z_2 - w_2|^2 |z_1 - z_2|^2 \\ \times < e^{-3\phi}(z_1) e^{\phi}(z_2) >< e^{-\bar\phi}\bar\psi^{m_1}(\bar w_1) e^{-\bar\phi}\psi^{n_1}(\bar w_2) > \\ \times < [: \psi^5_{t_1} : (\bar\partial X^{t_1} + i(k^{||}\bar\psi^{||})\bar\psi^{t_1}) e^{ik_1^{||}X} + c.c.](z_1, \bar z_1) \\ \times [: \psi^5_{t_2} : (\bar\partial X^{t_2} + (\bar\psi^{||})\bar\psi^{t_2}) e^{ik_2^{||}X} + c.c.](z_2, \bar z_2) \\ \times (\partial X_{m_1} + i(p_1\psi)\psi_{m_1}) e^{ip_1 X}(w_1, \bar w_1) \\ \times (\partial X_{m_1} - i((p_1 + k_1^{||} + k_2^{||})\psi)\psi_{m_1}) e^{-i(p_1 + k_1^{||} + k_2^{||})X}(w_2, \bar w_2) > \end{aligned} \tag{17}$$

where z_1, z_2 and w_2 are now fixed by the conformal invariance and will be later set to $0, 1$ and ∞ Evaluation the free theory correlators in (17) gives

$$A(p_1)$$

$$= \lambda^{(+1)}\lambda^{(-3)} \int \frac{d^4k_1^{||}}{k_1^{||^4}} \int \frac{d^4k_2^{||}}{k_2^{||^4}} \int d^2w_1|z_1-w_2|^2|z_2-w_2|^2|z_1-z_2|^2(\bar{w_1}-\bar{w_2})^{-2}$$
$$\times[\{\frac{2p_1^{t_1}p_2^{t_2}+\eta^{t_1t_2}(p_1p_2)^{||}}{(z_1-w_1)^2(z_2-w_2)^2}+\frac{2p_1^{t_2}p_2^{t_1}+\eta^{t_1t_2}(p_1^{||}p_2^{||})}{(z_1-w_2)^2(z_2-w_1)^2}$$
$$+\frac{2p_1^{t_1}p_2^{t_2}+2p_2^{t_1}p_1^{t_2}+\eta^{t_1t_2}}{(z_1-w_1)(z_1-w_2)(z_2-w_2)(z_2-w_1)}-\frac{(8(p_1^{||}k_1^{||})+8(p_1^{||}k_2^{||}))\eta^{t_1t_2}}{(z_1-z_2)^2(w_1-w_2)^2}$$
$$-\frac{\eta^{t_1t_2}(8(p_1^{||}k_1^{||})+8(p_1^{||}k_2^{||})-8(p_1^{||})^2)}{(z_1-z_2)(w_1-w_2)}(\frac{1}{(z_1-w_1)(z_2-w_2)}+\frac{1}{(z_1-w_2)(z_2-w_1)})\} \quad (18)$$
$$\times\{\frac{p_{1t_1}p_{1t_2}}{(\bar{z_1}-\bar{w_1})(\bar{z_2}-\bar{w_1})}+\frac{p_{2t_1}p_{2t_2}}{(\bar{z_1}-\bar{w_2})(\bar{z_2}-\bar{w_2})}$$
$$+\frac{p_{1t_1}p_{2t_2}}{(\bar{z_1}-\bar{w_1})(\bar{z_2}-\bar{w_2})}+\frac{p_{1t_2}p_{2t_1}}{(\bar{z_1}-\bar{w_2})(\bar{z_2}-\bar{w_1})}+\frac{\eta_{t_1t_2}}{(\bar{z_1}-\bar{z_2})^2}\}+c.c.]$$
$$\times[\frac{1}{|w_1-w_2|^{2(p_1p_2)}|z_1-z_2|^{2(k_1^{||}k_2^{||})}|z_1-w_1|^{2(k_1^{||}p_1^{||})}}$$
$$\times\frac{1}{|z_1-w_2|^{2(k_1^{||}p_2^{||})}|z_2-w_1|^{2(k_2^{||}p_1^{||})}|z_2-w_2|^{2(k_2^{||}p_2^{||})}}]$$

Here $p_{1,2}^{||}$ is the projection of the dilaton momentum on the four longitudinal directions (parallel to the D3-brane worldvolume). Setting $z_1 \to 0$, $z_2 \to 1$,$w_2 \to \infty$ and evaluating the integral over w_1 we find that the λ^2 contribution to the amplitude is given by:

$$A(p_1) = \lambda^{(+1)}\lambda^{(-3)} \int \frac{d^4k_1^{||}}{k_1^{||^4}} \int \frac{d^4k_2^{||}}{k_2^{||^4}} \int d^2w_1\{[(\frac{1}{w_1^2}+\frac{1}{\bar{w_1}^2})(2p_1^{t_1}p_2^{t_2}+(p_1^{||}p_2^{||})\eta^{t_1t_2})$$
$$+(\frac{1}{1-w_1^2}+\frac{1}{1-\bar{w_1}^2})(2p_1^{t_2}p_2^{t_1}+(p_1^{||}p_2^{||})\eta^{t_1t_2})$$
$$+(\frac{2}{w_1(1-w_1)}+\frac{2}{\bar{w_1}(1-\bar{w_1})})(p_1^{t_1}p_2^{t_2}+p_1^{t_2}p_2^{t_1}+(p_1^{||}p_2^{||})\eta^{t_1t_2})] \quad (19)$$
$$\times p_1^{t_1}p_1^{t_2}|w_1|^{2(k_1^{||}p_1^{||})}|1-w_1|^{2(k_2^{||}p_1^{||})}\}$$
$$= 4p_1^{||^4} \int \frac{d^4k_1^{||}}{k_1^{||^4}} \int \frac{d^4k_2^{||}}{k_2^{||^4}} \frac{\Gamma(1+(k_1^{||}p_1^{||}))\Gamma((p_1^{||}k_2^{||})-1)\Gamma(1-(p_1^{||}(k_1^{||}+k_2^{||})))}{\Gamma((p_1^{||}(k_1^{||}+k_2^{||})))\Gamma((-k_1^{||}p_1^{||}))\Gamma(2-(k_2^{||}p_1^{||}))}$$

where $\Gamma(p)$ is Euler gamma-function. It is remarkable that this amplitude depends exclusively on the longitudinal projection of the dilaton momentum p_1,i.e. the four-dimensional vector $p_1^{||}$. That means that , upon the Fourrier transform $A(p_1)$ will be the function of four space-time coordinates, corresponding to the AdS_5 boundary (or the polarization of the 5-form vertex).We observe that the structure of the two-point dilaton amplitude involves the fourth power of the four-dimensional momentum $p_1^{||}$ (which appears as kinematic factor in the NSR superstring four-point function) multiplied by the factor that will arise as a result of the integration of the product of gamma-functions in the Veneziano amplitude over the momenta of the exotic states. We will show that this factor is proportional to the $\sim ln(p_1^{||2})$, i.e. $A(p_1) \sim p_1^{||4} ln(p_1^{||2})$ indeed reproduces the two-point correlation function $< F^2(p)F^2(-p) >$ in the four-dimensional $N=4$ super Yang-Mills. Our aim now is to perform the integration over $k_1^{||}$ and $k_2^{||}$ in the amplitude (19). Let us cast the expression (19) for the dilaton amplitude into the following form:

$$A(p_1) = 2\lambda^{(+1)}\lambda^{(-3)}(p_1^{||})^2 \int \frac{d^2 w_1}{|1-w|^4} \int \frac{d^4 k_1^{||}}{k_1^{||4}} \int \frac{d^4 k_2^{||}}{k_2^{||4}} [2(p_1^{||})^2 + (p_1^{||}(k_1^{||} - k_2^{||}))] \times e^{-2(k_1^{||}p_1^{||})ln|w_1| + (k_2^{||}p_1^{||})ln|1-w_1|} \quad (20)$$

We start with the term proportional to $p_1^{||4}$ in (20). We need to evaluate the integral over $k_1^{||}$ (the integration over $k_2^{||}$ is totally similar). To make our notations more convenient, we denote $k_1^{||} \equiv k$ and $p_1^{||} \equiv p$ and write:

$$I_1(p_1, w_1) = \int \frac{d^4 k}{k^4} e^{-2(kp)ln|w|} = \int d^3\vec{k} \int_{-\infty}^{\infty} dk_0 \frac{e^{-2(k_0 p_0 - (\vec{k}\vec{p}))ln|w|}}{(k_0 - |\vec{k}| + i\epsilon)^2 (k_0 + |\vec{k}| - i\epsilon)^2} \quad (21)$$

Integrating over k_0 first by evaluating the residues we get the spatial integral over $\vec{k}$ which is not difficult to compute:

$$I_1(p_1,w_1) = -\int d^3\vec{k}\partial_{|k|}\frac{e^{-2|k|(p_0-|p|cos\theta)ln|w|}}{4|k^2|} = \int_0^\infty d|k||k|^2\int_0^\pi d\theta sin\theta$$
$$\times[\frac{e^{-2|k|(p_0-pcos\theta)ln|w|}}{2|k|^3} + \frac{2(p_0-pcos\theta)ln|w|e^{2i|k|(p_0-|p|cos\theta)ln|w|}}{4|k|^2}]$$
$$= \int_0^\infty d|k|(e^{-2(p_0+|p|)|k|ln|w|} - e^{-2(p_0-|p|)|k|ln|w|})$$
$$\times(\frac{1}{4|k|}\frac{-1}{8|p||k|^2ln|w|}(1-\frac{p_0}{|p|}) + \frac{1}{8p|k|^2ln|w|}) \quad (22)$$
$$= -\frac{1}{4|p|}(p_0+|p|)(ln[|2ln|w|(p_0+|p|)|]-1)$$
$$+\frac{1}{4|p|}(p_0-|p|)(ln[|2ln|w|(p_0-|p|)|]-1)$$
$$-\frac{1}{4}(1-\frac{p_0}{|p|})ln(p_0+|p|) - \frac{1}{4}(1+\frac{p_0}{|p|})ln(p_0-|p|)$$
$$= \frac{1}{2}(1+ln|2ln|w||-ln(p_0-|p|)-ln(p_0+|p|)) = \frac{1}{2}(1-ln|2ln|w||-\frac{1}{2}ln(p^2))$$

where we denoted $|p| \equiv |\vec{p}|$ and used that for $\alpha > 0$ the regularized integrals are given by:

$$\int_0^\infty \frac{e^{-\alpha x}}{x}dx = ln\alpha$$
$$\int_0^\infty \frac{e^{-\alpha x}}{x^2}dx = \alpha(ln\alpha - 1) \quad (23)$$

Using (20) and (22) we find that the two-point dilaton amplitude is given by:

$$A(p_1) = 2\lambda^{(+1)}\lambda^{(-3)}\int\frac{d^2w_1}{|1-w|^4}\{(p_1^{||})^4(1-ln(|2ln|w||)-\frac{1}{2}ln((p_1^{||})^2))$$
$$\times(1-ln(|2ln|1-w||)-\frac{1}{2}ln((p_1^{||})^2))\} \quad (24)$$

Since

$$\int d^2w\frac{1}{|w|^\alpha|1-w|^\beta} = \frac{\Gamma(1-\frac{\alpha}{2})\Gamma(1-\frac{\beta}{2})\Gamma(\frac{\alpha+\beta}{2}-1)}{\Gamma(2-\frac{\alpha+\beta}{2})\Gamma(\frac{\alpha}{2})\Gamma(\frac{\beta}{2})} \quad (25)$$

one has

$$\int\frac{d^2w_1}{|1-w|^4} = \frac{\Gamma(-1)\Gamma(1)\Gamma(1)}{\Gamma(2)\Gamma(0)\Gamma(0)} = \frac{1}{\Gamma(0)} = 0 \quad (26)$$

therefore the term proportional to the square of the logarithm $\sim (ln(p_1^{||})^2)^2$ vanishes. Then the leading non-analytic term in the amplitude is given by:

$$A(p_1)\sim 2\lambda^{(+1)}\lambda^{(-3)}(p_1^{||})^4 ln(p_1^{||})^2\int d^2w\frac{ln(|ln|w||)-ln(|ln|1-w||)}{|1-w|^4} \quad (27)$$

Fourier transforming (27) to the four-dimensional position space we get

$$< F^2(x)F^2(y) > \equiv < V_\varphi V_\varphi > \sim \frac{\lambda^{(+1)}\lambda^{(-3)}}{|x-y|^8} + ... \tag{28}$$

where we have skipped less singular terms, possibly arising from the $(p_1^{||})^2$-contribution. As we mentioned before, the dependence on N is actually hidden in the factor of λ. Since the two-point correlator is proportional to N^2 one has to impose the condition $\lambda^{(+1)}\lambda^{(-3)} \sim N^2$. We will choose

$$\begin{aligned} \lambda^{(+1)} &\sim \frac{\rho}{N} \\ \lambda^{(-3)} &\sim \rho N^3 \end{aligned} \tag{29}$$

where ρ is N-independent constant chosen in such a way that the product of ρ^2 with the regularized integral over w in (27) gives the correct normalization of the two-point function in the four-dimensional super Yang-Mills theory. Then we have

$$< F^2(x)F^2(y) > \sim \frac{N^2}{|x-y|^8} \tag{30}$$

At the first glance the choice (29) of N-dependence of $\lambda^{(+1)}$ and $\lambda^{(-3)}$ in (29) is not unique (since it is only important that their product is proportional to N^2). However we shall see that due to the ghost number conservation the N-dependence choice (29) is actually uniquely fixed by higher order corrections in λ^2 which we will discuss in the next section.

Corrections in λ as partial waves.

In this part we will make few comments on the structure of amplitudes arising as a result of further expansion in λ^2 the contributions of order λ^{2n} correspond to $2n+2$-point correlation functions integrated over $2n$ internal momenta of the five-form vertices. The kinematic factor correlation functions is some polynomial of degree $2n+2$ while the integration over the internal momenta should hopefully produce the logarithm similarly to the four-point case so the most singular part of the λ^{2k} contribution to the dilaton-dilaton amplitude in the model (13) is proportional to $\sim k^{2n+2} ln k^2$ In the AdS picture such a momentum dependence corresponds to the contribution of the dilaton partial wave with the angular momentum $l = 2n-2$. Therefore the expansion in λ^2 in (13) seems to correspond to the expansion in partial waves of the dilaton. Also the comment should be made about the N-dependence of the dilaton-dilaton amplitude. The choice (29) for the N-dependence insures the proper N-dependence for the 4n-point amplitudes (proportional to λ^{4n-2}), i.e. the partial waves with even angular momenta. As for the waves with odd angular momenta (corresponding to $4n+2$-point amplitudes) in the sigma-model, in order to take them into account with the proper weight one has to insert the picture-changing factor in the measure of functional integration so as to insure the odd momentum contributions are correctly weighted. Namely, one has to choose the picture-changing function $f(\Gamma)$ in the generating functional (13) as

$$f(\Gamma) = 1 + \frac{\Gamma^2}{N^2} \tag{31}$$

provided that the parameters $\lambda^{(-3)}$ and $\lambda^{(+1)}$ are defined as in (29). As we already mentioned before, there exists also an equivalent way of writing the generating functional (13) with only the picture $+1$ version of the five-form present in the action:

$$\begin{aligned} Z(\varphi, \lambda) = \int D[X]D[\Psi][ghosts]\frac{\Gamma^2}{\Gamma^2 - N^2}exp\{\int d^2z\frac{1}{2}(\partial X_m\bar{\partial}X^m + \psi_m\bar{\partial}\psi^m) \\ + \int \frac{d^4k^{||}}{k^{||4}}\epsilon^{p_1...p_4}\frac{\rho}{N}e^{\phi}\psi_{p_1}...\psi_{p_4}\psi_t(\bar{\partial}X^t + i(k^{||}\psi)\psi^t)e^{ik^{||}x} + cc \\ + \int d^{10}kV_{\varphi}(k)\varphi(k)\} \end{aligned} \tag{32}$$

It is easy to show that the functional (32) and the functional (13) with the measure function (31) and with $\lambda^{(-3)}$ and $\lambda^{(+1)}$ defined in (29) produce equivalent series in ρ.

Conclusion

We have shown that the dilaton-dilaton amplitude in the ten-dimensional branelike sigma-model (13), (32) reproduces the two-point correlation function $< F^2F^2 >$ of the $N = 4$ $D = 4$ super Yang-Mills theory. The natural project for the future is to compute three and four-point correlators in the branelike sigma-model model (13), (32) in order to verify their agreement with the corresponding computations in the AdS supergravity (see, for instance, [11,12,13,14,15]) The questions of particular interest would be reproducing the logarithmic singularities in four-point correlators in the AdS supergravity and the relation between logarithms in AdS correlators and anomalous dimensions of Yang-Mills operators. Another important object of interest is AdS S-matrix [16,17] which can be understood in terms of structure constants (or three-point correlation functions) of the conformal field theory on the worldsheet of superstring on $AdS_5 \times S^5$. In accordance with our model these functions are to be computed in the free theory with five-form insertions. The final observation to be made is that the fiveform state in the spectrum of NSR superstring in

$D = 10$ seems to play the crucial role in building the space-time geometry. Namely, the five-form vertex V_5 (which defines a BRST-invariant massless state in superstring theory in *flat* ten-dimensional space-time) integrated over its four-dimensional momentum , transforms the flat maximally supersymmetric space-time vacuum into the one of $AdS_5 \times S^5$. Thus the Anti-de-Sitter structure of our space-time appears to have dynamical origin: it is the consequence of the presence of exotic brane-like states in the spectrum of a superstring. We hope that the computations outlined above will strengthen the ground for this hypothesis.

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References

[1] J.Maldacena, Adv.Theor.Math.Phys.2 (1998) 231-252, hep-th/9711200

[2] S.Gubser,I.Klebanov, A.M.Polyakov, **Phys.Lett.B428:105-114**

[3] A.M.Polyakov,hep-th/9809057

[4] E.Witten**Adv.Theor.Math.Phys.2:253-291,1998**

[5] R.Metsaev,A.Tseytlin,hep-th/9806095

[6] R.Kallosh, A.Tseytlin,**J.High Energy Phys.9810:016,1998**

[7] I.Pesando,**JHEP11(1998)002**

[8] R.Kallosh,J.Rahmfeld, A.Rajaraman,hep-th/9805217

[9] R.Kallosh, J.Rahmfeld, hep-th/9808038

[10] D.Polyakov, hep-th/9812044

[11] E.D'Hoker, D.Freedman, L.Rastrelli, hep-th/9905049

[12] By E.D'Hoker, D.Freedman, S.Mathur, A.Matusis, L. Rastelli, MIT-CTP-2843 hep-th/9903196

[13] D.Freedman, S.Mathur, A.Matusis, L.Rastrelli, hep-th/980458

[14] S.Lee, S.Minwalla, M.Rangamani, N.Seiberg, Adv.Theor.Math.Phys 2, 697 (1998), hep-th/9806074

[15] M.Bianchi, M.B.Green, S.Kovacs, G.Rossi, JHEP**08**, 013 (1998), hep-th/9807033

[16] J. Polchinski (Santa Barbara, ITP). NST-ITP-99-02, hep-th/9901076

[17] J.Polchinski, L.Susskind, N.Toumbas, hep-th/9903228