JP0050020

8. 核子の海にαクラスターの生まれるまで

Alpha-clustering in dilute nucleonic sea

Akihiro TOHSAKI

Department of Fine Materials Engineering, Shinshu University, Ueda, 386-8567

α-clusters are expected to come out here and there in nucleonic sea owing to energetic benefit as its density is diluted. We propose a precise treatment to elucidate α-clusterized process in nucleonic sea after the breakdown of the uniformness. In order to do this, an infinite number of nucleons are considered by taking account of both the Pauli exclusion principle and effective internucleon forces. This method is called a microscopic approach, which has been successful in an α-cluster structure in light nuclei. In particular, we shed light on overcoming difficulties in a static model within the microscopic framework. This improvement is verified by using the empirical value in Weizaecker's mass formula.

§1. Introduction

In general, in spite of the divergence of the binding energy of an aggregation of an infinite number of nucleons, each cluster comprising a few nucleons may have a definite value of binding energy. However, the Coulomb interaction between protons can give a logarithmic divergence to the binding energy even of an individual cluster. Nevertheless, a residual value which remains after subtracting the divergence is easily evaluated to be considered. It is also a serious problem that the binding energy inevitably includes the spurious energy coming from a static treatment of aggregation. This approaches a zero-point oscillation energy of isolated α-particle in the most dilute sea, which can be estimated only in a dynamical treatment. We show how to overcome such serious difficulties in quantum mechanically precise treatment. In this report, we represent a brief indication how we obtain the binding energy per a cluster, and we give the verification of this approach seeing effects which take place in nucleonic sea interior and surface.

§2. Method

We briefly recapitulate the static model which has been proposed in ref.(1). Imagine nucleonic sea in the generator coordinate space(GC space) in which treatment is equivalent to that of resonating group method [2]. As shown in Fig.1. we cut out a certain domain Γ in nucleonic sea where $4N$ nucleons exist. It is possible to contain the surface of sea by this cutting which allows us to estimate surface effects of semi-infinite aggregation of nucleons. The total wave function is written by

$$\Psi = \frac{1}{\sqrt{(4N)!}} \prod_{p=1}^{4} \mathcal{A}\{\phi_1^{(p)}, \cdots, \phi_N^{(p)}\}\chi_p, \qquad (2\cdot1)$$

where $\mathcal{A}$ is the antisymmetrizer for all the nucleons with the same spin-isospin state, χ_p. The spatial wave function, $\phi_i^{(p)}$, of the i-th nucleon is expressed relative to the

dimensionless spatial parameter $\mathbf{S}_i^{(p)}$ by

$$\phi_i^{(p)} = (\pi)^{-3/4} \exp\{-\frac{1}{2}(\mathbf{q} - \mathbf{S}_i^{(p)})^2\}, \tag{2·2}$$

where the parameter $\mathbf{S}_i^{(p)}$ is regarded as the GC of the i-th nucleon. The realistic variable, $\mathbf{q}$, is integrated over infinite space. The parameter $\mathbf{S}_i^{(p)}$ is directly related to the spatial generator coordinate, $\mathbf{R}_i^{(p)} = b\mathbf{S}$, where b is the size parameter of $(0s)$ h.o. wave function with $\hbar\omega = \hbar^2/(Mb^2)$ (M is a nucleon mass). When the labels, p, differ from each other, the GCs are also different due to the spatial distribution of nucleons although they have the same label, i. In Fig.1, the nucleons with labels, i_1, i_2, i_3 and i_4 are placed on the same position, then four nucleons make an α-cluster in nucleonic sea with random arrangement of nucleons as shown in Fig.1.

After inserting a microscopic Hamiltonian, $\mathcal{H}$, the energy overlapping is written by

$$< \Psi|\mathcal{H}|\Psi > = < \Psi|\Psi > [T_r(TB^{-1}) + T_r\{(V_d - V_e)(B^{-1} \otimes B^{-1})\}], \tag{2·3}$$

where the matrices are defined as

$$B = \left(< \phi_i^p | \phi_j^q > \right), \tag{2·4}$$

$$T = \left(< \phi_i^{(p)} | - \frac{\hbar^2}{2M} \nabla_j^2 | \phi_j^{(q)} > \right), \tag{2·5}$$

$$\frac{1}{2} V_d = \left(< \phi_i^{(p)} \phi_j^{(q)} | v_{kl} | \phi_k^{(r)} \phi_l^{(s)} > \right), \tag{2·6}$$

and

$$\frac{1}{2} V_e = \left(< \phi_i^{(p)} \phi_j^{(q)} | v_{kl} | \phi_l^{(r)} \phi_k^{(s)} > \right), \tag{2·7}$$

where V_d and V_e have a tensor representation, and the symbol $\otimes$ means the tensor product. Besides, the matrix element with three-body operator can be estimated by an extended formalism from eq.(2.3). The respective binding energies of nucleons are straightforwardly regarded as corresponding elements of traces in eq.(2.3). The domain Γ' is given by being slightly expanded from Γ. If the binding energy of a nucleon in the center of Γ' is almost unchanged from that in Γ, we can regard the convergence of the value as its binding energy.

Nonetheless, we have two difficulties : that is,

1. how to get the convergence with the Coulomb force which inevitably leads to logarithmic divergence, and
2. how to remove the spurious energy which is included in such a static approach.

The first difficulty is eliminated by subtracting the Coulomb direct energy without the Pauli exchange effects. This term and the background coming from negative charges are canceled out each other. The remaining value is just regarded as the Coulomb exchange term which rapidly converges into a definite value. We think that this value should be included in the volume term of the Weizaecker's mass formula

as suppressing the binding energy. The second difficulty is fundamental because it is impossible to treat dynamically an infinite number of nucleons. Therefore, we propose an approximate method to remove the spurious energy. We assume that the α-cluster in a cavity, which is surrounded by the remaining nucleons, moves freely. Then we solve the equation of motion for α-cluster in the cavity in Γ using the Hill-Wheeler variational method as

$$\int < \Psi(\mathbf{S})|\mathcal{H} - \lambda|\Psi(\mathbf{S}') > f(\mathbf{S}')d\mathbf{S}' = 0. \qquad (2\cdot8)$$

The deviation of $\sum_i E_i$ and λ_{min} may correspond to the spurious energy for the centered α-cluster, thus the spurious free energy is written in $E_i^{(ns)}$ in the following section. The weight function, f(**S**), to be solved expresses characteristics of motion through the transform into a wave function. This method can exactly remove the spurious energy in the rarefied limit of nucleon gas. One of the developments of this treatment is to enable us to investigate intercluster potential in a medium within a microscopic framework.

§3. Results and conclusion

As a preliminary step, we employ Brink-Bocker No.1 force[3] as an effective internucleon force which can reproduce the saturation property of nuclear matter. Unfortunately, the aggregation of nucleons always makes α-clusters over wide range of nucleon density because this force is α-clustering favorable. Therefore, we cannot find out the uniform density region in the nucleonic sea. The most likely α-clusterized nucleonic sea is composed of fcc (or ccp) lattice configuration of an infinite number of α-clusters. The difference between fcc and ccp appears only in the structure of surface. In Fig.2, we show the energy quantities of α-clusters which belong to three positions such as the sea interior, the surface of fcc and the surface of ccp. Here, we use $b = 1.4fm$ which reproduces the minimum binding energy of an isolating α-particle. We summarize the traits in Fig.2 as follows

1. The Coulomb exchange energy is about $-0.22MeV$ per nucleon which is attractive against the Coulomb repulsion. This value seems to be independent of the positions of α-clusters.
2. The spurious energies are different from each other for the positions of α-cluster, and $-0.09MeV$ per nucleon for sea interior and $-0.42MeV$ per nucleon for sea surface. But the difference for surface between fcc and ccp is almost negligible.
3. The method of removing the spurious energy works well because of the energy α-cluster can be reproduced in dilute density.

Finally we list the bulk properties of nucleonic sea comparing the stable binding energy and empirical values of Weizaecher's mass formula. Here we obtain the surface term b_s by

$$b_s = (\sqrt{3}\pi)^{\frac{1}{3}} 4E_s, \qquad (3\cdot1)$$

where

$$E_s = \sum_{i=surface}^{\infty} (E_i - E_{interior}), \qquad (3\cdot2)$$

for the static model and

$$E_s = \sum_{i=surface}^{\infty} (E_i^{(ns)} - E_{interior}^{(ns)}), \tag{3·3}$$

for the dynamic model. The incompressibility is estimated by diagonalizing the Hessian matrix for the partial differentials of the second order with respect to the density of nucleonic sea and the size of α-cluster.

Table.I

	Static model	Dynamic model	Ordinary nuclear matter	Empirical value
$b_v(MeV)$	14.1	14.4	15.7	15.6
$b_s(MeV)$	36.6	22.8	-	17.2
$K(MeV)$	181	185	184	~ 300

The b_s for the dynamic model is remarkably improved from that for the static model.

We anticipate to clarify the α-clusterized process in nucleonic sea by the most realistic internucleon force. The details in the static model appear in Refs(4-6).

References

1) A.Tohsaki, Prog. Theor. Phys. Suppl. No.132(1998),213.
2) D.M.Brink,*Proceeding of the International school of physics, 'Enrico Fermi' course 36, Verenna,1966*,ed.C.Bloch (Academic Press, New York and London,1966),p247.
3) D.M.Brink and E.Boeker, Nucl. Phys. **A91** (1967),1.
4) A.Tohsaki, Prog. Theor. Phys. **88**(1992),1119.
5) A.Tohsaki, Prog. Theor. Phys. **90**(1993),871.
6) A.Tohsaki, Phys. Rev. Lett. **76**(1996),3518.

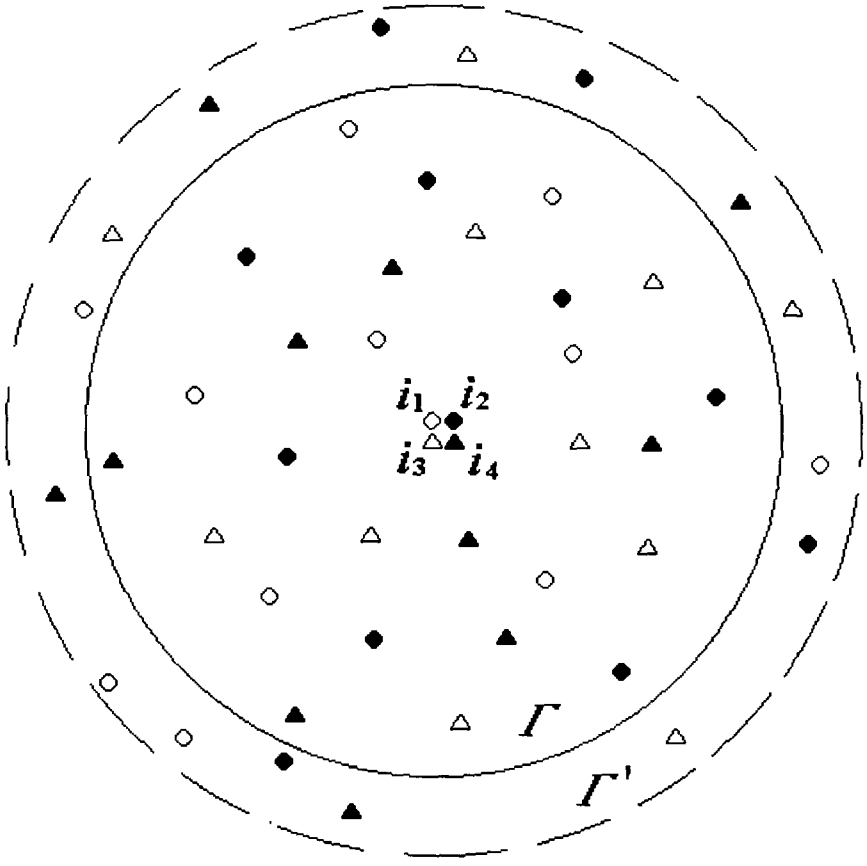

Fig. 1

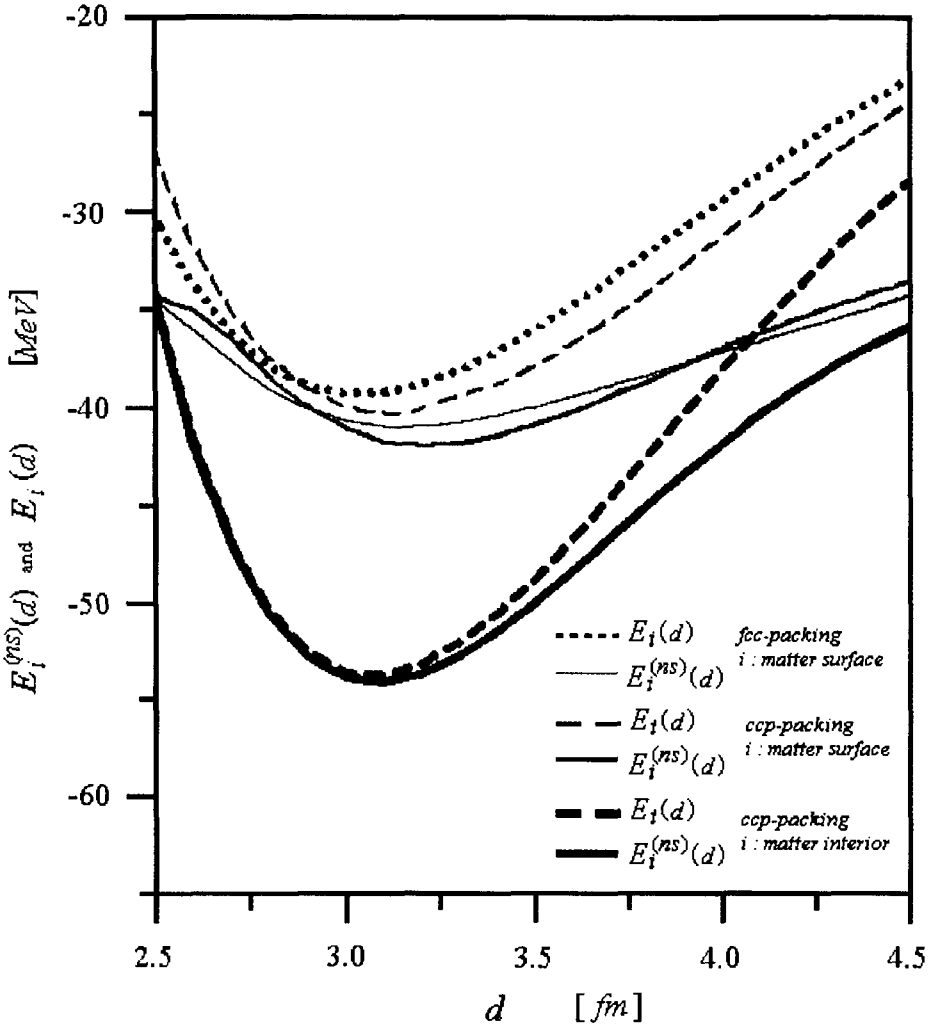
-20
-30
-40
-50
-60
2.5
3.0
3.5
4.0
4.5
d [fm]
$E_i^{(ns)}(d)$ and $E_i(d)$ [MeV]
$E_i(d)$ fcc-packing
$E_i^{(ns)}(d)$ i : matter surface
$E_i(d)$ ccp-packing
$E_i^{(ns)}(d)$ i : matter surface
$E_i(d)$ ccp-packing
$E_i^{(ns)}(d)$ i : matter interior

Fig. 2