Calculating hadronic properties in strong QCD

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This talk gives a brief review of the progress that has been made in calculating the properties of hadrons in strong QCD. In keeping with this meeting I will concentrate on those properties that can be studied with electromagnetic probes. Though perturbative QCD is highly successful, it only applies in a limited kinematic regime, where hard scatterings occur, and the quarks move in the interaction region as if they are free, pointlike objects. However, the bulk of strong interactions are governed by the long distance regime, where the strong interaction is strong. It is this regime of length scales of the order of a fermi, that determines the spectrum of light hadrons and their properties. The calculation of these properties requires an understanding of non-perturbative QCD, of confinement and chiral symmetry breaking.
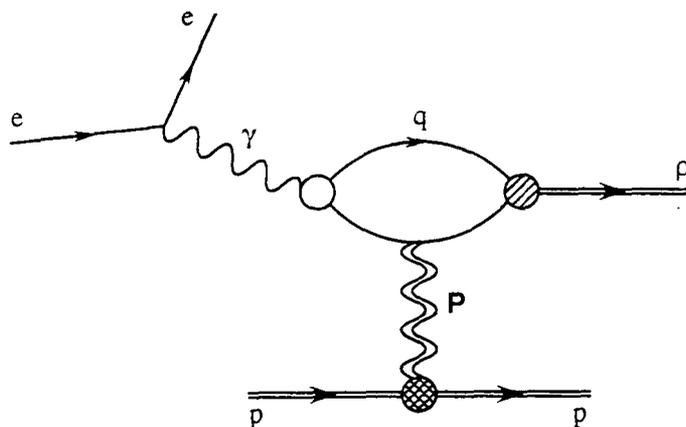

Figure 1: *Modelling ρ electroproduction.*

As an example, let us consider a process much discussed at this meeting, the electroproduction of vector mesons, e.g. $ep \to e\rho p$. Can we calculate this in strong QCD ? The diffractive mechanism, the Pomeron part **P**, is largely phenomenological [1], and, at present, not amenable to calculation. Nevertheless, we can imagine modelling the process, Fig. 1. in terms of factorisable components, some of which are under control [2]. First the $\gamma^* \to q\bar{q}$: the quarks are fully dressed with gluon clouds giving the whole interaction a spatial extent. Following this, the quark and antiquark propagate

and then after the Pomeron interaction form a bound state, Fig. 1. Importantly, each of these building blocks depends on the even more basic Green's function, the gluon propagator, but how are they to be calculated ?

Perhaps the best known way of performing computations in strong QCD involves the lattice construct. However, this is not well suited to these particular problems. Firstly, the up and down quarks, having current masses of a few MeV, do not fit on a lattice of the size of several fermis. Moreover, as a result of the small u, d quark masses, the pion is light — the Goldstone boson of chiral symmetry breaking. Consequently, the emission of pions is the commonest of all hadronic processes. However, the creation of $q\bar{q}$ pairs involves a complex determinant on the lattice and so this key physics is often deliberately suppressed in the *quenched* approximation. Consequently QCD in the continuum is the natural way to study such problems [3] and progress in this direction using the Schwinger-Dyson equations is what I will briefly outline.
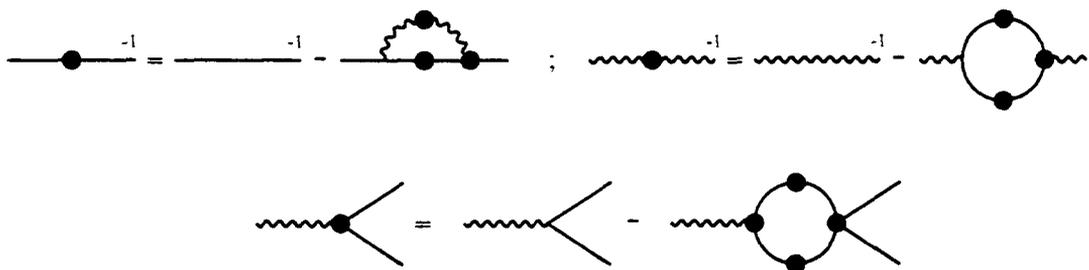

Figure 2: *Schwinger-Dyson equations for fermion, boson and vertex in QED. The dots mean the Green's functions are fully dressed.*

To explain the technique and illustrate recent theoretical progress, I will begin by describing a slightly different strong physics problem : dynamical mass generation. For pedagogical simplicity I will consider QED, but we shall shortly see this problem is closely related to chiral symmetry breaking in QCD. We study the field equations of the theory, the Schwinger-Dyson equations (SDEs). We begin with the 2-point functions, the fermion and boson propagators, Fig. 2. We can imagine solving these coupled equations for the full (dressed) fermion and boson propagators — provided we have an ansatz for the fermion-boson vertex. However, this 3-point function satisfies an SDE too relating it to 2.3 and 4-point functions and the 4-point function satisfies its own SDE ... and so on *ad infinitum*. Consequently, the theory is defined by an infinite set of nested integral equations, which of course cannot be solved without some truncation. The best understood truncation scheme is perturbation theory, in which each Green's function

is expanded in powers of the coupling. This not only gives a systematic procedure, but importantly this respects gauge invariance and multiplicative renormalizability order-by-order. However, we want to know when a fermion mass can be dynamically generated, even when it's bare mass is zero. This is a non-perturbative (strong physics) problem, since it is well-known that if the bare mass is zero, then the mass is zero to all orders in perturbation theory. We therefore need a non-perturbative truncation. It is here that progress has been made by recognising that ensuring gauge covariance and multiplicative renormalisability at each level of truncation, the physics of the neglected higher point Green's functions is effectively pulled into the lower ones. This is readily illustrated in quenched QED. If a crude truncation is made and the vertex treated as bare (the so called *rainbow* approximation), then it is well-known that a non-zero mass is generated if the interaction is strong enough, $\alpha \equiv e^2/4\pi > \alpha_c = \pi/3$ in the Landau gauge, as first found by Miransky [4]. This means a massless electron would, in the field of a highly charged nucleus with effective coupling $Z\alpha > \pi/3$, no longer move at the speed of light (in hypothetically quenched QED). The mass and critical coupling should be gauge independent. However, explicit calculation shows these to be strongly gauge dependent. This is a consequence of the unphysical truncation of the fermion SDE. If instead, one ensures that the vertex, the fermion-boson interaction, respects gauge covariance and multiplicative renormalizability of the fermion propagator, a gauge independent mass and critical coupling occur [5]. This marks progress in the study of the SDEs and ensures basic physics is retained in the truncation.
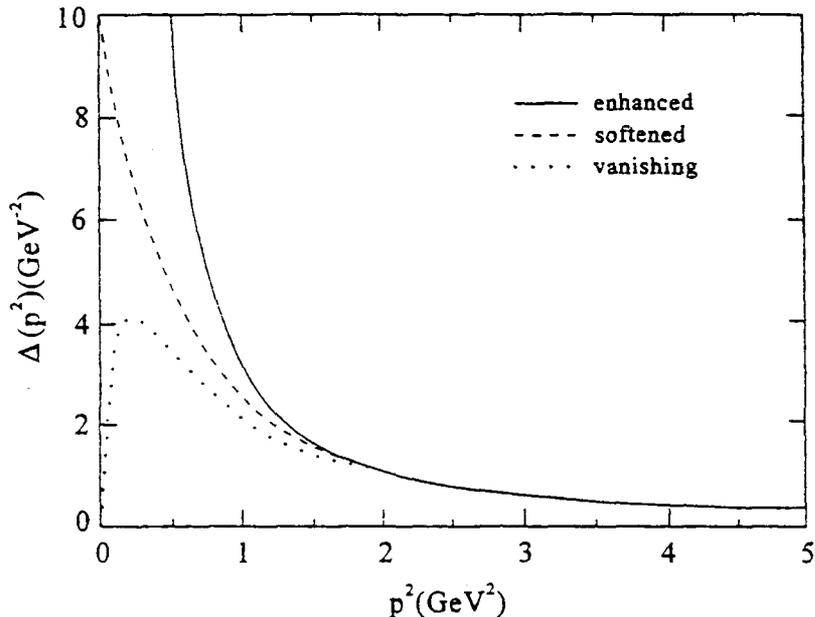

Figure 3: *Representation of the alternative claims for the momentum dependence of the Landau gauge gluon propagator.*

Within this framework, we turn to QCD and its first building block, the gluon prop-

agator. In covariant gauges, a full treatment is not yet possible : the quartic gluon coupling is suppressed and the ghosts treated perturbatively, for example. The pure glue theory is first considered and three distinct behaviours have been claimed for the momentum dependence of the (gauge variant) gluon propagator $\Delta(p^2)$. While each agrees, thanks to asymptotic freedom, with renormalization-group improved perturbation theory at large momenta, in the infrared they are distinct : (i) is enhanced [6,7], (ii) is softer than the bare propagator [8] and (iii) is vanishing when $p^2 \to 0$ [9] (Fig. 3). What has been understood in the last few years is that only the enhanced form is a solution of the truncated SDE. The softened behaviour is not possible with the correct sign for the loop corrections [10] and the infrared vanishing propagator occurs if the vertex has massless coloured singularities and the Green's functions are complex in the spacelike region, where they should be real. Moreover, the infrared enhancement has a scale directly proportional to Λ_{QCD}, i.e. $\Delta(p^2) \sim \Lambda_{QCD}^2/p^4$ [7], so that the stronger the interaction the smaller the size of hadrons. Such a gluon propagator generates a Wilson area law [11] and does not have a Lehmann representation required of an asymptotic state [3]. Consequently, the gluon is confined by this strong self-interaction. While lattice results claim to support a gluon mass and not this enhanced behaviour, they do not yet reliably probe momenta low enough to differentiate.

Now what effect does the infrared enhanced behaviour have on the quark propagator? Just as in strong QED, the interaction is sufficient to generate a non-zero dynamical mass : the virtual gluon cloud gives weight to a quark, even if it's bare mass is zero. A non-zero $\langle q\bar{q}\rangle$ condensate is created and the quark propagator is found to have no timelike pole : again signalling confinement [3]. A gluon with infrared behaviour other than enhanced fails to produce this confinement property [12]. While complex singularities do arise in the momentum plane, these may be merely due to imperfections in the truncation or the numerical procedures and so suggest that the quark propagator is an entire function. This would mean that it had an essential singularity that would prohibit the Wick rotation from Minkowski to Euclidean space used in many calculations here and in lattice work. This is an issue requiring more study, but such an essential singularity may be the price to pay for having no free coloured states.

In principle, a complete coupling of the quark and gluon equations is really needed and steps have been made in this direction (at least in the case of unbroken chiral symmetry [13]). This would allow the determination of the u and d quark propagators in terms of Λ_{QCD} and the current masses, m_R, in some renormalization scheme. However, pending such detailed computations, a model gluon propagator has been used with a phenomenological scale, κ, marking the divide between the strong coupling and perturbative regimes [14]. One can then use the output quark propagator to build the Bethe-Salpeter (or bound state) amplitudes for meson states by a suitable modelling of the 4-quark kernel (something in principle determined by the SDE for the 4-point function). To date approximating this by dressed one gluon exchange is all that has been used. However, as shown long ago by Delbourgo and Scadron [15], this is sufficient to ensure that, with a non-zero $\langle q\bar{q}\rangle$-condensate, massless pseudoscalar bound states arise and chiral symmetry is spontaneously broken. This is unique to an infrared

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enhanced gluon. The two parameters, κ and m_R, are fixed by the pion mass and its decay constant [14].

We can then switch on electromagnetic interactions and calculate the pion e.m. formfactor. Again in principle this really requires a computation of the full $\pi\pi\gamma$ interaction. However, in practice, we may model this in the impulse approximation by the graph of Fig. 4, in which we now know all the components, the vertices and propagators. As shown by Craig Roberts [16], this gives a good description of experiment in the spacelike region, Fig. 5, and the behaviour is like $\sim 1/Q^4$ (modulo logarithms) even out to $Q^2 \simeq 35(\mathrm{GeV/c})^2$. This beautifully highlights how non-perturbative, long range, interactions play a role out to large momenta and explains why the perturbative result does not apply at present momenta.
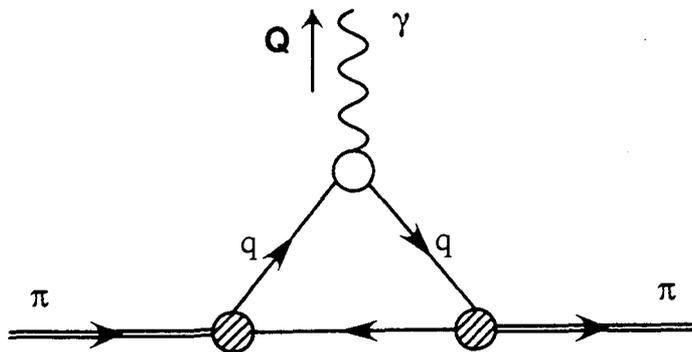

Figure 4: *Pion electromagnetic form-factor in the impulse approximation.*

We can in turn compute the Q^2-dependence of the pion transition form-factor for $\pi\gamma \to \gamma^*$, in a similar approximation. Frank et al. [17] found this to be in reasonable agreement with data both old and new [18]. In contrast, Al Mueller has emphasised [18] that these data present an *uncontroversial* and *unambiguous* test of perturbative QCD. It is clear that data at larger momenta, plus refinements of the calculation presented here, e.g. a more complete scattering kernel, as well as higher orders in perturbation theory, are all needed before such claims and counter-claims can be regarded as substantiated.

Lastly, we can go back to electroproduction of the ρ-meson. This has been modelled by Pichowsky and Lee [2]. The ρ-Bethe-Salpeter amplitude has parameters fixed by $\rho \to e^+ e^-$ and $\rho \to \pi\pi$ decay rates. The agreement with the t and Q^2-dependences is good, but clearly this is crucially affected by the assumed coupling of the Pomeron to dressed quarks. The result is encouraging, but not yet a definitive prediction. The pion electromagnetic and transition form-factors are much more direct. They provide an experimental probe of the infrared behaviour of the gluon that controls confinement and chiral symmetry breaking : properties fundamental to the hadron world. This
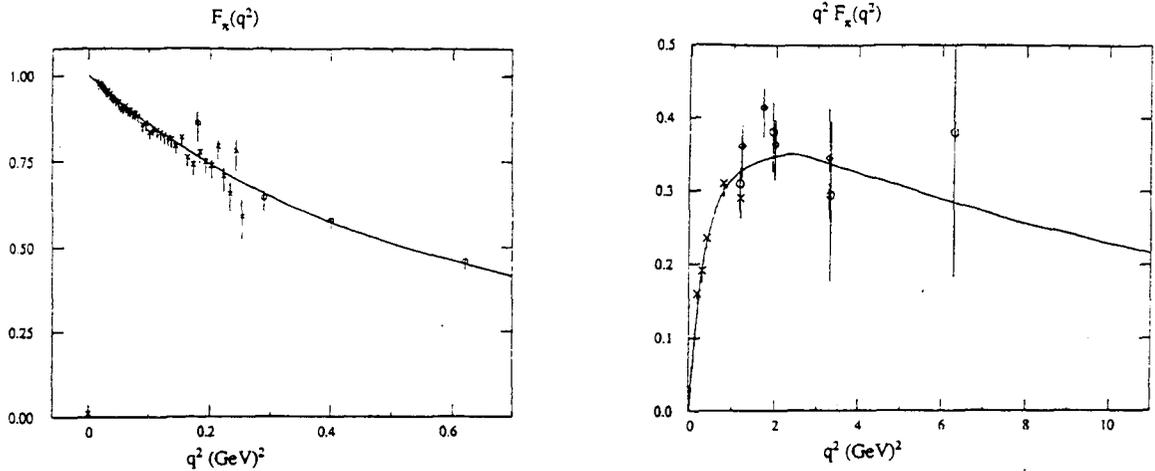
Figure 5: *Pion electromagnetic form-factor prediction compared with experiment [16].*

programme has a long way to go, but I hope you are convinced it has come far.

Acknowledgements

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N.B. This list of references is far from exhaustive.