7. Estimation of Delayed Neutron Emission Probability by Using the Gross Theory of Nuclear β–Decay

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The delayed neutron emission probabilities (P_n-values) of fission products are necessary in the study of reactor physics; e.g. in the calculation of total delayed neutron yields and in the summation calculation of decay heat. In this report, the P_n-values estimated by the gross theory for some fission products are compared with experiment, and it is found that, on the average, the semi-gross theory somewhat underestimates the experimental P_n-values. A modification of the β-decay strength function is briefly discussed to get more reasonable P_n-values.

1. Introduction

Either in the 2nd generation of the gross theory [1] (referred to as GT2 hereafter) or in the semi-gross theory [2] (referred to as SGT hereafter), the strength function of the β-decay $|M_\Omega(E)|^2$ is assumed to be given as

$$|M_\Omega(E)|^2 = \int_{\varepsilon_{\min}}^{\varepsilon_{\max}} D_\Omega(E,\varepsilon)W(E,\varepsilon)\frac{dn_1}{d\varepsilon}d\varepsilon, \qquad (1)$$

where Ω denotes the type of β-transition (Fermi, Gamow-Teller, or the 1st forbidden transition), ε the single-particle energy, and E the transition energy measured from the parent state. The function $D_\Omega(E,\varepsilon)$ is the one-particle strength function, $W(E,\varepsilon)$ a weight function to take into account the Pauli exclusion principle, and $\frac{dn_1}{d\varepsilon}$ the one-particle energy distribution of the decaying nucleon.

The characteristics of GT2 in comparison with SGT are as follows.

(1) The function $D_\Omega(E,\varepsilon)$ is a superposition of two parts. For example, in the case of the Gamow-Teller transition, one is a function with a large peak corresponding to the giant resonance, and the other is a widely-spreading distribution with long tails.

(2) The Fermi gas model with an effective mass is used for the calculation of the function

$\frac{dn_1}{d\varepsilon}$, and pairing gaps are taken into account for the nucleons near the Fermi surface.

(3) The UV factor of the BCS model is taken into account by a somewhat crude method.

In the case of GT2, the shell effect is taken into account only through the Q-value which is an input data of the model.

On the contrary, the characteristics of SGT are as follows.

(1) The function $D_\Omega(E,\varepsilon)$ is assumed to be a superposition of several functions which reflect the effects of spin-flip and change of the oscillator quanta by the transitions.

(2) The energy distribution $\frac{dn_1}{d\varepsilon}$ is a non-uniform and discrete function taking into account the shell effects and pairing effects in the parent nucleus.

(3) The UV factor is calculated by using this energy distribution $\frac{dn_1}{d\varepsilon}$.

In the case of SGT, having the above characteristics, some shell effects are taken into account in the strength function around the ground state of the daughter nucleus.

The delayed neutron emission probability, P_n, is calculated with the use of the total β-strength function, S_β, which is the sum of the allowed-equivalent strengths of all transitions $|M_\Omega(E)|^2$:

$$P_n = \frac{\lambda_n}{\lambda}, \tag{2}$$

with

$$\lambda_n = \int_{-Q+S_n}^{0} S_\beta(E)\ f(-E)\ \frac{\Gamma_n}{\Gamma_n+\Gamma_\gamma} dE\ , \tag{3}$$

and

$$\lambda = \int_{-Q}^{0} S_\beta(E)\ f(-E)\ dE = \frac{\ln 2}{T_{1/2}}. \tag{4}$$

Here, Q is the β-decay Q-value, S_n the neutron separation energy of the daughter nucleus, f the integrated Fermi function, $T_{1/2}$ the β-decay half-life, and $\Gamma_n/(\Gamma_n+\Gamma_\gamma)$ the competition factor with γ-radiation. In this report, we put this factor to unity for simplicity.

2. Calculated P_n-values and modified strength function

The calculated P_n- and λ_n - values are compared with experimental data [3] in fig. 1. In order to get the 'experimental' λ_n - values, we adopt the experimental $T_{1/2}$ [4] and P_n-values [3] in the equations (2) and (4). The calculated λ_n - values, on the average, somewhat overestimate the experimental values. This tendency is not bad if we notice that real competition factor should be less than unity. In GT2, reasonable P_n- and λ_n - values are

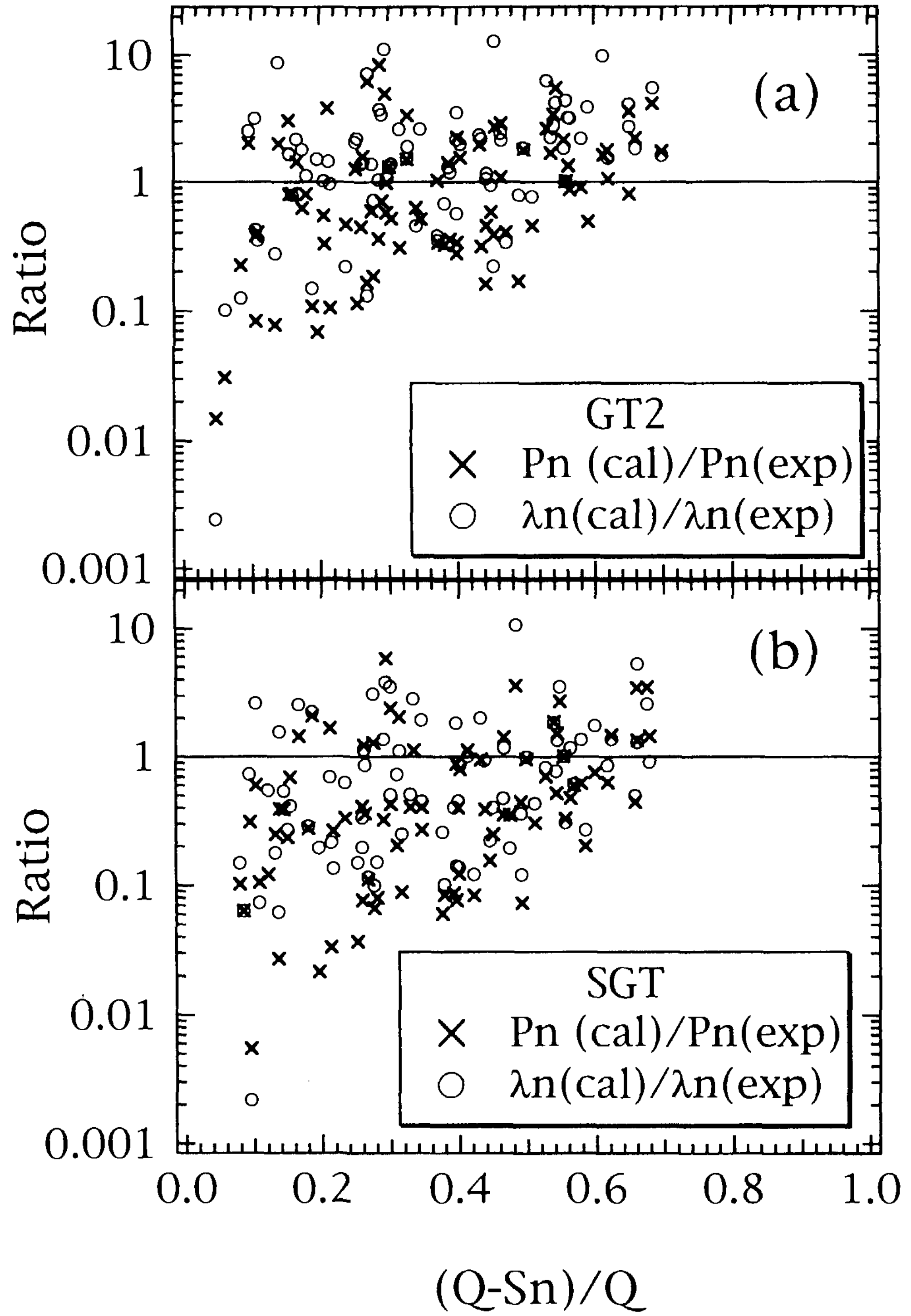

Fig. 1 Comparison between the calculated P_n-values (λ_n - values) and experimental ones. Figure (a) is for GT2 and (b) is for SGT.

obtained on the average except for the nuclei with small $(Q\text{-}Sn)/Q$ values. On the other hand, in the case of SGT, many P_n-values and λ_n - values are lower than the experimental ones. This means that the strength functions estimated by using SGT are smaller than the experimental strengths in the delayed neutron window.

In order to get more reasonable P_n - and λ_n - values in SGT, we introduce the spreading function $G(E-E_0;\ E_0+Q)$ [see Ref. [5] for more detail]. The β-strength function of SGT is spread in each small interval around the energy E_0 with the use of this function $G(E-E_0;\ E_0+Q)$. The modified strength function is obtained as

$$S_\beta(E)=\int_{-Q}^{\infty} S_\beta^{\rm SGT}(E_0)\ G(E-E_0;\ E_0+Q)\ dE_0, \qquad (5)$$

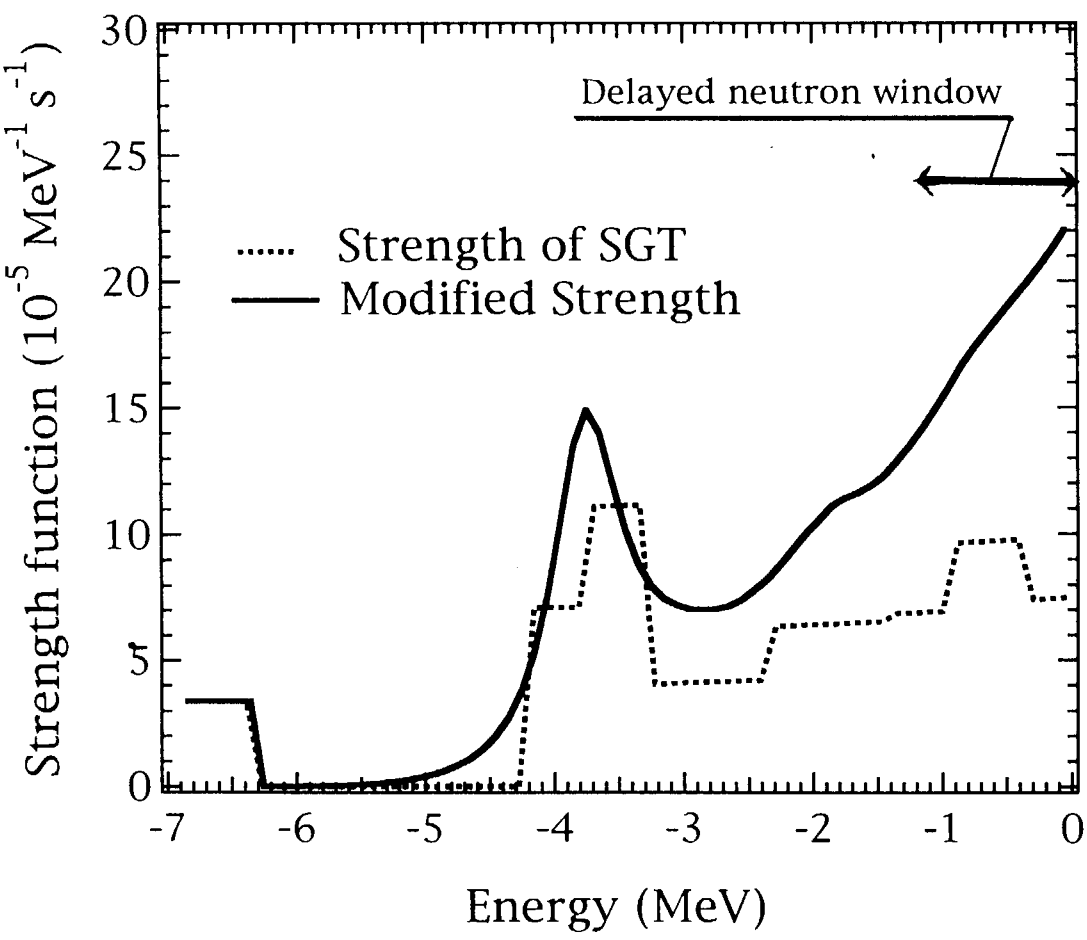

Fig. 2 The Gamow-Teller strength function of ^{87}Br. The delayed neutron window is between −1.3 (MeV) and 0 (MeV). δ-functions in the strengths are drawn with a half-width of 0.5 MeV.

with

$$G(E-E_0;\ E_0+Q)=\frac{C\ \exp(-[E-E_0]^2/2a_G^2[E_0+Q]^2)}{\sqrt{2\pi}a_G(E_0+Q)}\times\frac{a_L(E_0+Q)}{2\pi([E-E_0]^2+a_L^2[E_0+Q]^2/4)} \tag{6}$$

where S_β^{SGT} is the strength function calculated by SGT. This spreading function G is the product of a Lorentzian function with a parameter a_L and a Gaussian function with a normalization constant C and a parameters a_G. The parameters a_L and a_G define the absolute values of the widths and are fixed to reproduce reasonable half-lives and P_n-values in the whole nuclidic region. After some examinations, we have taken a_G=0.3 and a_L=0.2. The example of the modified strength function shown in fig. 2 has been obtained in this way. It should be noted that the strength function at the ground state of the daughter nucleus does not change because the function G is a δ-function at $E=-Q$. As seen in fig. 2, the modified strength function increases very much in the delayed neutron window but does not change so much around the ground state of the daughter nucleus.

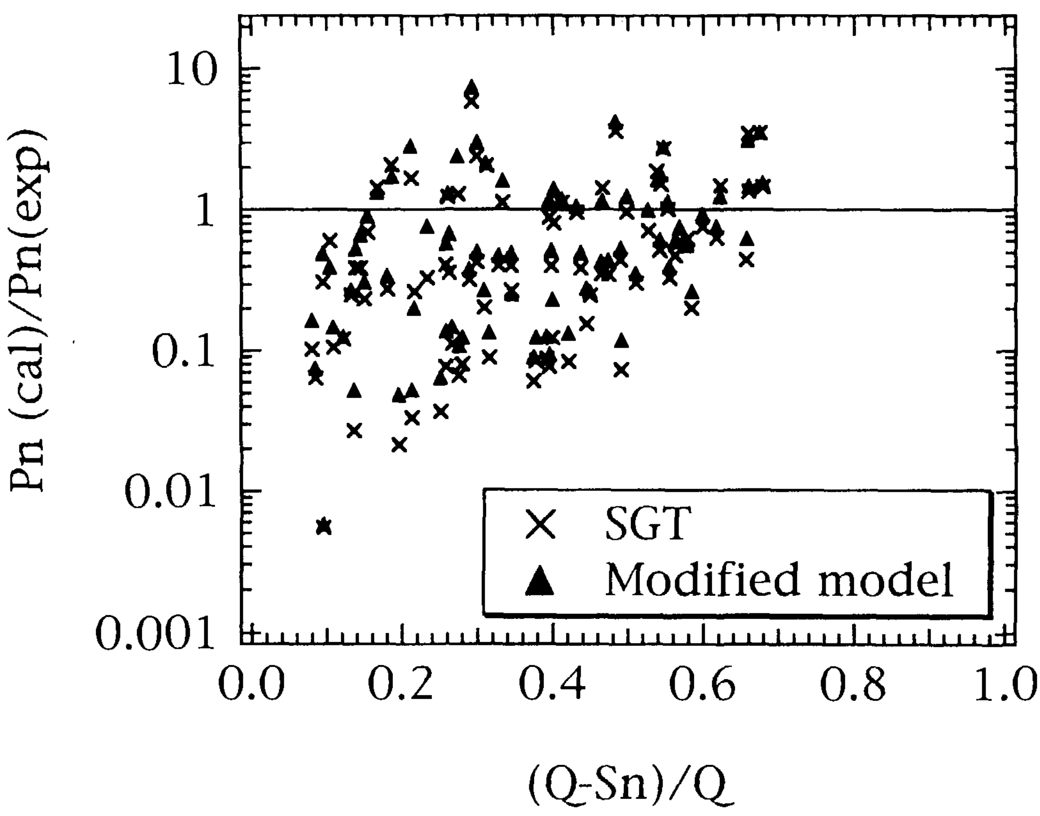

Fig. 3 The ratios between calculated and experimental P_n-values.

In fig. 3, we show how the calculated P_n-values change with the use of the modified strength functions. Almost all the new P_n-values have increased to the right direction, but the magnitudes of the modification are not sufficient. In the case of λ_n - values, the effect of the modification is more clearly seen [5].

3. Conclusion

The P_n-values calculated with the use of GT2 are, on the average, in fairly good agreement with the experiment, although, in the case of SGT, the P_n-values are somewhat underestimated. We have proposed a method to modify the strength function to get more reasonable P_n-values.

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