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TENSOR SUSCEPTIBILITIES OF THE VACUUM  
FROM CONSTITUENT QUARKS\*

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# Tensor susceptibilities of the vacuum from constituent quarks<sup>\*</sup>

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## Abstract

We show that the constituent quark model leads to simple expressions for the isoscalar and isovector tensor susceptibilities of the vacuum. The found values are negative and of magnitude compatible with QCD-sum-rule parameterizations of spectral densities in appropriate  $L = 1$ -meson channels.

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Susceptibilities of the vacuum are important quantities of strong interaction physics. They directly enter in the determination of hadron properties in the QCD sum rule approach. In particular, tensor susceptibilities of the vacuum [1–3] are relevant for the determination of tensor charges of the nucleon [4,5]. Widely different values for these susceptibilities, all obtained by QCD sum rules techniques, have been reported in the literature [1–3]. In this paper we address the issue from a completely different viewpoint, using the concept of *constituent quarks*. It is believed that chiral constituent quark models [6,7] describe properly the essential physics in the *shallow Euclidean* region, with momentum transfers of the order of  $|q^2| \ll \Lambda^2$ , where  $\Lambda$  is the ultraviolet cut off, typically of the order of a few hundred MeV. This is strongly supported by the instanton-liquid model [8,9]. In that region spontaneous breakdown of chiral symmetry yields a constituent mass for quarks,  $M \sim (250 \div 400)\text{MeV}$ . Susceptibilities are quantities defined with  $q^2 = 0$ , hence they seem to perfectly fit in the applicability range of the constituent quark model.

We define the isoscalar and isovector tensor susceptibilities of the vacuum as

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$$\Pi_{I=0} \equiv \langle 0 | i \int d^4x T \{ \bar{\psi}(x) \sigma^{\mu\nu} \psi(x), \bar{\psi}(0) \sigma_{\mu\nu} \psi(0) \} | 0 \rangle, \quad (1)$$

$$\Pi_{I=1} \equiv \langle 0 | i \int d^4x T \{ \bar{\psi}(x) \sigma^{\mu\nu} \tau^3 \psi(x), \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) \} | 0 \rangle, \quad (2)$$

where  $\psi = (u, d)$  is the quark field,  $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ , and  $\tau^a$  denote Pauli isospin matrices. Related definitions are present in the literature: He and Ji [1] introduce  $\Pi(0) = \frac{1}{4}(\Pi_{I=0} + \Pi_{I=1})$ , and Belyaev and Oganessian [2] consider  $\Pi_1(0) \equiv \frac{1}{12}\Pi(0)$ . The main result of this paper is that the constituent quark model (CQM) predicts:

$$\Pi_{I=0}^{\text{CQM}} = -24F_\pi^2 \frac{1 + \frac{1}{2}\kappa_{I=0}^Q}{g_A^Q}, \quad \Pi_{I=1}^{\text{CQM}} = -24F_\pi^2 \frac{1 + \frac{1}{2}\kappa_{I=1}^Q}{g_A^Q}, \quad (3)$$

where  $F_\pi = 93\text{MeV}$  is the pion decay constant,  $g_A^Q$  is the axial charge of the constituent quark, and  $\kappa_{I=0,1}^Q$  denote its isoscalar and isovector anomalous magnetic moments. The result (3) is the leading- $N_c$  result, with  $g_A^Q$  and  $\kappa_{I=0,1}^Q$  entering as model parameters. The expected range for  $g_A^Q$  is  $0.75 < g_A^Q \leq 1$ , with the lower value naively helping to reproduce  $g_A$  of the nucleon in the non-relativistic quark model. Such lower values are obtained when constituent quarks attract in the vector-isovector channel [7,10,11]. In the absence of such interactions  $g_A^Q = 1$ . It is commonly accepted that the magnitudes of  $\kappa_{I=0}^Q$  and  $\kappa_{I=1}^Q$  are tiny, less than  $\sim 5\%$ . In constituent quark models these quantities depart from zero when tensor interactions among quarks are introduced [10,11]. Using above estimates we find

$$-0.25\text{GeV}^2 \leq \Pi_{I=0,1}^{\text{CQM}} \leq -0.2\text{GeV}^2. \quad (4)$$

This value is very close to the estimate of Belyaev and Oganessian [2] made in the framework of the QCD sum rules:  $(\Pi_{I=1} + \Pi_{I=0})/2 = -0.2\text{GeV}^2$ .

Now we pass to the derivation of Eq. (3). For our purpose no details of a specific constituent quark model are needed, such as the way of introducing the ultraviolet cut off, its value, or the value of the constituent quark mass. It is enough that the spontaneous symmetry has been broken and quarks are massive. In addition, our calculation is made in the *leading- $N_c$  approximation*, which is equivalent to the one-quark-loop approximation in the effective action. Corrections to this leading result could in principle be included. We begin with the simplest case of no tensor nor vector interactions. Then Eqs. (1-2) are evaluated by the one-quark-loop diagram, which in the momentum space gives

$$\begin{aligned} \Pi_{I=0,1}^{\text{CQM}} &= N_c N_f i \int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}_{\text{Dirac}}[(\not{k} + M)\sigma^{\mu\nu}(\not{k} + M)\sigma_{\mu\nu}]}{(k^2 - M^2 + i\delta)^2} = \\ &= 48N_c N_f i \int \frac{d^4k}{(2\pi)^4} \frac{M^2}{(k^2 - M^2 + i\delta)^2} = -48N_c N_f \int_{\Lambda} \frac{d^4k_E}{(2\pi)^4} \frac{M^2}{(k_E^2 + M^2)^2}, \end{aligned} \quad (5)$$

where  $N_c = 3$  is the number of colors,  $N_f = 2$  is the number of flavors, and  $M$  is the mass of the constituent quark. In the last equality we have Wick-rotated to Euclidean momentum  $k_E$ . The integral in Eq. (5) is divergent, as is inherent in effective models, hence a regulator is introduced in the integral in Eq. (5), indicated by the subscript  $\Lambda$  in the integral. There

are many ways of introducing the cut-off, the details, however, are irrelevant. The crucial point is that on the RHS of Eq. (5) we recognize the well-known result of constituent quark models [6,7,10,11] for the square of the pion decay constant:

$$F_\pi^2 = 4N_c \int_\Lambda \frac{d^4 k_E}{(2\pi)^4} \frac{M^2}{(k_E^2 + M^2)^2}. \quad (6)$$

Combining Eqs. (5-6) proves Eq. (3) for the case of no vector-isovector ( $g_A^Q = 1$ ) and no tensor ( $\kappa_{I=0,1}^Q = 0$ ) interaction. Note that our result holds also for the case where  $M$  depends on the momentum,  $M = M(k^2)$ , as in the case of the instanton-vacuum model described in Ref. [6].

In the general case interactions in the vector and tensor channels can be present. The following terms in the effective chiral  $SU(2) \otimes SU(2)$  Lagrangian density are relevant for our purpose:

$$-\frac{1}{2}G_\rho \left[ \left( \bar{\psi}(x) \gamma^\mu \tau^a \psi(x) \right)^2 + \left( \bar{\psi}(x) \gamma^\mu \gamma_5 \tau^a \psi(x) \right)^2 \right] \\ + \frac{1}{2}G_T \left[ \left( \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) \right)^2 - \left( \bar{\psi}(x) \sigma^{\mu\nu} \tau^a \psi(x) \right)^2 \right]. \quad (7)$$

The signs inside brackets are fixed by the chiral invariance, and the signs outside brackets are conventional. The presence of interactions (7) causes rescattering of quarks in vector and tensor channels: for instance, the  $q\bar{q}$  pair induced by the tensor current interacts, while propagating, by the tensor interactions. The resulting Bethe-Salpeter chain can be summed up as a geometric series. Details of this procedure are well known [7] and we give no further details here. The effects of finite  $G_\rho$  and  $G_T$  are particularly simple for the vanishing external momentum,  $q = 0$ , which is the case needed here. We find

$$\Pi_{I=0,1}^{\text{CQM}} = -24 \frac{J(0)}{1 \pm 4G_T J(0)}, \quad (8)$$

where  $J(0) = 4N_c \int_\Lambda \frac{d^4 k_E}{(2\pi)^4} M^2 / (k_E^2 + M^2)^2$ . Also, the expressions for the pion decay constant and the weak charge of the constituent quark are modified appropriately by the presence of vector-isovector interactions [7,10,11],

$$F_\pi^2 = g_A^Q J(0), \quad g_A^Q = \frac{1}{1 + 4G_\rho J(0)}. \quad (9)$$

The anomalous magnetic moments of the constituent quark are equal to [7,10,11]

$$\kappa_{I=0,1}^Q = \frac{\mp 8G_T J(0)}{1 \pm 4G_T J(0)}, \quad \text{or:} \quad 1 + \frac{\kappa_{I=0,1}^Q}{2} = \frac{1}{1 \pm 4G_T J(0)}. \quad (10)$$

With the help of relations (9,10) we rewrite Eq. (8) as Eq. (3), completing the proof.

In the remaining part of this paper we argue that the sign and size of  $\Pi_{I=0}$  and  $\Pi_{I=1}$ , as given by Eq. (4), agree with expectations based on dispersion relations and simple models of spectra in appropriate channels. Let us begin with  $\Pi_{I=1}$ , for which the relevant intermediate states have quantum numbers of  $\rho$  ( $I^G(J^{PC}) = 1^+(1^{--})$ ) and  $b_1(1235)$  ( $I^G(J^{PC}) = 1^+(1^{+-})$ ). With the help of the spectral decomposition we can write the imaginary part of the correlator  $\Pi_{I=1}(q) \equiv i\langle 0 | \int d^4x e^{iq \cdot x} T \{ \bar{\psi}(x) \sigma^{\mu\nu} \tau^3 \psi(x), \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) \} | 0 \rangle$  as

$$\begin{aligned} \text{Im}\Pi_{I=1}(q) = & \pi \sum_{\rho} \delta(q^2 - m_{\rho}^2) \sum_{\lambda} \langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | \rho; q, \lambda \rangle \langle \rho; q, \lambda | \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) | 0 \rangle + \\ & + \pi \sum_b \delta(q^2 - m_b^2) \sum_{\lambda} \langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | b; q, \lambda \rangle \langle b; q, \lambda | \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) | 0 \rangle, \end{aligned} \quad (11)$$

where  $\sum_{\rho,b}$  denote the sum/integral over all physical states with quantum numbers of neutral  $\rho$  and  $b_1$ , quantities  $m_{\rho}$  and  $m_b$  denote the invariant mass of these states, and  $\lambda$  are their polarizations. The Lorentz structure of the matrix elements consistent with quantum numbers of the state is<sup>1</sup>

$$\langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | \rho; q, \lambda \rangle = i\sqrt{2} f_{\rho}(q^2) (\epsilon_{\lambda}^{\mu} q^{\nu} - \epsilon_{\lambda}^{\nu} q^{\mu}), \quad (12)$$

$$\langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | b; q, \lambda \rangle = i\sqrt{2} f_b(q^2) \epsilon^{\mu\nu\alpha\beta} (\epsilon_{\lambda})_{\alpha} q_{\beta}, \quad (13)$$

where  $\epsilon_{\lambda}$  is the polarization vector. Using the identity  $\sum_{\lambda} \epsilon_{\lambda}^{\mu} (\epsilon_{\lambda}^{\nu})^* = -g^{\mu\nu} + q^{\mu} q^{\nu} / q^2$  we find that

$$\sum_{\lambda} \langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | \rho; q, \lambda \rangle \langle \rho; q, \lambda | \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) | 0 \rangle = -12q^2 |f_{\rho}(q^2)|^2, \quad (14)$$

$$\sum_{\lambda} \langle 0 | \bar{\psi}(0) \sigma^{\mu\nu} \tau^3 \psi(0) | b; q, \lambda \rangle \langle b; q, \lambda | \bar{\psi}(0) \sigma_{\mu\nu} \tau^3 \psi(0) | 0 \rangle = +12q^2 |f_b(q^2)|^2, \quad (15)$$

and, consequently, Eq. (11) can be written as

$$\text{Im}\Pi_{I=1}(q) = -12\pi \sum_{\rho} \delta(q^2 - m_{\rho}^2) m_{\rho}^2 |f_{\rho}(m_{\rho}^2)|^2 + 12\pi \sum_b \delta(q^2 - m_b^2) m_b^2 |f_b(m_b^2)|^2. \quad (16)$$

Note the negative sign of the  $\rho$  contribution, and the positive sign of the  $b_1$  contribution. The dispersion relation gives

$$\Pi_{I=1}(q) = \frac{12}{\pi} \int_0^{\infty} ds \frac{\sigma_b(s) - \sigma_{\rho}(s)}{s - q^2 - i\delta}, \quad (17)$$

where in general the spectral densities collect the contribution from all poles (and cuts) in the appropriate channel:  $\sigma(s) = \sum_a \delta(s - m_a^2) m_a^2 |f_a(m_a^2)|^2$ . Dispersion relation (17) requires no subtraction in the chiral limit. This follows from the fact that in QCD the asymptotic

<sup>1</sup> The factors of  $\sqrt{2}$  in Eqs. (12-13) make our expressions consistent with the definition  $\langle 0 | \bar{u}(0) \sigma^{\mu\nu} d(0) | \rho^+; q, \lambda \rangle = i f_{\rho}(q^2) (\epsilon_{\lambda}^{\mu} q^{\nu} - \epsilon_{\lambda}^{\nu} q^{\mu})$  of Ref. [12].

forms at large Euclidean momenta ( $q^2 \rightarrow -\infty$ ) of tensor correlators in the  $\rho$  and  $b_1$  channels are equal [13]. Indeed, the tensor correlator

$$\Pi_{I=1}^{\mu\nu,\alpha\beta}(q) \equiv i\langle 0 | \int d^4x e^{iq\cdot x} T \{ \bar{\psi}(x) \sigma^{\mu\nu} \tau^3 \psi(x), \bar{\psi}(0) \sigma^{\alpha\beta} \tau^3 \psi(0) \} | 0 \rangle \quad (18)$$

can be decomposed as follows in structures involving the contributions of  $\rho$  and  $b_1$  states:

$$\Pi_{I=1}^{\mu\nu,\alpha\beta}(q) = -4T_{(-)}^{\mu\nu,\alpha\beta} \Pi_{\rho}(q) + 4T_{(+)}^{\mu\nu,\alpha\beta} \Pi_b(q), \quad (19)$$

where the factors of 4 are conventional<sup>2</sup>. The tensor structures are defined as

$$T_{(-)}^{\mu\nu,\alpha\beta} = \frac{1}{2q^2} (q^\alpha q^\mu g^{\beta\nu} - q^\beta q^\mu g^{\alpha\nu} + q^\beta q^\nu g^{\alpha\mu} - q^\alpha q^\nu g^{\beta\mu}), \quad (20)$$

$$T_{(+)}^{\mu\nu,\alpha\beta} = -T_{(-)}^{\mu\nu,\alpha\beta} + \frac{1}{2} (g^{\alpha\mu} g^{\beta\nu} - g^{\alpha\nu} g^{\beta\mu}), \quad (21)$$

and satisfy  $T_{(-)}^{\mu\nu}{}_{\mu\nu} = T_{(+)}^{\mu\nu}{}_{\mu\nu} = 3$ . In the deep Euclidean region QCD gives [12,13]<sup>3</sup>.

$$\Pi_{\rho,b}^{\text{QCD}}(q) = -\frac{q^2}{8\pi^2} \left\{ \left( 1 \pm \frac{6m^2}{q^2} \right) \log \frac{-q^2}{\mu^2} + \mathcal{O}(\alpha_s) \right\} + \mathcal{O}\left(\frac{1}{q^2}\right), \quad (22)$$

where  $m$  is the current quark mass, which is neglected. The form (22) implies that at large  $s$  we have asymptotically  $\sigma_{\rho}(s) \sim \sigma_b(s) \sim s/(8\pi)$ , and the difference  $\sigma_{\rho}(s) - \sigma_b(s) \sim 1/s$ . This means that the unsubtracted dispersion relation (17) holds, and we obtain immediately

$$\Pi_{I=1} = \frac{12}{\pi} \int_0^{\infty} ds \frac{\sigma_b(s) - \sigma_{\rho}(s)}{s}. \quad (23)$$

Repeating the above analysis for the isovector channel, where the relevant physical states have the quantum numbers of  $\omega$  ( $I^G(J^{PC}) = 0^-(1^{--})$ ) and  $h_1(1170)$  ( $I^G(J^{PC}) = 0^-(1^{+-})$ ), with conventionally the same factors as in Eqs. (12,13,19,22) gives

$$\Pi_{I=0} = \frac{12}{\pi} \int_0^{\infty} ds \frac{\sigma_h(s) - \sigma_{\omega}(s)}{s}. \quad (24)$$

Note that in Eqs. (23-24) there is cancellation between spectral densities corresponding to opposite parity states.

<sup>2</sup> We note that the constituent quark model gives  $\Pi_{\rho}^{\text{CQM}}(0) + \Pi_b^{\text{CQM}}(0) = 0$ , in agreement with the general requirement due to the absence of zero-mass states in the tensor channel [12,14].

<sup>3</sup> Ref. [13] has an error in the overall factor in Eq. (A3). Their tensors in Eqs. (2.3) relate to ours as follows:  $P^{(1)} = \frac{1}{8}T_{(-)}$ ,  $P^{(2)} = -\frac{1}{8}T_{(+)}$ , and Eq. (A3) should read:  $\Pi(q) = 24P^{(1)}\Pi^- + 24P^{(2)}\Pi^+$ , in order to agree with our Eq. (19)

Now, following Refs. [1,2,12], we can estimate the contributions to Eq. (23-24). We expect that the following pole + continuum parameterization of the spectral densities should give at least a correct order-of-magnitude:

$$\sigma_a(s) = \pi m_a^2 f_a^2 \delta(s - m_a^2) + \frac{s}{8\pi} \theta(s_a - s), \quad (25)$$

where  $a$  labels the channel. Substituting Eq. (25) into Eqs. (23-24) gives

$$\Pi_{I=0} = -12(f_\omega^2 - f_h^2 + \frac{1}{8\pi^2}(s_h - s_\omega)), \quad (26)$$

$$\Pi_{I=1} = -12(f_\rho^2 - f_b^2 + \frac{1}{8\pi^2}(s_b - s_\rho)) \quad (27)$$

Let us begin with the last term in Eq. (27). The  $s_b$  parameter has been obtained from QCD sum rules in Refs. [2,12,13] giving  $s_b \sim (2.3 \div 3)\text{GeV}^2$ . Using derivative coupling Ref. [15] reports  $s_b \sim 2.5\text{GeV}^2$ . The QCD sum rule fit for the  $\rho$  meson spectrum in the tensor channel gives [12,14,16]  $s_\rho \sim (1.3 \div 1.7)\text{GeV}^2$ . Thus we have  $s_b - s_\rho \sim 0.6 \div 1.7\text{GeV}^2$ , and the continuum contribution to  $\Pi_{I=1}$  is of the order of  $-0.1\text{GeV}^2$  to  $-0.25\text{GeV}^2$ . The  $b_1$  pole contribution is, according to Ref. [2,12],  $12f_b^2 \sim 0.4\text{GeV}^2$ , and the  $\rho$  pole contribution is, according to Ref. [12],  $12f_\rho^2 \sim 0.3\text{GeV}^2$ , according to [16],  $12f_\rho^2 \sim 0.35\text{GeV}^2$ , and according to [14],  $12f_\rho^2 \sim 0.5\text{GeV}^2$ . Taking a “global” average yields  $\Pi_{I=1} \sim (-0.1 \div -0.25)\text{GeV}^2$ . Note that the order of magnitude of all contributions to Eq. (27) is the same as in Eq. (4), indicating our constituent-quark-model value has the expected size. The sign of Eq. (4) shows that the  $\rho$  channel “wins” over the  $b_1$  channel in Eq. (27). In fact, as already pointed out in Ref. [1], cancelations in Eqs. (26-27) make any precise estimates of tensor susceptibilities based on models of spectra difficult if not impossible. Note, however, as pointed out by Belyaev and Oganessian [2], that Ref. [1] does not include the continuum contribution in Eq. (27), which causes their value to be much smaller, and of opposite sign compared to Ref. [2] and to our estimate (4). The value for  $\Pi(0)$  obtained by Kisslinger [3] from the QCD sum rules for three-point functions are of opposite sign and large in magnitude. In view of these controversies, direct model estimates, such as that of Eq. (4) gain significance.

We point out that the mentioned QCD sum rule calculations do not distinguish between the isoscalar and isovector channels. Surprisingly, it is supported by our result (3), since, due to the smallness of the quark anomalous magnetic moments,  $\Pi_{I=0}^{\text{CQM}}$  is practically equal to  $\Pi_{I=1}^{\text{CQM}}$ . A corollary of this observation is the approximate relation  $\int_0^\infty ds(\sigma_h - \sigma_\omega)/s \simeq \int_0^\infty ds(\sigma_b - \sigma_\rho)/s$ .

We conclude with several comments:

- (1) The obtained values for tensor susceptibilities of the vacuum have direct relevance for the calculations of tensor charges of the nucleon [1,4,5].
- (2) The simple result (3) and the following numerical estimate (4) are leading- $N_c$  results. Formally suppressed effects, such as meson (especially pion) exchange, may be important, as is the case *e.g.* in the calculation of the quark condensate in the Nambu–Jona-Lasinio model [17].

- (3) Quark-model relations, such as our Eq. (3), bear some similarity to the relations obtained by Leutwyler [18] and Mallik [19] on the basis of the SU(6) symmetry of meson wave functions. In fact, more relations of the type of Eq. (3) can be obtained in our approach [20].
- (4) Our relations of susceptibilities to the  $L = 1$  meson spectra, Eqs.(23-24), are obtained via dispersion relations. This is in the spirit of QCD sum rules, where one compares the deep Euclidean region to the physical region via dispersion relations. In our method we compare the shallow Euclidean region to the physical region, following the idea described in Ref. [21]. That way we can infer useful information concerning meson excitations without leaving the validity range of the constituent chiral quark model.

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