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Turbulence of High-Beta Plasma

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Abstract.

Principals of numerical modelling of turbulence in high-beta plasma (β>0.1) are discussed. Creation of transport model for axial symmetric nonuniform confining magnetic field is considered. Numerical model of plasma turbulence in FRC is presented. The physical and mathematical models are formulated from nonuniform axial symmetric high-beta plasma. It is shown that influence of waves arise under this plasma conditions lead to chaotic motion of charged particles across magnetic field.

1. Introduction.

In this work we consider principals of numerical modelling of turbulence in high-beta plasma (β>0.1). This problem arises for FRC, Tandem Mirror and other magnetic systems confining high-beta plasma.

High-beta plasma has a number of features in comparison with low-beta plasma. It is well-known for high-beta plasma it is necessary to take into account nonuniformity of plasma density, plasma temperature and magnetic field [1]. Investigations of microinstabilities inside a such plasma show the possibilities of arising of following modes [1]: Tserkovnikov and Alfven waves, low-hybrid, low-frequency and ion-cyclotron waves. But increasing plasma beta leads to suppression of some modes of oscillations [1]. Consequently careful analysis of possible microinstabilities for every concrete conditions is required. Here we do not consider this problem but discuss some features of anomalous transport. In this work the method of numerical analysis of the transport processes inside nonuniform high-beta plasma (β>0.1) induced by microinstabilities is discussed. The simple examples are considered.

The important example of considered nonuniform plasma conditions is FRC plasma. In Fig. 1 the radial distributions of the plasma pressure and plasma beta for Hill's vortex (in plane z=0) are plotted. One can see strong nonuniformity and big values of beta. Naturally in real experimental situations these distributions differ on plotted in Fig. 1. But indicated features are true in any case.

We consider the axial cylindrical radial nonuniform plasma with magnetic field $B(r)$ directed along axis of symmetry (z-axis). The electrostatic and electromagnetic waves propagate inside plasma in azimuthal and longitudinal directions.

In order to estimate numerical values of diffusion coefficient based on [2,3] method developed in [5,4] was used. We derived a set of non-linear equations describing motion of particles under influence of electric field of wave inside axial symmetric trap. Therefore we can estimate the confinement time for a plasma inside magnetic trap. Preliminary calculations show good agreement between theoretical and experimental data.

Detail investigations also indicate another scenario of radial transport. If oscillations are long-wave and the electric field of the wave is parallel to the wave vector, then it is possible convective losses of the particles across the magnetic field. In this case the particles accomplish the electric drift in the radial direction under the influence of the magnetic field of the trap and the electric field of the wave. This motion is not stochastic, but in this case lines in phase plane directed to plasma boundary are exists.

Therefore numerical calculations allow to investigate the important features of radial transport inside axial symmetric magnetic trap and separate convective and turbulent transport.

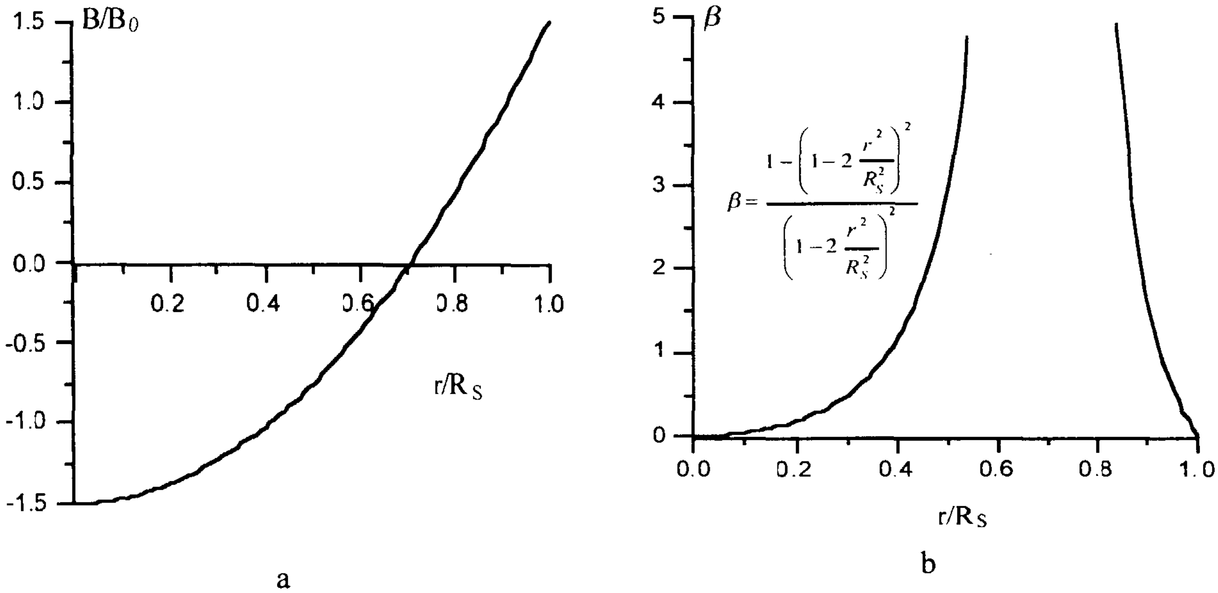

Fig. 1. Magnetic field (a) and beta (b) distributions for Hill's vortex FRC.

2. Microinstabilities and anomalous transport in nonuniform plasma.

The anomalous transport is a result of simultaneous joint influence on the particles of the static magnetic field and the propagating waves fields. This results is complex particles motion across magnetic field. Solution of this problem allows to estimate the anomalous transport properties of the plasma.

Analysis shows it is non-linear problem. The main parameters determining results of numerical solution are as follows:

- drift frequency of the particles in the static nonuniform magnetic field;
- speed and direction of propagation of waves;
- frequency, magnitude and polarisation of electromagnetic waves.

Investigations [1,6] have shown the most probable oscillations inside the nonuniform high-beta plasma are:

- low frequency oscillations ($\omega << \Omega_i$, where ω is the wave frequency, Ω_i is the ion-cyclotron frequency);

- ion-cyclotron oscillations ($\omega\sim\Omega_i$);
- low-hybrid instabilities;
- Tserkovnikov waves;
- Alfven instabilities;
- oscillations, induced by transversal current.

The features of waves propagation depends on different parameters, such as magnetic field B and ∇B, plasma temperature T and ∇T, plasma density n and ∇n, $k_\perp\rho_e$ and $k_\perp\rho_i$ ($k_\perp$ is perpendicular to magnetic field component of wave vector, ρ_e and ρ_i are electron and ion Larmor radiuses), etc. Analysis of microinstabilities allows to define characteristics of oscillation modes for different regions of nonuniform plasma.

3. Non-linear analysis of charged particles dynamics under influence of wave.

We consider cylindrical configuration, where magnetic field $B(r)$ directed along axis z and depends on radius r. Gradient dB/dr has radial direction. Electric field of the wave $E(r,\theta,t)$ in our model is azimuthal and depends on co-ordinates as

$$E(r,\theta,t)=\frac{\phi_0}{r}\sin(\omega_0 t-k_\theta\theta), \qquad (1)$$

where ϕ_0 is corresponding potential amplitude ω_0 is the wave frequency, k_θ is azimuthal wave number. We consider wave mode with k_θ=1. Value of wave potential amplitude usually can be estimated as $e\phi_0\sim kT$.

To obtain particles trajectories in (r,θ) co-ordinates we use dynamics equations

$$\frac{dv_r}{dt}=-\frac{qB(r)}{m}v_\theta, \qquad (2a)$$

$$\frac{dv_\theta}{dt}=\frac{qB(r)}{m}v_r+\frac{q}{m}E(r,\theta,t), \qquad (2b)$$

where m and $q=Ze$ are particle mass and charge correspondingly, $v_r=dr/dt$ and $v_\theta=rd\theta/dt$.

To obtain model of stochastic diffusion we consider influence of low frequency electrostatic wave on particle motion. In this case ω_0 more less then frequency of synchrotron rotation of the particle. Consequently guiding center approach to particle motion can be used. Equations for guiding center motion in cylindrical geometry in system, connected with wave and rotating with frequency ω_0, are

$$\frac{d\alpha}{dt}=\omega_{dr}(r)-\omega_0, \qquad (3a)$$

$$\frac{dr}{dt}=\frac{\phi_0}{B(r)r}\sin\alpha, \qquad (3b)$$

where α is angular co-ordinate,

$$\omega_{dr}(r) = \frac{\varepsilon_{\perp}}{qrB^2}\frac{dB}{dr} \tag{4}$$

is gradient drift frequency, $\varepsilon_{\perp}=m(v_r^2+v_\theta^2)/2$. In order to stochasticity in particle motion can appear, drift frequency and wave frequency mast satisfy resonance condition

$$\omega_{dr} = n\omega_0,\ n = 1,2,3,... \tag{5}$$

Radial locations of resonance drift surfaces r_n (n=1,2,3,...) are defined by (4) and (5).

We used new variable

$$X = \frac{q}{\varepsilon_{\perp}}\int_{r}^{r_l} B(r)rdr, \tag{6}$$

where r_l is radius of the nearest to separatrix resonance surface in close lines region (l is number of the surface). This variable transforms system (3) to non-linear pendulum like [2,3] equations

$$\frac{d\alpha}{dt} = AX, \tag{7a}$$

$$\frac{dX}{dt} = -C\sin\alpha, \tag{7b}$$

where

$$C = \frac{q\phi_0}{\varepsilon_{\perp}}\omega_0, \tag{8}$$

$$A = \omega_0 \frac{\left.\frac{d\ln|B|}{dX}\right|_{X(r_l)} - \left.\frac{d\ln|B|}{dX}\right|_{X(r)}}{X} \approx \frac{\omega_0}{4}. \tag{9}$$

After these transformations it is easy to obtain phase planes of system (7) and consequently system (3) by standard mapping [7]. For variables α and $I=AT_0X$, where $T_0=2\pi/\Delta\omega$ is the iteration time, $\Delta\omega$ is interval between resonance frequencies standard mapping is

$$\bar{I} = I + K_0 \sin\alpha, \tag{10a}$$

$$\bar{\alpha} = \alpha + \bar{I}, \qquad \mathrm{mod}(2\pi), \tag{10b}$$

where $\bar{\alpha}$ and $\bar{I}$ are values of α and I at new iteration,

$$K_0 = ACT_0^2 = 4\pi^2 \frac{AC}{(\Delta\omega)^2} \tag{11}$$

is Chirikov's coefficient. In case $K_0>1$ significant stochasticity take place. In considering case $\Delta\omega=\omega_0$ and

$$K_0 = \pi^2 \frac{q\phi_0}{\varepsilon_\perp}. \tag{12}$$

In case of wave potential amplitude estimated as $q\phi_0 \sim kT$ Chirikov's criterion value is

$$K_0 = \pi^2 \frac{kT}{\varepsilon_\perp}. \tag{13}$$

For majority of plasma particles $\varepsilon_\perp \sim kT$ and, as Eq. (13) shows, $K_0 \sim 10$. Consequently their motion is significantly stochastic in case of action of considered low frequency electrostatic wave.

Presented above analysis was carried out for general form of magnetic field distribution. Examples of phase plans in (r,α) co-ordinates for different value K_0 are plotted in Fig. 2 for Hill's vortex FRC.

In Fig. 2 change of the nature of the particles motion for increasing magnitude of wave is shown. The important feature of these drawings is the presence of resonance surfaces between magnetic drift frequency of the particles and wave rotation frequency around the axis of the magnetic system. In Fig. 2a shift of the particles in radial direction is restricted by some distance in the vicinity of a resonance surface. This distance is less than distance between neighbouring resonance surfaces. In Fig. 2b the areas of excited motion are overlapped. In this case motion of particles becomes stochastic in whole considered space field. Corresponding particles can move freely in this field and leave the plasma. These conditions corresponds to turbulent transport inside plasma. Example of particle trajectory is plotted in Fig. 3.

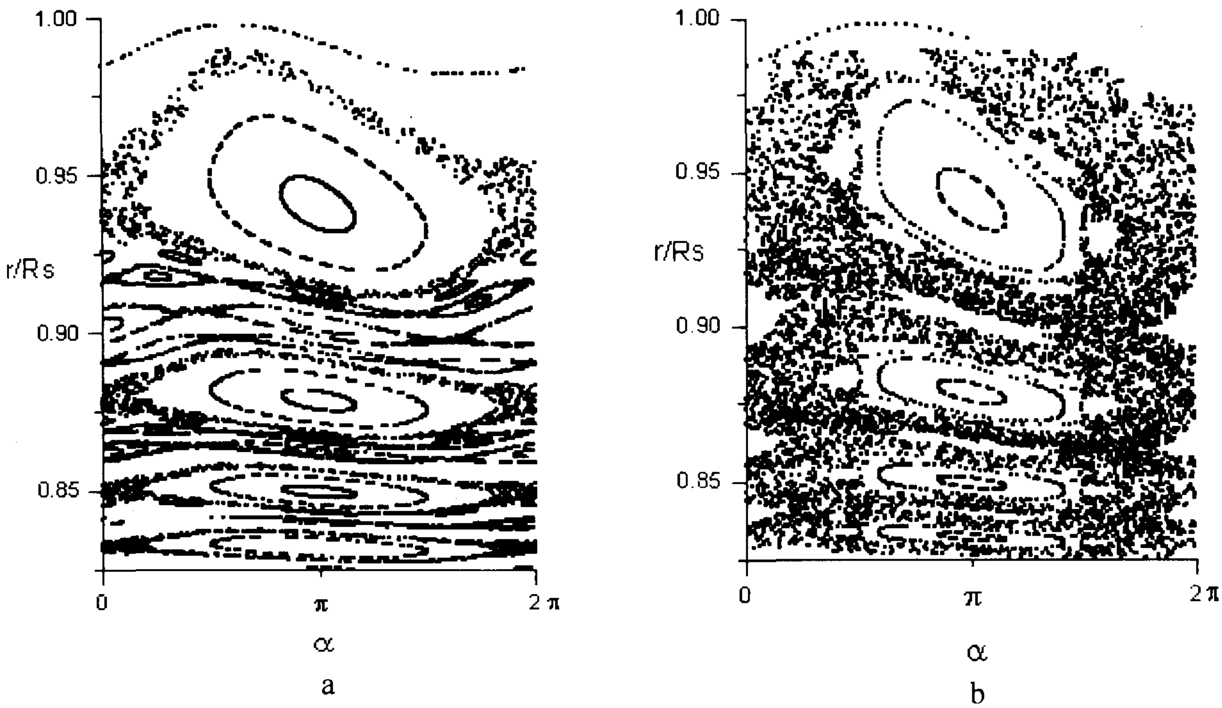

Fig. 2. Phase planes for particles in FRC plasma with magnetic field B_0=1 T, separatrix radius R_s=0.5 m, wave frequency $\omega_0=4\times10^4$ rad/s for K_0=1 (a) and K_0=2 (b).

4. Stochastic Diffusion Coefficient.

For K_0>>1 motion in phase plane can be considered as diffusion process. Corresponding stochastic radial diffusion coefficient is [7]

$$D_r = \frac{1}{\pi}\Delta\omega(\Delta r)^2_m, \tag{14}$$

where $(\Delta r)_m$ corresponds to coefficient $\phi_0/B(r)r$ in Eq. (3b).
Similarly [4,5] for conditions $q\phi_0 \sim kT$, l=1 we obtain averaged value of diffusion coefficient

$$D_r \approx \frac{1}{5}\frac{kT}{qB}, \tag{15}$$

where T is the overaged value of plasma temperature, B is characteristic value of magnetic field (for example, value at the separatrix).

Comparison corresponding stochastic diffusion time

$$\tau_{SD} = \frac{R_s^2}{D_r} = \frac{5qBR_s^2}{kT} \tag{16}$$

with experimental data [8] for particles confinement times τ_P are presented in Fig. 4. Obtained τ_{SD} depends on B, T and R_s similarly to presented in [6] confinement time.

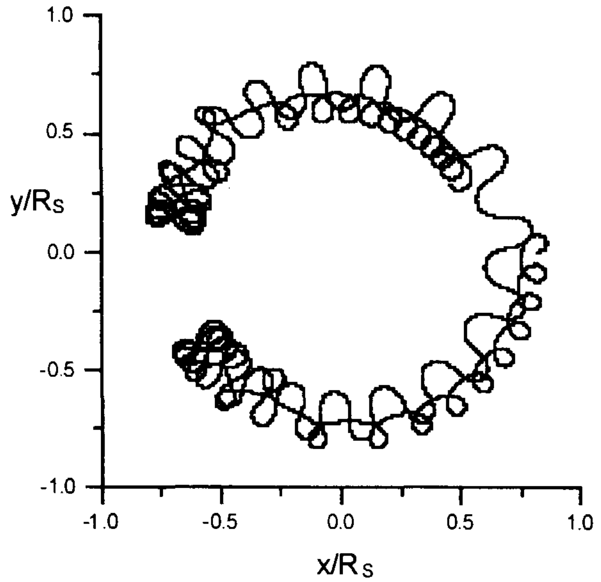

Fig. 3. Fragment of proton trajectory in Hill's vortex FRC (B_0=1 T, $\varepsilon_\perp$=10 keV).

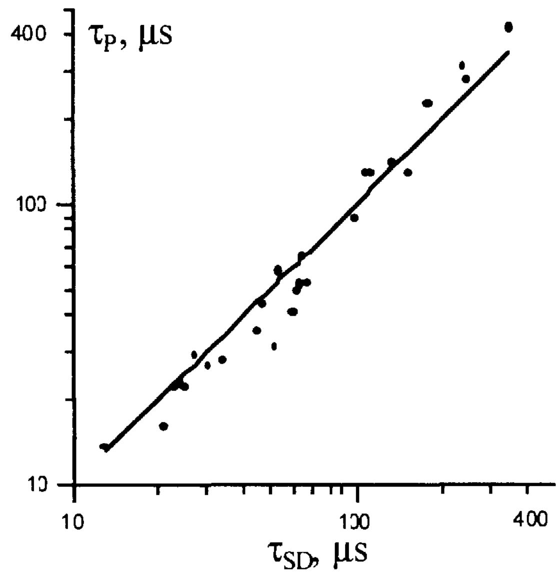

Fig. 4. Comparison between experimental data and theoretical results.

5. Conclusions.

As carried out above analysis shown, in nonuniform high-beta plasma non-linear interaction between particles and waves can induce strong turbulence and influence on particles motion and transport. Therefore problem of calculations of transport processes must include following stages.

1. Analysis of possible modes of microinstabilities for nonuniform plasma (β>0.1), including nonuniformity both magnetic field and plasma density and temperature.
2. Numerical calculations of stochastic motion of particles for axial symmetric magnetic field taking into account azimuthal electrostatic and electromagnetic waves, including non-linear complex interactions between waves and particles. This analysis allows to separate "clean" turbulent transport and convective transport. These tasks demand non-linear consideration, taking into account resonance processes including overlap of the resonance areas.

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